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A generalization of the Zeeman-Gossard perspector theorem

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Abstract. In this note, we introduce a generalization of the Zeeman-Gossard perspector theorem and a generalization of the Dao's twelve Euler lines point $X(4240)$ in the Kimberling's Encyclopedia of Triangle Centers. We present problems related to the concurrence of four Newton lines in the generalized Zeeman-Gossard perspector configuration.

Keywords. Zeeman-Gossard perspector, Dao's twelve Euler lines point, triangle geometry, Euclidean geometry.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

The following theorem is well known:

Theorem 1 ([1], Zeeman-Gossard perspector theorem). *Let the Euler line of triangle ABC meets the sidelines BC, CA, AB at A_0, B_0, C_0 respectively. The three Euler's lines of triangles AB_0C_0, BC_0A_0 and CA_0B_0 form triangle $A_gB_gC_g$. Then triangles $A_gB_gC_g$ and ABC are homothetic and congruent, and the homothetic center (called Zeeman-Gossard's perspector) lies on the Euler line.*

You can see details about the Zeeman-Gossard perspector in [2], [3]. We present the following problem:

Problem 1 ([4],[5], A generalization of the Zeeman-Gossard perspector theorem). *Let ABC be a triangle. Let H and O be two points, and let the line HO meets BC, CA, AB at A_0, B_0, C_0 respectively. Let A_H and A_O be two points such that $C_0A_H \parallel BH$, $B_0A_H \parallel CH$ and $C_0A_O \parallel BO, B_0A_O \parallel CO$. Define B_H, B_O, C_H, C_O cyclically. Then the triangle formed by the lines A_HA_O, B_HB_O, C_HC_O and triangle ABC are homothetic and congruent, and the homothetic center lies on the line OH.*

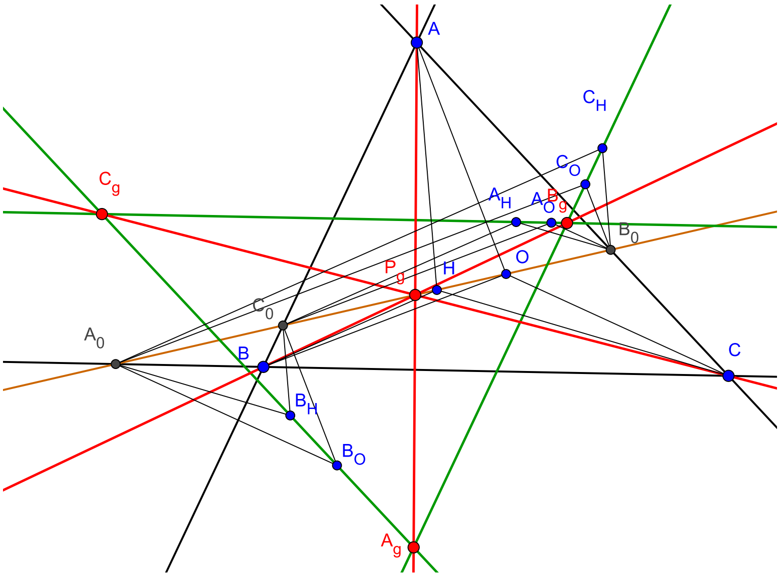

FIGURE 1. A generalization of Zeeman-Gossard perspector theorem

Special Cases.

- If OH is any line parallel to the Euler line, this problem is the Zeeman-Gossard theorem.
- If OH is any line through the centroid of triangle ABC, this problem is the Yiu's generalization of the Gossard perspector theorem [2].

Simple statement of problem 1:

Let ABC be a triangle, let a line L meets BC, CA, AB at A_0, B_0, C_0 respectively. Let P be a point on the line L. Let P_A be a point such that $B_0P_A \parallel CP$ and $C_0P_A \parallel BP$. Define P_B and P_C cyclically. Show that the three lines through P_A, P_B, P_C and parallel to BC, CA, AB respectively, form a triangle congruent and homothetic with ABC. The homothetic center lies on the line L.

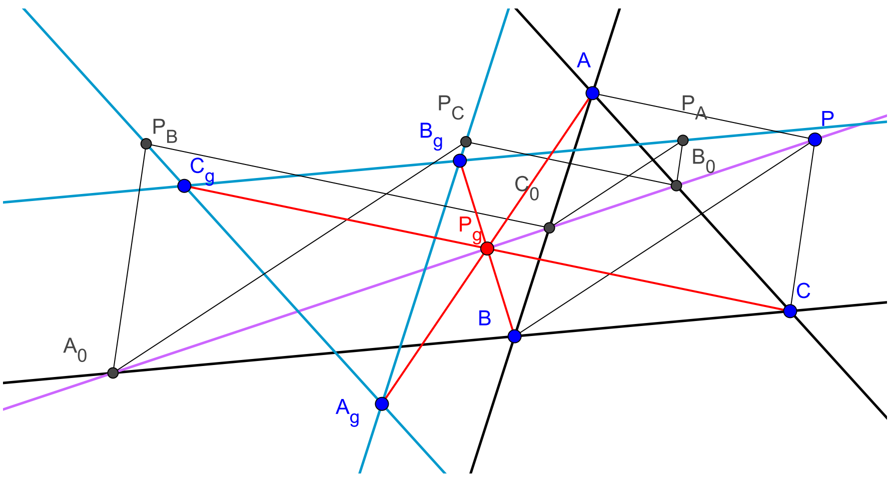

FIGURE 2. Simple statement of problem 1

Problem 2 ([4])**.** *We use the notation of Problem 1. Then the Newton lines of the four quadrilaterals formed by the four lines AB, AC, A_HA_O, HO, the four lines*

BC, BA, B_HB_O, HO, the four lines CA, CB, C_HC_O, HO and the four lines AB, BC, CA, HO, pass through the homothetic center of triangles ABC and $A_gB_gC_g$.

