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Computer Discovered Mathematics: Euler Anticevian Triangles

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Abstract. By using the computer program “Discoverer”, we study the Euler Anticevian Triangles.

Keywords. Euler Anticevian triangle, triangle geometry, remarkable point, computer-discovered mathematics, Euclidean geometry, Discoverer.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. INTRODUCTION

The computer program “Discoverer”, created by the authors, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science. See e.g. [3].

In this paper, by using the “Discoverer”, we define and study the Euler Anticevian triangles. Given triangle ABC with side lengths $a = BC$, $b = CA$ and $c = AB$. Denote by $PaPbPc$ the anticevian triangle of an arbitrary point P and denote by Ea the midpoint of P and Pa , by Eb the midpoint of P and Pb and by Ec the midpoint of P and Pc . Then $EaEbEc$ is the Euler Anticevian Triangle of P (see Figure 1). If we replace the “Anticevian Triangle” by the “Cevian Triangle”, we obtain the definition of the Euler Cevian Triangle of P (see [4]).

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In this paper we study the special cases $P = \text{Incenter}$ and $P = \text{Centroid}$. We invite the reader to investigate other special cases.

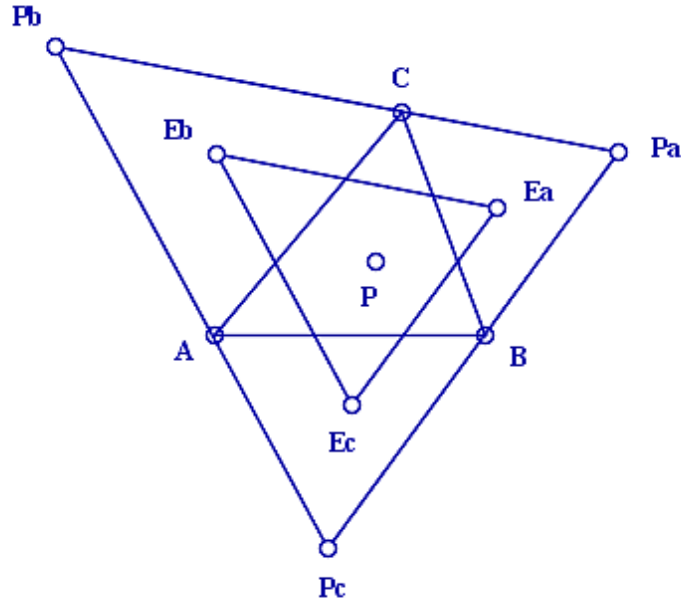


FIGURE 1. Euler Anticevian Triangle $EaEbEc$ of P .

We refer the reader for barycentric coordinates to [5], [3], [6], [7], [10], [11], [13], [1], [2], [9], [12]. We use the 10200 points version of 2016 of the Kimberling's ETC [8].

2. EULER ANTICEVIAN TRIANGLE OF P

2.1. Barycentric Coordinates, area, side lengths.

Theorem 2.1. *The barycentric coordinates of the Euler Anticevian Triangle $EaEbEc$ of $P(u, v, w)$ are as follows:*

$$(1) \quad \begin{aligned} Ea &= (-u^2, v(v+w), w(v+w)), \\ Eb &= (u(u+w), -v^2, w(u+w)), \\ Ec &= (u(u+v), v(u+v), -w^2). \end{aligned}$$

Theorem 2.2. *The area of the Euler Anticevian Triangle $EaEbEc$ of $P(u, v, w)$ is as follows:*

$$(2) \quad \text{area}(EaEbEc) = \frac{uvw\Delta}{(u+v-w)(v+w-u)(w+u-v)},$$

where Δ is the area of triangle ABC .

