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Proofs of computer discovered theorems about Yiu Transform

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Abstract. In 2015 Paul Yiu has published a transform which we call the Yiu transform. We study the Yiu Transform of several triangle centers. We prove seven theorems discovered by the program of artificial intelligence "Discoverer". This paper is a shortened version of the talk presented by the author in April 2016 at the EUROMATH 2016, the European Student Conference in Mathematics at Thessaloniki, Greece.

Keywords. Yiu transform, triangle geometry, remarkable point, computer discovered mathematics, Discoverer, Euclidian geometry.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

From early times people have been trying to make their life easier by inventing different helpers – we call them robots. In many areas of activity these robots replace people and successfully compete with us. Presently robots perform many of the physical work. They build engines, construct other mechanisms, clean our house. But it was not enough and scientists decided to pull intellectual work on robots, because robot's working capacity is higher than people's. Robots calculate, think and process information very quickly.

Is it possible to invent a robot, for instance researcher or mathematician, which can find new and extraordinary prospects to sort out academic task and find new theorems? Finally, world's academics have a potential to answer this question. Mathematicians from Bulgaria, Grozdev and Dekov, created the computer program "Discoverer". This is the first computer program able easily to discover new ways, methods, decisions, and approaches in science. Person using a computer should check it, as there may be a failure in the operation of the system or a mistake in a loop. And we decided to check the extent of approval's accurateness manually, by using Wolfram alpha and Geogebra. Evidence takes very important part in mathematical discoveries, which people frequently can not prove.

For example, Fermat's Last Theorem scientists were not able to give a proof for 300 years. We proved theorems, which were produced by artificial intelligence in human math language.

Our scientific work will help people better to understand these theorems.

In 2015 Paul Yiu has published a transform which we call the *Yiu transform*. In this paper, by using the "Discoverer", we investigate theorems about the Yiu transform. We expect that the majority of these theorems are new, discovered by a computer.

The reader may find more information about the "Discoverer" e.g. in [2], [3], [4].

This paper is a shortened version of the talk presented in April 2016 by the author at the EUROMATH 2016, the European Student Conference in Mathematics in Thessaloniki, Greece [10].

2. Yiu transform

In the plane of a triangle ABC, consider a line $\mathscr{L}$ containing the Incenter I and a point P. The line $\mathscr{L}$ intersects the sidelines BC, CA, AB at X, Y, Z respectively. Construct the reflection X' of X in the bisector of angle A, and similarly the reflection Y' of Y in the bisector of angle B, and Z' of Z in angle C. The three reflections X', Y', Z' lie on a line denoted by $\mathscr{L}'$. The line $\mathscr{L}'$ is tangent to the incircle of triangle ABC [12]. Denote by Q the tangency point of the line $\mathscr{L}'$ and the incircle.

Hence, in the plane of triangle ABC to the point P correspond point a Q, the tangency point of the line $\mathscr{L}'$ and the incircle. We call this correspondence the *Yiu transform*.

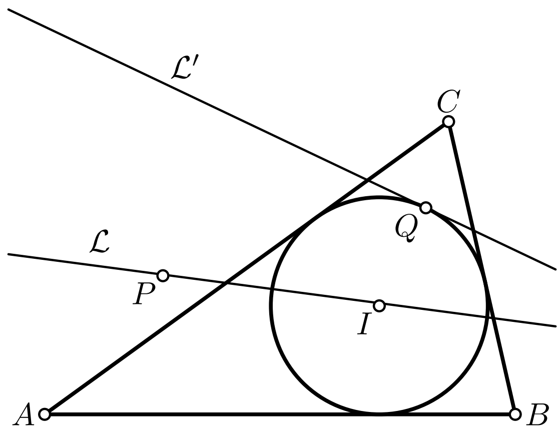

Figure 1.

Figure 1 illustrates the Yiu transform. In Figure 1, I is the incenter, the circle is the incircle, P is an arbitrary point, $\mathscr{L}$ is the line through points I and P, the line $\mathscr{L}'$ contain reflections X', Y', Z' (these reflections are not drawn in the figure). Then the line $\mathscr{L}'$ is tangent to the incircle, and the point of tangency is denoted by Q.

We will work with barycentric coordinates with reference to triangle ABC. See [5] [6] [9] [11] [1] [8] for basic terminology and results.

