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Computer Discovered Mathematics: The Mittenpunkt

SAVA GROZDEV^a AND DEKO DEKOV^{b2}

^a University of Finance, Business and Entrepreneurship,
Gusla Street 1, 1618 Sofia, Bulgaria
e-mail: sava.grozdev@gmail.com

^bZahari Knjazheski 81, 6000 Stara Zagora, Bulgaria
e-mail: ddekov@ddekov.eu
web: <http://www.ddekov.eu/>

Abstract. We present results about the Mittenpunkt which are discovered by the computer program “Discoverer” and possibly are not available in the literature. These results extend the article “X(9) Mittenpunkt” in the Kimberling’s Encyclopedia of Triangle Centers (ETC). Many of proofs of theorems in this paper are not presented here. We recommend to the reader to consider these theorems as problems and to find the proofs.

Keywords. Mittenpunkt, triangle geometry, remarkable point, computer discovered mathematics, Euclidean geometry, “Discoverer”.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. INTRODUCTION

The computer program “Discoverer”, created by the authors, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science. See [1].

The Mittenpunkt is the Complement of the Gergonne Point. It is point X(9) in the Kimberling’s Encyclopedia of Triangle Centers (ETC) [2]. The article “X(9) = Mittenpunkt” in [2] contains a number of theorems about the Mittenpunkt. In this paper we extend the results of this article by adding new theorems discovered by the “Discoverer”.

Many of the proofs of theorem are not presented here. We recommend the reader to find the proofs.

The reader may find the definitions in [3].

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²Corresponding author

2. THEOREMS

The following theorem is available in [2]:

Theorem 2.1 ([2], Mittenpunkt). *The Mittenpunkt is the Symmedian Point of the Excentral Triangle.*

We say that "the Symmedian Point of the Excentral Triangle" is a *role of the Mittenpunkt*. Additional roles of the Mittenpunkt, not available in [2] are as follows:

Theorem 2.2. *The Mittenpunkt is the*

- (1) *Centroid of the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Incenter.*
- (2) *Orthocenter of the Triangle of the Centroids of the Anticevian Corner Triangles of the Incenter.*
- (3) *Orthocenter of the First Brocard Triangle of the Hexyl Triangle.*
- (4) *Symmedian Point of the Intouch Triangle of the Medial Triangle.*
- (5) *Symmedian Point of the Half-Cevian Triangle of the Nagel Point.*
- (6) *Symmedian Point of the Inner Grebe Triangle of the Excentral Triangle.*
- (7) *Symmedian Point of the Outer Grebe Triangle of the Excentral Triangle.*
- (8) *Symmedian Point of the Fourth Brocard Triangle of the Excentral Triangle.*
- (9) *Gergonne Point of the Medial Triangle.*
- (10) *Gergonne Point of the Half-Median Triangle of the Antimedial Triangle.*
- (11) *Gergonne Point of the Half-Anticevian Triangle of the Centroid.*
- (12) *Gergonne Point of the Tangential Triangle of the Excentral Triangle.*
- (13) *Gergonne Point of the Triangle of the Centroids of the Triangulation Triangles of the Mittenpunkt.*
- (14) *Retrocenter of the Circum-Anticevian Triangle of the Incenter.*
- (15) *Retrocenter of the Medial Triangle of the Excentral Triangle.*
- (16) *Steiner Point of the First Brocard Triangle of the Excentral Triangle.*
- (17) *Center of the Brocard Circle of the Antimedial Triangle of the Hexyl Triangle.*
- (18) *Second Rigby Point of the Lucas Central Triangle of the Excentral Triangle.*

The following theorem is available in [2]:

Theorem 2.3 ([2], Mittenpunkt). *The Mittenpunkt is the Perspector of Triangle ABC and the Medial Triangle of the Extouch Triangle.*

The above theorem is attributed by Kimberling to Randy Hutson (9/23/2011). In fact, this theorem is not discovered by a man. It is discovered by the prototype of the "Discoverer" and published in 2006 in the first edition of the Computer-Generated Encyclopedia of Euclidean Geometry. See the page http://www.ddekov.eu/e1/htm/04_Kimberling.htm.