Theorem 2.3. *The side lengths of the Euler Anticevian Triangle $EaEbEc$ of $P(u, v, w)$ are as follows:*

$$(3) \quad |EbEc| = \frac{u\sqrt{a^2vw - b^2vw - c^2vw + b^2w^2 + c^2v^2}}{(w + u - v)(u + v - w)},$$

$$(4) \quad |EbEc| = \frac{v\sqrt{b^2wu - c^2uw - a^2wu + c^2u^2 + a^2w^2}}{(u + v - w)(v + w - u)},$$

$$(5) \quad |EbEc| = \frac{w\sqrt{c^2uv - a^2vu - b^2uv + a^2v^2 + b^2u^2}}{(v + w - u)(w + u - v)}.$$

2.2. Perspectors.

Theorem 2.4. *The Euler Anticevian Triangle of Point P is perspective with the (different perspectors)*

- (1) *Medial Triangle.*
- (2) *Triangle of Reflections of the Vertices of Triangle ABC in the Complement of Point P .*
- (3) *Euler Triangle of the Anticomplement of the Anticomplement of Point P .*
- (4) *Triangle of the Centroids of the Triangulation Triangles of Point P .*
- (5) *Stevanovic Triangle of the Orthocenters of the Triangulation Triangles of Point P .*
- (6) *Inner Monge Triangle of the Circumcircles of the Pedal Corner Triangles of Point P .*
- (7) *Inner Monge Triangle of the Second Brocard Circles of the Pedal Corner Triangles of Point P .*

Figure 2 illustrates part (1) of Theorem 2.3.

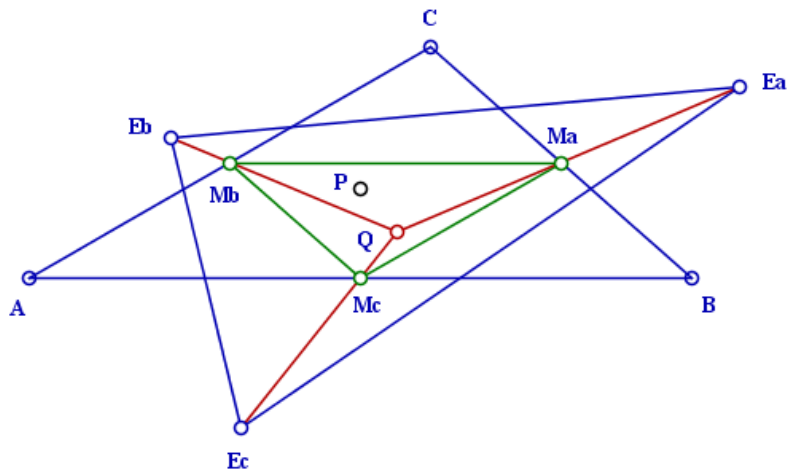


FIGURE 2. Triangle $EaEbEc$ is the Euler Anticevian Triangle of Point P and triangle $MaMbMc$ is the Medial Triangle of triangle ABC . Then lines $EaMa$, $EbMb$ and $EcMc$ concur in a point, denoted by Q . Hence Point Q is perspector of triangles $EaEbEc$ and $MaMbMc$.

Figure 3 illustrates part (2) of Theorem 2.3.

Figure 4 illustrates part (3) of Theorem 2.3.

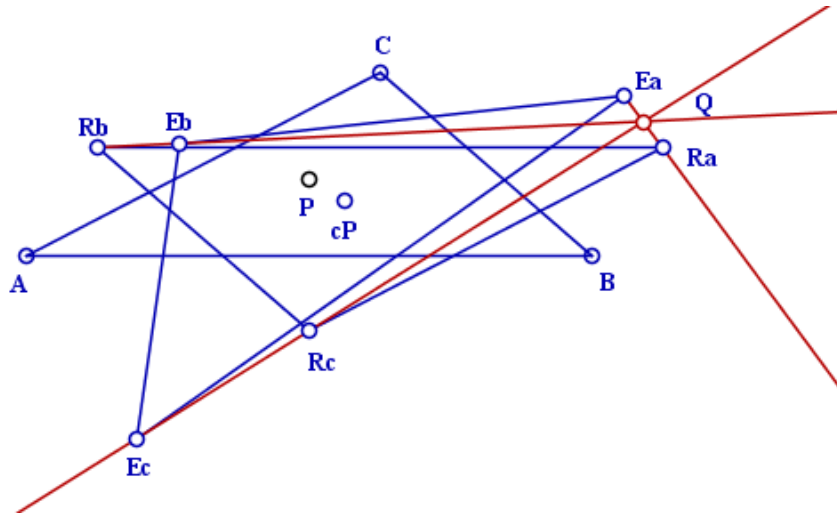


FIGURE 3. Point cP is the complement of point P , triangle $EaEbEc$ is the Euler Anticevian triangle of Point P , and points Ra , Rb and Rc are reflection points of vertices A , B , C , respectively, about point cP . Then lines $EaRa$, $EbRb$ and $EcRc$ concur in a point, denoted here as Q .