We use the labeling of triangle centers and the barycentric coordinates of points in accordance with the Kimberling's Encyclopedia of Triangle Centers [7].

Let $P = (u : v : w)$. We denote by $YT(P)$ the Yiu transform of the point P. It is known that the barycentric coordinates of $YT(P) = (\tilde{u} : \tilde{v} : \tilde{w})$ are as follows [12]:

$$\tilde{u} = a^2\frac{(cv - bw)^2}{-a + b + c}, \tilde{v} = b^2\frac{(aw - cu)^2}{-b + a + c}, \tilde{w} = c^2\frac{(bu - av)^2}{-c + a + b}. \tag{1}$$

Here $a = BC$, $b = AC$, $c = AB$.

We are going to prove seven theorems obtained by the computer program "Discoverer". These theorems are as follows:

Theorem 2.1. *The Yiu Transform of the $X(6)$ Symmedian Point is the $X(1358)$.*

Theorem 2.2. *The Yiu Transform of the $X(7)$ Gergonne Point is the $X(3022)$.*

Theorem 2.3. *The Yiu Transform of the $X(8)$ Nagel Point is the $X(1357)$.*

Theorem 2.4. *The Yiu Transform of the $X(9)$ Mittenpunkt is the $X(1358)$.*

Theorem 2.5. *The Yiu Transform of the $X(10)$ Spieker Center is the $X(1357)$.*

Theorem 2.6. *The Yiu Transform of the $X(11)$ Feuerbach Point is the $X(3025)$.*

Theorem 2.7. *The Yiu Transform of the $X(12)$ Feuerbach Perspector is the $X(3025)$.*

3. Proof of the theorems

3.1. **Proof of Theorem 2.1.** In the Kimberling's Encyclopedia of Triangle Centers [7] the symmedian point appears as the sixth point, $X(6)$.

The barycentric coordinates of the Symmedian point $X(6) = (u_6 : v_6 : w_6)$ are as follows:

$$u_6 = a^2,\ v_6 = b^2,\ w_6 = c^2. \tag{2}$$

Let $YT(X(6)) = (\tilde{u}_6, \tilde{v}_6, \tilde{w}_6)$ be the Yiu transform of $X(6)$.

Substituting (2) in (1) we get

$$\tilde{u}_6 = \frac{a^2(cv_6 - bw_6)^2}{-a + b + c} = \frac{a^2(cb^2 - bc^2)^2}{-a + b + c} = \frac{a^2b^2c^2(b - c)^2}{-a + b + c},$$

$$\tilde{v}_6 = \frac{b^2(aw_6 - cu_6)^2}{-b + a + c} = \frac{b^2(ac^2 - ca^2)^2}{-b + a + c} = \frac{a^2b^2c^2(c - a)^2}{-b + c + a},$$

$$\tilde{w}_6 = \frac{c^2(bu_6 - av_6)^2}{-c + a + b} = \frac{c^2(ba^2 - ab^2)^2}{-c + a + b} = \frac{a^2b^2c^2(a - b)^2}{-c + a + b}.$$

Hence,

$$YT(X(6)) = \left(\frac{a^2b^2c^2(b - c)^2}{-a + b + c} : \frac{a^2b^2c^2(c - a)^2}{a - b + c} : \frac{a^2b^2c^2(a - b)^2}{-c + a + b}\right) =$$

$$= \left(\frac{(b - c)^2}{-a + b + c} : \frac{(c - a)^2}{a - b + c} : \frac{(a - b)^2}{-c + a + b}\right). \tag{3}$$

Point $YT(X(6))$ coincides with $X(1358)$. The Theorem 2.1 is proved.

3.2. **Proof of Theorem 2.2.** The barycentric coordinates of the Gergonne point are as follows:

$$X(7) = \left(\frac{1}{b+c-a} : \frac{1}{c+a-b} : \frac{1}{a+b-c}\right). \tag{4}$$

We will find $YT(X(7)) = (\tilde{u}_7, \tilde{v}_7, \tilde{w}_7)$. By using (1), we obtain:

$$\tilde{u}_7 = \frac{a^2(cv_7 - bw_7)^2}{-a+b+c} = \frac{a^2(\frac{c}{c+a-b} - \frac{b}{a+b-c})^2}{-a+b+c} =$$

$$a^2(b-c)^2(b+c-a)^3,$$

$$\tilde{v}_7 = \frac{b^2(aw_7 - cu_7)^2}{-b+a+c} = b^2(c-a)^2(a-b+c)^3,$$

$$\tilde{w}_7 = \frac{c^2(bu_7 - av_7)^2}{-c+a+b} = c^2(a-b)^2(a+b-c)^3.$$

To conclude the proof, it remains to note that the 6th Stevanovic point X(3022) has the same barycentric coordinates. The theorem 2.2 is proved.