Additional theorems about the Mittenpunkt, not available in [2] are as follows:

Theorem 2.4. *The Mittenpunkt is the*

- (1) *Perspector of the Excentral Triangle and the Medial Triangle.*
- (2) *Perspector of the Excentral Triangle and the Medial Triangle.*
- (3) *Perspector of the Medial Triangle and the Half-Cevian Triangle of the Nagel Point.*

- (4) *Perspector of the Half-Bisector Triangle and the Anticevian Triangle of the Bevan Point.*
- (5) *Perspector and Homothetic Center of the Excentral Triangle and the Half-Cevian Triangle of the Nagel Point.*
- (6) *Perspector of the Excentral Triangle and the Half-Anticevian Triangle of the Centroid.*
- (7) *Perspector and Homothetic Center of the Half-Median Triangle and the Euler Triangle of the Gergonne Point.*
- (8) *Perspector of the Medial Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter.*
- (9) *Perspector of the Excentral Triangle and the Triangle of the Orthocenters of the Triangulation Triangles of the Incenter.*
- (10) *Perspector of Triangle ABC and the Triangle of the Centroids of the Cevian Corner Triangles of the Nagel Point.*
- (11) *Perspector and Homothetic Center of the Half-Median Triangle and the Triangle of the Gergonne Points of the Cevian Corner Triangles of the Centroid.*
- (12) *Perspector and Homothetic Center of the Triangle ABC and the Triangle of the Gergonne Points of the Anticevian Corner Triangles of the Centroid.*
- (13) *Perspector of the Medial Triangle and the Triangle of the Centroids of the Anticevian Corner Triangles of the Incenter.*
- (14) *Perspector of the Excentral Triangle and the Triangle of the Centroids of the Anticevian Corner Triangles of the Incenter.*
- (15) *Perspector of Triangle ABC and the Triangle of the Centroids of the Pedal Corner Triangles of the Bevan Point.*
- (16) *Perspector and Homothetic Center of the Half-Median Triangle and the Triangle of the Circumcenters of the Pedal Corner Triangles of the Gergonne Point.*
- (17) *Perspector and Homothetic Center of the Triangle ABC and the Triangle of the Gergonne Points of the Antipedal Corner Triangles of the Orthocenter.*
- (18) *Perspector of the Medial Triangle and the Triangle of the Centroids of the Antipedal Corner Triangles of the Incenter.*
- (19) *Perspector of the Excentral Triangle and the Triangle of the Centroids of the Antipedal Corner Triangles of the Incenter.*

Theorem 2.5. *The Mittenpunkt is Product of the*

- (1) *Incenter and the Nagel Point.*
- (2) *Schiffler Point and the Spieker Center.*
- (3) *Gergonne Point and the Perspector of the Extouch Triangle and the Intangents Triangle.*
- (4) *Internal Center of Similitude of the Incircle and the Circumcircle and the Isotomic Conjugate of the Incenter.*
- (5) *Retrocenter and the Perspector and Homothetic Center of the Intangents Triangle and the Orthic Triangle.*

Theorem 2.6. *The Mittenpunkt is Quotient of the*

- (1) *Incenter and the Gergonne Point.*
- (2) *Second Power Point and the External Center of Similitude of the Incircle and the Circumcircle.*

- (3) *Internal Center of Similitude of the Incircle and the Circumcircle and the Incenter.*
- (4) *Nagel Point and the Isotomic Conjugate of the Incenter.*
- (5) *Schiffler Point and the Isotomic Conjugate of the Spieker Center.*
- (6) *Grinberg Point and the Perspector of the Half-Bisector Triangle and the Intouch Triangle.*
- (7) *Schroder Point and the Inverse of the Gergonne Point in the Incircle.*
- (8) *Centroid of the Extouch Triangle and the Spieker Center.*
- (9) *Perspector of the Extouch Triangle and the Intangents Triangle and the Nagel Point.*
- (10) *Perspector and Homothetic Center of the Intangents Triangle and the Orthic Triangle and the Orthocenter.*