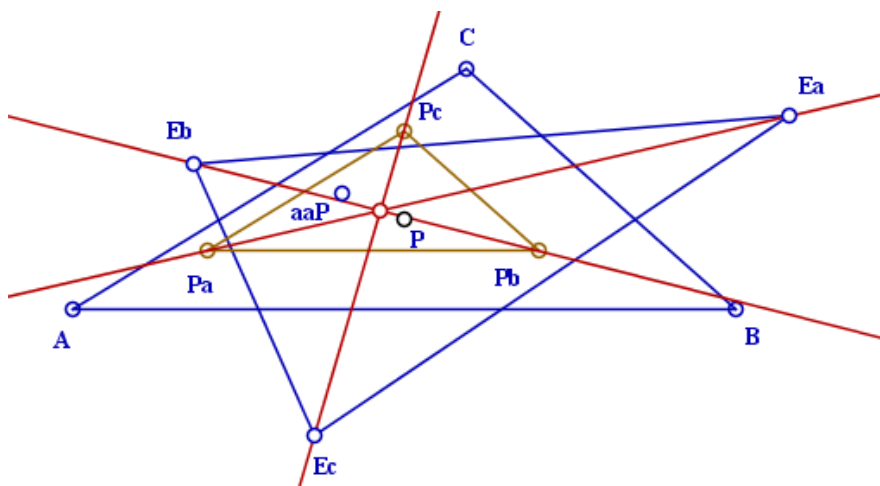


FIGURE 4. Point aaP is the anticomplement of anticomplement of point P , triangle $EaEbEc$ is the Euler Anticevian triangle of Point P , and $PaPbPc$ is the Euler triangle of point aaP . Then lines $EaPa$, $EbPb$ and $EcPc$ concur in a point.

3. EULER ANTICEVIAN TRIANGLE OF INCENTER

Note that the Euler Anticevian Triangle of the Incenter coincides with the Circum-Incentral triangle of triangle ABC .

3.1. Barycentric Coordinates, area, side lengths.

Theorem 3.1. *The barycentric coordinates of the Euler Anticevian Triangle $EaEbEc$ of Incenter $I = (a, b, c)$ are as follows:*

$$(6) \quad \begin{aligned} Ea &= (-a^2, b(b+c), c(b+c)), \\ Eb &= (a(a+c), -b^2, c(a+c)), \\ Ec &= (a(a+b), b(a+b), -c^2). \end{aligned}$$

Theorem 3.2. *The area of the Euler Anticevian Triangle $EaEbEc$ of Incenter $I = (a, b, c)$ is as follows:*

$$(7) \quad \text{area}(EaEbEc) = \frac{abc\Delta}{(a+b-c)(b+c-a)(c+a-b)},$$

where Δ is the area of triangle ABC .

Theorem 3.3. *The side lengths of the Euler Anticevian Triangle $EaEbEc$ of $P(u, v, w)$ are as follows:*

$$\begin{aligned} |EbEc| &= \frac{a\sqrt{bc}}{\sqrt{(a+b-c)(a+c-b)}}, \\ |EbEc| &= \frac{b\sqrt{ca}}{\sqrt{(b+c-a)(b+a-c)}}, \\ |EbEc| &= \frac{c\sqrt{ab}}{\sqrt{(c+a-b)(c+b-a)}}. \end{aligned}$$

3.2. Compare with Other Triangles.

Theorem 3.4. *The Euler Anticevian Triangle of Incenter is the:*

- (1) *Circum-Incentral Triangle.*
- (2) *Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (3) *Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*

Theorem 3.5. *The Euler Anticevian Triangle of Incenter is congruent (but not the same) to the:*

- (1) *Circum-Anticevian Triangle of the Incenter.*
- (2) *Circumcevian Triangle of the Inverse of the Incenter in the Circumcircle.*

Theorem 3.6. *The Euler Anticevian Triangle of Incenter is similar to the:*

- (1) *Intouch Triangle.*
- (2) *Excentral Triangle.*
- (3) *Hexyl Triangle.*
- (4) *Fuhrmann Triangle.*
- (5) *Yff Central Triangle.*
- (6) *Pedal Triangle of the Inverse of the Incenter in the Circumcircle.*
- (7) *Triangle of Reflections of the Incenter in the Sidelines of Triangle ABC .*
- (8) *Triangle of Reflections of the Inverse of the Incenter in the Circumcircle in the Sidelines of Triangle ABC .*