3.3. **Proof of Theorem 2.3.** The barycentric coordinates of the Nagel point: $X(8) = (u_8 : v_8 : w_8)$ are as follows:

$$u_8 = b+c-a, v_8 = c+a-b, w_8 = a+b-c. \tag{5}$$

Let $YT(X(8)) = (\tilde{u}_8, \tilde{v}_8, \tilde{w}_8)$ be the Yiu transform of X(8). Substituting (5) in (1) we obtain:

$$\tilde{u}_8 = \frac{a^2(cv_8 - bw_8)^2}{-a+b+c} = \frac{a^2(c^2+ac-ab-b^2)^2}{-a+b+c} = \frac{a^2(b-c)^2(a+b+c)^2}{-a+b+c},$$

$$\tilde{v}_8 = \frac{b^2(aw_8 - cu_8)^2}{-b+a+c} = \frac{b^2(a^2+ab-bc-c^2)^2}{-b+a+c} = \frac{b^2(c-a)^2(a+b+c)^2}{-b+a+c},$$

$$\tilde{w}_8 = \frac{c^2(bu_8 - av_8)^2}{-c+a+b} = \frac{c^2(c^2-ac-ab+b^2)^2}{-c+a+b} = \frac{c^2(a-b)^2(a+b+c)^2}{-c+a+b}.$$

Therefore,

$$YT(X(8)) = \left(\frac{a^2(b-c)^2(a+b+c)^2}{-a+b+c} : \frac{b^2(a-c)^2(a+b+c)^2}{-b+a+c} : \right.$$

$$\left. \frac{c^2(a-b)^2(a+b+c)^2}{-c+a+b}\right)$$

$$= \left(\frac{a^2(b-c)^2}{-a+b+c}, \frac{b^2(c-a)^2}{-b+c+a}, \frac{c^2(a-b)^2}{-c+a+b}\right)$$

Point X(1357) coincides with $YT(X(8))$. This completes the proof of theorem 2.3.

3.4. **Proof of Theorem 2.4.** The barycentric coordinates of the Mittenpunkt $X(9) = (u_9, v_9, w_9)$ are as follows:

(6) $$u_9 = a(b + c - a), v_9 = b(c + a - b), w_9 = c(a + b - c).$$

Let $YT(X(9)) = (\tilde{u}_9, \tilde{v}_9, \tilde{w}_9)$ be the Yiu transform of X(9). By 1, 6 we obtain:

$$\tilde{u}_9 = \frac{a^2(cv_9 - bw_9)^2}{-a + b + c} = \frac{a^2b^2c^2 4(b - c)^2}{-a + b + c},$$

$$\tilde{v}_9 = \frac{b^2(aw_9 - cu_9)^2}{-b + a + c} = \frac{a^2b^2c^2 4(c - a)^2}{-b + c + a},$$

$$\tilde{w}_9 = \frac{c^2(bu_9 - av_9)^2}{-c + a + b} = \frac{c^2a^2b^2 4(a - b)^2}{-c + a + b}.$$

Hence,

(7) $$YT(X(9)) = \left(\frac{(b - c)^2}{-a + b + c} : \frac{(c - a)^2}{-b + c + a} : \frac{(a - b)^2}{-c + a + b}\right).$$

Point X(1358) coincides with $YT(X(9))$.The theorem 2.4 is proved.