Theorem 2.7. *The Mittenpunkt is the Prasolov Product of the Isotomic Conjugate of the Mittenpunkt and the Mittenpunkt of the Medial Triangle.*

Theorem 2.8. *The Mittenpunkt is the Kosnita Product of the Gergonne Point of the Antimedial Triangle and the Centroid (Homothetic Center).*

Theorem 2.9. *The Mittenpunkt is the*

- (1) *Cevian Corner Product of the Centroid and the Mittenpunkt (Homothetic Center).*
- (2) *Cevian Corner Product of the Nagel Point and the Centroid.*

Theorem 2.10. *The Mittenpunkt is the Anticevian Corner Product of the Centroid and the Gergonne Point (Homothetic Center).*

Theorem 2.11. *The Mittenpunkt is the Pedal Corner Product of the*

- (1) *Circumcenter and the Mittenpunkt (Homothetic Center).*
- (2) *Mittenpunkt and the Circumcenter (Homothetic Center).*
- (3) *Bevan Point and the Centroid.*
- (4) *Isogonal Conjugate of the Mittenpunkt and the Orthocenter.*
- (5) *Perspector of Triangle ABC and the Hexyl Triangle and the Symmedian Point.*

Theorem 2.12. *The Mittenpunkt is the Antipedal Corner Product of the*

- (1) *Orthocenter and the Gergonne Point (Homothetic Center).*
- (2) *Gergonne Point and the Orthocenter (Homothetic Center).*

Theorem 2.13. *The Mittenpunkt is the*

- (1) *Internal Center of Similitude of the Excentral Circle and the Spieker Circle.*
- (2) *Internal Center of Similitude of the Nine-Point Circle and the Tangential Circle of the Excentral Triangle.*
- (3) *External Center of Similitude of the Excentral Circle and the Half-Moses Circle of the Excentral Triangle.*

Theorem 2.14. *The Mittenpunkt is the Center of the following Circles:*

- (1) *Cosine Circle of the Excentral Triangle.*
- (2) *Cosine Circle of the Half-Cevian Triangle of the Nagel Point.*
- (3) *Antimedial Circle of the Triangle of the Centroids of the Anticevian Corner Triangles of the Incenter.*

- (4) *Cosine Circle of the Inner Grebe Triangle of the Excentral Triangle.*
- (5) *Cosine Circle of the Outer Grebe Triangle of the Excentral Triangle.*
- (6) *Cosine Circle of the Fourth Brocard Triangle of the Excentral Triangle.*
- (7) *Brocard Circle of the Antimedial Triangle of the Hexyl Triangle.*
- (8) *Lemoine Circle of the Antimedial Triangle of the Hexyl Triangle.*

Theorem 2.15. *The Mittenpunkt lies on the following circles:*

- (1) *Brocard Circle of the Excentral Triangle.*
- (2) *Brocard Circle of the Half-Cevian Triangle of the Nagel Point.*
- (3) *Orthocentroidal Circle of the Triangle of the Centroids of the Anticevian Corner Triangles of the Incenter.*
- (4) *Orthocentroidal Circle of the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Incenter.*
- (5) *Parry Circle of the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Incenter.*
- (6) *Brocard Circle of the Inner Grebe Triangle of the Excentral Triangle.*
- (7) *Brocard Circle of the Outer Grebe Triangle of the Excentral Triangle.*
- (8) *Brocard Circle of the Fourth Brocard Triangle of the Excentral Triangle.*
- (9) *Image of the Orthocentroidal Circle under the Homothety with Center the Gergonne Point and Ratio $3/2$.*
- (10) *Image of the Parry Circle under the Homothety with Center the Gergonne Point and Ratio $3/2$.*

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