Theorem 3.7. *The Euler Anticevian Triangle of Incenter has the same area as the:*

- (1) *Circum-Anticevian Triangle of the Incenter.*
- (2) *Circumcevian Triangle of the Inverse of the Incenter in the Circumcircle.*
- (3) *Triangle of the Circumcenters of the Triangulation Triangles of the Perspector of Triangle ABC and the Hexyl Triangle.*
- (4) *Anticevian Euler Triangle of the Mittenpunkt.*

3.3. Homothetic Triangles. Below we select a few triangles homothetic with the Euler Anticevian Triangle of Incenter.

Theorem 3.8. *The Homothetic Center of the Euler Anticevian Triangle of Incenter and the*

- (1) *Intouch Triangle is the X(56) External Center of Similitude of Circumcircle and Incircle.*
- (2) *Excentral Triangle is the X(1) Incenter.*
- (3) *Hexyl Triangle is the X(3576).*
- (4) *Yff Central Triangle is the X(7587).*
- (5) *Pedal Triangle of the Incenter is the X(56) External Center of Similitude of Circumcircle and Incircle.*
- (6) *Antipedal Triangle of the Incenter is the X(1) Incenter.*
- (7) *Circum-Anticevian Triangle of the Incenter is the X(3) Circumcenter.*

3.4. Perspective Triangles with a Set of Different Pectorsors. We have discovered a set of 138 different pectorsors (but not homothetic centers) of Euler Anticevian Triangle of Incenter. Of these 25 are Kimberling points. The Kimberling pectorsors are listed below. The Supplementary material contains the rest of the set of the different pectorsors which are new points.

Theorem 3.9. (1) *X(1) Incenter is the Pectorsor of the Euler Anticevian Triangle of Incenter and Triangle ABC.*

- (2) *X(3) Circumcenter is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Medial Triangle.*
- (3) *X(21) Schiffler Point is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Cevian Triangle of the Isotomic Conjugate of the Spieker Center.*
- (4) *X(28) Quotient of the Clawson Point and the Spieker Center is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Triangle of the Prasolov Points of the Triangulation Triangles of the Incenter.*
- (5) *X(56) External Center of Similitude of Circumcircle and Incircle is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Circum-Anticevian Triangle of the Isogonal Conjugate of the Mittenpunkt.*
- (6) *X(58) Isogonal Conjugate of the Spieker Center is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Circum-Anticevian Triangle of the Isogonal Conjugate of the Spieker Center.*
- (7) *X(100) Anticompliment of the Feuerbach Point is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Triangle of Reflections of the Vertices of Triangle ABC in the Spieker Center.*
- (8) *X(101) is the Pectorsor of the Euler Anticevian Triangle of Incenter and the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Incenter.*

- (9) $X(108)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Prasolov Points of the Anticevian Corner Triangles of the Incenter.
- (10) $X(109)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Centers of the Sine-Triple-Angle Circles of the Anticevian Corner Triangles of the Incenter.
- (11) $X(110)$ Euler Reflection Point is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Kosnita Points of the Anticevian Corner Triangles of the Incenter.
- (12) $X(901)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Parry Reflection Points of the Anticevian Corner Triangles of the Incenter.
- (13) $X(927)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Third Brocard Points of the Anticevian Corner Triangles of the Incenter.
- (14) $X(934)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Retrocenters of the Anticevian Corner Triangles of the Incenter.
- (15) $X(999)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Stevanovic Triangle of the de Longchamps Points of the Triangulation Triangles of the Incenter.
- (16) $X(1001)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Monge Triangle of the Incircles of the Cevian Corner Triangles of the Centroid.
- (17) $X(1385)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of Reflections of the Vertices of the Anticevian Triangle of the Grinberg Point in the Circumcenter.
- (18) $X(1394)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Antipedal Triangle of the Bevan Point.
- (19) $X(2360)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Cevian Triangle of the Schiffler Point.
- (20) $X(3576)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of Reflections of the Vertices of the Cevian Triangle of the Mittenpunkt in the Circumcenter.
- (21) $X(3659)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Incenters of the Anticevian Corner Triangles of the Incenter.
- (22) $X(6135)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Outer Vecten Points of the Anticevian Corner Triangles of the Incenter.
- (23) $X(6136)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Inner Vecten Points of the Anticevian Corner Triangles of the Incenter.
- (24) $X(6265)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Triangle of the Centers of the Fuhrmann Circles of the Anticevian Corner Triangles of the Centroid.
- (25) $X(7290)$ is the Perspector of the Euler Anticevian Triangle of Incenter and the Stevanovic Triangle of the Exeter Points of the Triangulation Triangles of the Incenter.