3.5. **Proof of Theorem 2.5.** The barycentric coordinates of the Speiker center $X10 = (u_{10}, v_{10}, w_{10})$ are as follows:

(8) $$u_{10} = b + c, v_{10} = c + a, w_{10} = a + b.$$

Let $YT(X(10)) = (\tilde{u}_{10}, \tilde{v}_{10}, \tilde{w}_{10})$ be the Yiu transform of X(10). Now if we recall (1) and 8, we get:

$$\tilde{u}_{10} = \frac{a^2(cv_{10} - bw_{10})^2}{-a + b + c} = \frac{a^2(c^2 - b^2 + a(c - b))}{-a + b + c} = \frac{a^2(b - c)^2(c + b + a)^2}{-a + b + c},$$

$$\tilde{v}_{10} = \frac{b^2(aw_{10} - cu_{10})^2}{-b + a + c} = \frac{b^2(a^2 - cc^2 + b(a - c)^2)}{-b + a + c} = \frac{b^2(c - a)^2(a + b + c)^2}{a + b - c},$$

$$\tilde{w}_{10} = \frac{c^2(bu_{10} - av_{10})^2}{-c + a + b} = \frac{c^2(a - b)^2(a + b + c)^2}{-c + a + b}.$$

Hence,

$$YT(X(10)) = \left(\frac{a^2(b - c)^2}{-a + b + c} : \frac{b^2(a - c)^2}{-b + a + c} : \frac{c^2(a - b)^2}{-c + a + b}\right).$$

Point $X(1357)$ coincides with $YT(X(10))$. The theorem 2.5 is proved.

3.6. **Proof of Theorem 2.6.** The barycentric coordinates of the Feuerbach center $X(11)$ are as follows:

(9) $$X(11) = \left((b + c - a)(b - c)^2 : (c + a - b)(c - a)^2 : (a + b - c)(a - b)^2\right).$$

Let us find $YT(X(11)) = (\tilde{u}_{11}, \tilde{v}_{11}, \tilde{w}_{11})$. Substituting (9) in (1), we obtain:

$$\tilde{u}_{11} = \frac{a^2(cv_{11} - bw_{11})^2}{-a + b + c} = \frac{a^2((a - b)^2(a + b - c)c - b(a - c)^2(a - b + c))^2}{-a + b + c} =$$
$$a^2(b + c - a)(b - c)^2(b^2 + c^2 - a^2 - bc)^2,$$

$$\tilde{v}_{11} = \frac{b^2(aw_{11} - cu_{11})^2}{-b + a + c} = \frac{b^2((a - c)^2(a - b + c)a - c(b - c)^2(b - a + c))^2}{-b + c + a} =$$
$$b^2(c + a - b)(c - a)^2(c^2 + a^2 - b^2 - ca)^2,$$

$$\tilde{w}_{11} = \frac{c^2(bu_{11} - av_{11})^2}{-c + b + a} = \frac{c^2((a - b)^2(a - b + c)a - b(b - c)^2(b - a + c))^2}{-c + a + b} =$$

$$c^2(a+b-c)(a-b)^2(a^2+b^2-c^2-ab)^2.$$

In conclusion, it remains to note that the 9th Stevanovic point X(3025) has the same barycentric coordinates. The theorem 2.6 is proved.

3.7. **Proof of Theorem 2.7.** The X(12) Feuerbach Perspector is the perspector of triangle ABC and its Feurbach triangle.

$$X(12) = \left(\frac{(b+c)^2}{b+c-a} : \frac{(c+a)^2}{c+a-b} : \frac{(a+b)^2}{a+b-c}\right). \tag{10}$$

Let us find $YT(X(12)) = (\tilde{u}_{12}, \tilde{v}_{12}, \tilde{w}_{12})$. If we recall (1), we obtain:

$$\tilde{u}_{12} = \frac{a^2(cv_{12}-bw_{12})^2}{-a+b+c} = a^2(b+c-a)(b-c)^2(b^2+c^2-a^2-bc)^2(a+b+c)^2,$$

$$\tilde{v}_{12} = \frac{b^2(aw_{12}-cu_{12})^2}{-b+a+c} = b^2(c+a-b)(c-a)^2(c^2+a^2-b^2-ca)^2(a+b+c)^2,$$

$$\tilde{w}_{12} = \frac{c^2(bu_{12}av_{12})^2}{-c+b+a} = c^2(a+b-c)(a-b)^2(a^2+b^2-c^2-ab)^2(a+b+c)^2.$$

We divide $\tilde{u}_{12}, \tilde{v}_{12}$ and $\tilde{w}_{12}$ by $(a+b+c)^2$ and we see that point $X(12)$ has the same barycentric coordinates as point $X(3025)$. Theorem 2.7 is proved.

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