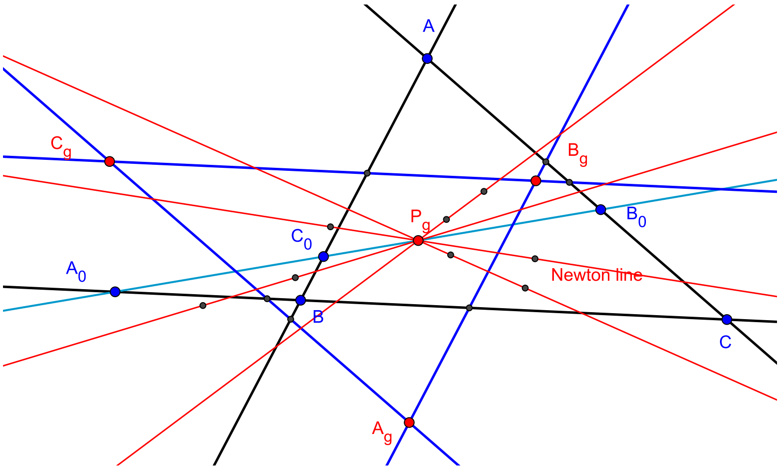

FIGURE 3. Problem 2: Concurrence of four Newton lines

See [6], [7] for description of point X(4240).

Problem 3 ([8], A generalization of point X(4240))**.** *Let ABC be a triangle. Let H and O be two points, let the line HO meets BC, CA, AB at A_0, B_0, C_0 respectively. Let A'_H, A'_O be two points such that $C_0A'_H \parallel CH$, $B_0A'_H \parallel BH$ and $C_0A'_O \parallel CO$, $B_0A_O \parallel BO$. Define B'_H, B'_O, C'_H, C'_O cyclically. Then the four lines $A'_HA'_O$, $B'_HB'_O$, $C'_HC'_O$ and OH are concurrent. If OH is the Euler line, then the point of concurrence is point $X(4240)$.*

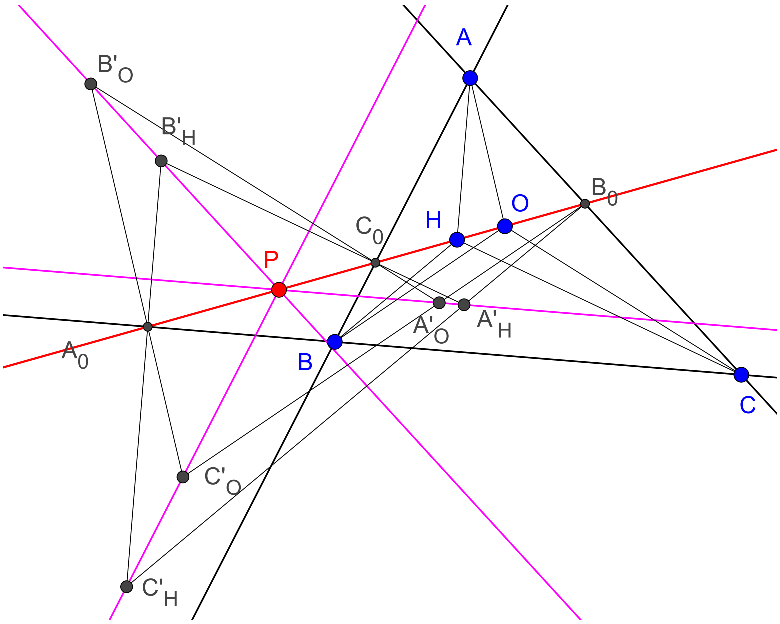

FIGURE 4. Problem 3: A generalization of point $X(4240)$.

Simple statement of problem 3:

Let ABC be a triangle, let a line L meets BC, CA, AB at A_0, B_0, C_0 respectively. Let P be a point on the line L. The three lines through A_0, B_0, C_0 and parallel to AP, BP, CP form triangle $P_AP_BP_C$ respectively. Show that the three lines through P_A, P_B, P_C and parallel to BC, CA, AB respectively are concurrent. The point of concurrence lies on line L.

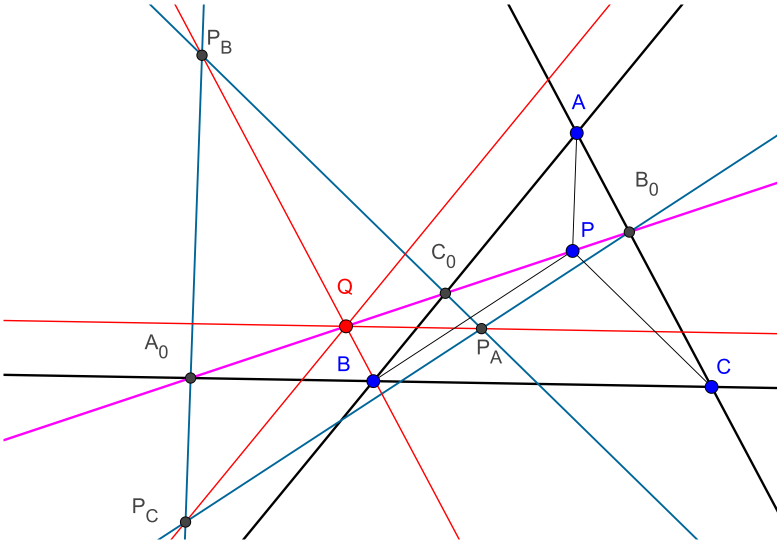

FIGURE 5. Simple statement of problem 3.

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