3.5. Kimberling's Points. We have investigated 195 remarkable points of the Euler Anticevian Triangle of the Incenter. Of these 34 points are available in the Kimberling's ETC [8] and the rest of 161 points are new points, not available in [8].

Table 1 gives a few of the centers of the Euler Anticevian triangle of the Incenter in terms of the centers of the reference triangle for Kimberling centers $X(n)$. The reader may find additional notable points of the Euler Anticevian triangle of the Incenter in the Supplementary Material.

$X(n)$	Notable Points of the Euler Anticevian Triangle of Incenter	$X(n)$
$X(2)$	Centroid	$X(3576)$
$X(3)$	Circumcenter	$X(3)$ Circumcenter
$X(4)$	Orthocenter	$X(1)$ Incenter
$X(5)$	Nine-Point Center	$X(1385)$
$X(6)$	Symmedian Point	$X(1001)$
$X(20)$	de Longchamps Point	$X(40)$ Bevan Point
$X(22)$	Exeter Point	$X(3428)$
$X(24)$	Perspector of Kosnita Triangle and Orthic Triangle	$X(56)$ External Similitude Center of Circumcircle and Incircle
$X(25)$	Product of Orthocenter and Symmedian Point	$X(999)$
$X(51)$	Centroid of Orthic Triangle	$X(551)$
$X(52)$	Orthocenter of Orthic Triangle	$X(946)$
$X(53)$	Symmedian Point of Orthic Triangle	$X(1386)$

TABLE 1.

We can use the properties of the corresponding notable points given in Kimberling's ETC [8], but also we can use the Discoverer in order to discover new properties. For example, we find new properties of point $X(3576)$, the Centroid of the Euler Anticevian Triangle of Incenter. A few of the new properties are given below.

Theorem 3.10. *The Centroid of the Euler Anticevian Triangle of Incenter is the*

- (1) *Center of the Orthocentroidal Circle of the Excentral Triangle.*
- (2) *Centroid of the Circum-Incentral Triangle.*
- (3) *Centroid of the Circumcevian Triangle of the Inverse of the Incenter in the Circumcircle.*
- (4) *Orthocenter of the Triangle of the Centroids of the Triangulation Triangles of the Incenter.*
- (5) *Centroid of the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (6) *Bevan Point of the Triangle of the Centroids of the Triangulation Triangles of the de Longchamps Point.*

- (7) *Nagel Point of the Triangle of the Centroids of the Triangulation Triangles of the Bevan Point.*
- (8) *Center of the Fuhrmann Circle of the Triangle of the Centroids of the Triangulation Triangles of the Circumcenter.*
- (9) *Centroid of the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
- (10) *Centroid of the Triangle of the Circumcenters of the Antipedal Corner Triangles of the Incenter.*
- (11) *Centroid of the Euler Triangle of the Excentral Triangle.*
- (12) *Circumcenter of the Fourth Brocard Triangle of the Excentral Triangle.*
- (13) *Homothetic Center of the Hexyl Triangle and the Circum-Incentral Triangle.*
- (14) *Perspector and Homothetic Center of the Hexyl Triangle and the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (15) *Homothetic Center of the Hexyl Triangle and the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
- (16) *Homothetic Center of the Hexyl Triangle and the Triangle of the Circumcenters of the Antipedal Corner Triangles of the Incenter.*
- (17) *Harmonic Conjugate of the Incenter with respect to the Bevan-Schroder Point and the Internal Center of Similitude of the Incircle and the Circumcircle (the Incenter is between the Bevan-Schroder Point and Internal Center of Similitude of the Incircle and the Circumcircle).*
- (18) *Harmonic Conjugate of the Bevan Point with respect to the Circumcenter and the Incenter (the Bevan Point is outside the segment from the Circumcenter and Incenter).*
- (19) *Harmonic Conjugate of the Pohoata Point with respect to the External Center of Similitude of the Incircle and the Circumcircle and the Incenter (the Pohoata Point is between the External Center of Similitude of the Incircle and the Circumcircle and Incenter).*
- (20) *Harmonic Conjugate of the Isogonal Conjugate of the Mittenpunkt with respect to the Incenter and the Inverse of the Incenter in the Circumcircle (the Isogonal Conjugate of the Mittenpunkt is between the Incenter and Inverse of the Incenter in the Circumcircle).*
- (21) *Point Dividing Internally the Directed Segment from $X(1)$ Incenter to the $X(3)$ Circumcenter in Ratio of 2:1.*
- (22) *Point Dividing Internally the Directed Segment from $X(40)$ Bevan Point to the $X(1)$ Incenter in Ratio of 2:1.*

3.6. New Notable Points. In our investigation we have found 161 new notable points of the Euler Anticevian Triangle of Incenter which are not available in Kimberling's ETC [8]. By using the Discoverer, we can investigate the properties of these new notable points. Below we give a few theorems about 3 new points which are not available in the Kimberling [8]. Clearly, by using the Discoverer we can find a number of additional new theorems about new points.

Theorem 3.11. *The Incenter of the Euler Anticevian Triangle of the Incenter is the*

- (1) *Spieker Center of the Hexyl Triangle.*
- (2) *Incenter of the Circum-Incentral Triangle.*

- (3) *Bevan Point of the Circum-Anticevian Triangle of the Incenter. Incenter of the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (4) *Incenter of the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
- (5) *Incenter of the Triangle of the Circumcenters of the Antipedal Corner Triangles of the Incenter.*
- (6) *Incenter of the Euler Triangle of the Excentral Triangle.*
- (7) *Nagel Point of the Half-Median Triangle of the Hexyl Triangle.*
- (8) *Bevan Point of the Medial Triangle of the Excentral Triangle.*
- (9) *Center of the Taylor Circle of the Excentral Triangle of the Hexyl Triangle.*
- (10) *Midpoint of the Incenter and the Incenter of the Excentral Triangle.*
- (11) *Reflection of the Spieker Center of the Excentral Triangle in the Circumcenter.*
- (12) *Incenter wrt the Medial Triangle of the Hexyl Triangle.*

Theorem 3.12. *The Incenter of the Euler Anticevian Triangle of Incenter lies on the following circles:*

- (1) *Nine-Point Circle of the Fuhrmann Triangle of the Excentral Triangle.*
- (2) *Symmedian Circle of the Fuhrmann Triangle of the Hexyl Triangle.*

Theorem 3.13. *The Incenter of the Euler Anticevian Triangle of Incenter lies on the following lines:*

- (1) *Line through the External Center of Similitude of the Incircle and the Circumcircle and the First Mid-Arc Point.*
- (2) *Line through the Incenter and the Incenter of the Excentral Triangle.*
- (3) *Line through the Circumcenter and the Spieker Center of the Excentral Triangle.*

Theorem 3.14. *The Gergonne Point of the Euler Anticevian Triangle of Incenter is the:*

- (1) *Mittelpunkt of the Hexyl Triangle.*
- (2) *Gergonne Point of the Circum-Incentral Triangle.*
- (3) *Gergonne Point of the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (4) *Gergonne Point of the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
- (5) *Gergonne Point of the Euler Triangle of the Excentral Triangle.*
- (6) *Symmedian Point of the Excentral Triangle of the Hexyl Triangle.*
- (7) *Midpoint of the Incenter and the Gergonne Point of the Excentral Triangle.*
- (8) *Reflection of the Mittelpunkt of the Excentral Triangle in the Circumcenter.*
- (9) *Image of the Gergonne Point of the Excentral Triangle under the Homothety with Center at the Incenter and Ratio $1/2$.*
- (10) *Gergonne Point wrt the Medial Triangle of the Hexyl Triangle.*

Theorem 3.15. *The Nagel Point of the Euler Anticevian Triangle of Incenter is the:*

- (1) *Center of the Fuhrmann Circle of the Excentral Triangle.*
- (2) *Incenter of the Hexyl Triangle.*

- (3) *Nagel Point of the Circum-Incentral Triangle.*
- (4) *Nagel Point of the Triangle of the Circumcenters of the Triangulation Triangles of the Incenter.*
- (5) *Nagel Point of the Triangle of the Circumcenters of the Anticevian Corner Triangles of the Incenter.*
- (6) *Nagel Point of the Triangle of the Circumcenters of the Antipedal Corner Triangles of the Incenter.*
- (7) *Nagel Point of the Euler Triangle of the Excentral Triangle.*
- (8) *Circumcenter of the Fuhrmann Triangle of the Excentral Triangle.*
- (9) *Circumcenter of the Intouch Triangle of the Hexyl Triangle.*
- (10) *Orthocenter of the Excentral Triangle of the Hexyl Triangle.*
- (11) *Spieker Center of the Antimedial Triangle of the Hexyl Triangle.*
- (12) *Incenter of the Intangents Triangle of the Hexyl Triangle.*
- (13) *Circumcenter of the Hexyl Triangle of the Hexyl Triangle.*
- (14) *Orthocenter of the Fuhrmann Triangle of the Hexyl Triangle.*
- (15) *Incenter of the Inner Yff Triangle of the Hexyl Triangle.*
- (16) *Incenter of the Outer Yff Triangle of the Hexyl Triangle.*
- (17) *Bevan Point of the Yff Central Triangle of the Hexyl Triangle.*
- (18) *First Mid-Arc Point of the Tangential Triangle of the Hexyl Triangle.*
- (19) *Inner Vecten Point of the de Villiers Triangle of the Hexyl Triangle.*
- (20) *Midpoint of the Incenter and the Nagel Point of the Excentral Triangle.*
- (21) *Reflection of the Incenter of the Excentral Triangle in the Circumcenter.*
- (22) *Reflection of the Bevan Point in the Spieker Center of the Excentral Triangle.*
- (23) *Image of the Nagel Point of the Excentral Triangle under the Homothety with Center at the Incenter and Ratio $1/2$.*
- (24) *External Center of Similitude of the Excentral Circle and the Inner Johnson-Yff Circle of the Hexyl Triangle.*
- (25) *Internal Center of Similitude of the Excentral Circle and the Outer Johnson-Yff Circle of the Hexyl Triangle.*
- (26) *Nagel Point wrt the Medial Triangle of the Hexyl Triangle.*

3.7. Internal Similitude Centers. We have discovered 840 internal centers of similitude between circles of Triangle ABC and circles of the Euler Anticevian triangle of Incenter. Of these 49 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 3.16. *The Internal Center of Similitude of the*

- (1) *Circumcircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the $X(3576)$.*
- (2) *Circumcircle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the $X(3576)$.*
- (3) *Circumcircle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the $X(3428)$.*
- (4) *Circumcircle of Triangle ABC and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the $X(993)$.*
- (5) *Circumcircle of Triangle ABC and the Second Brocard Circle of the Euler Anticevian Triangle of the Incenter is the $X(3)$.*
- (6) *Circumcircle of Triangle ABC and the Cosine Circle of the Euler Anticevian Triangle of the Incenter is the $X(8225)$.*

- (7) *Incircle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(55).*
- (8) *Incircle of Triangle ABC and the Incircle of the Euler Anticevian Triangle of the Incenter is the X(7588).*
- (9) *Incircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(2646).*
- (10) *Incircle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (11) *Nine-Point Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(2).*
- (12) *Nine-Point Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(1125).*
- (13) *Nine-Point Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(8227).*
- (14) *Nine-Point Circle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(958).*
- (15) *Excentral Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(165).*
- (16) *Excentral Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(7987).*
- (17) *Excentral Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (18) *Antimedial Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(2).*
- (19) *Antimedial Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(3616).*
- (20) *Antimedial Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(946).*
- (21) *Antimedial Circle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(2975).*
- (22) *Tangential Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(24).*
- (23) *Tangential Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9625).*
- (24) *Spieker Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(958).*
- (25) *Spieker Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9623).*
- (26) *Sine-Triple-Angle Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1147).*
- (27) *Sine-Triple-Angle Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9621).*
- (28) *Adams Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (29) *Conway Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (30) *Conway Circle of Triangle ABC and the Taylor Circle of the Euler Anticevian Triangle of the Incenter is the X(3616).*

- (31) *Brocard Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1340).*
- (32) *Second Brocard Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (33) *Second Brocard Circle of Triangle ABC and the Second Brocard Circle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (34) *Orthocentroidal Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1344).*
- (35) *Moses Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(574).*
- (36) *Moses Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9619).*
- (37) *Half-Moses Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(5013).*
- (38) *Half-Moses Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9592).*
- (39) *Inner Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(12).*
- (40) *Inner Johnson-Yff Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9612).*
- (41) *Outer Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(6284).*
- (42) *Outer Johnson-Yff Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(3586).*
- (43) *Lemoine Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1342).*
- (44) *Gallatly Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1690).*
- (45) *Kenmotu Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3385).*
- (46) *Cosine Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(371).*
- (47) *Cosine Circle of Triangle ABC and the Cosine Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (48) *Apollonius Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(573).*
- (49) *Apollonius Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9548).*

We have discovered 812 internal centers of similitude between circles of the Euler Anticevian triangle. Of these 20 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 3.17. *The Internal Center of Similitude of the*

- (1) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (2) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(3576).*

- (3) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(3428).*
- (4) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(993).*
- (5) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Second Brocard Circle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (6) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Cosine Circle of the Euler Anticevian Triangle of the Incenter is the X(8225).*
- (7) *Incircle of the Euler Anticevian Triangle of the Incenter and the Spieker Circle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (8) *Nine-Point Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (9) *Nine-Point Circle of the Euler Anticevian Triangle of the Incenter and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (10) *Nine-Point Circle of the Euler Anticevian Triangle of the Incenter and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(21).*
- (11) *Antimedial Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (12) *Tangential Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3428).*
- (13) *Tangential Circle of the Euler Anticevian Triangle of the Incenter and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (14) *Spieker Circle of the Euler Anticevian Triangle of the Incenter and the Incircle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (15) *Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(993).*
- (16) *Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(21).*
- (17) *Second Brocard Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3).*
- (18) *Lemoine Circle of the Euler Anticevian Triangle of the Incenter and the Gallatly Circle of the Euler Anticevian Triangle of the Incenter is the X(8225).*
- (19) *Gallatly Circle of the Euler Anticevian Triangle of the Incenter and the Lemoine Circle of the Euler Anticevian Triangle of the Incenter is the X(8225).*

- (20) *Cosine Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(8225).*

3.8. External Similitude Centers. We have discovered 829 internal centers of similitude between circles of Triangle ABC and circles of the Euler Anticevian triangle. Of these 46 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 3.18. *The External Center of Similitude of the*

- (1) *Circumcircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (2) *Circumcircle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(40).*
- (3) *Circumcircle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(56).*
- (4) *Circumcircle of Triangle ABC and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(5450).*
- (5) *Circumcircle of Triangle ABC and the Half-Moses Circle of the Euler Anticevian Triangle of the Incenter is the X(1001).*
- (6) *Incircle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(56).*
- (7) *Incircle of Triangle ABC and the Incircle of the Euler Anticevian Triangle of the Incenter is the X(8077).*
- (8) *Incircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(1319).*
- (9) *Incircle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(56).*
- (10) *Nine-Point Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(4).*
- (11) *Nine-Point Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(5587).*
- (12) *Nine-Point Circle of Triangle ABC and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(36).*
- (13) *Excentral Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (14) *Excentral Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (15) *Excentral Circle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(36).*
- (16) *Antimedial Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(20).*
- (17) *Antimedial Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(5731).*
- (18) *Tangential Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(22).*
- (19) *Tangential Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9626).*
- (20) *Spieker Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1376).*

- (21) *Spieker Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(936).*
- (22) *Spieker Circle of Triangle ABC and the Half-Moses Circle of the Euler Anticevian Triangle of the Incenter is the X(8299).*
- (23) *Sine-Triple-Angle Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(184).*
- (24) *Sine-Triple-Angle Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9622).*
- (25) *Adams Circle of Triangle ABC and the Lemoine Circle of the Euler Anticevian Triangle of the Incenter is the X(7677).*
- (26) *Conway Circle of Triangle ABC and the Taylor Circle of the Euler Anticevian Triangle of the Incenter is the X(2).*
- (27) *Brocard Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1341).*
- (28) *Orthocentroidal Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1345).*
- (29) *Moses Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(32).*
- (30) *Moses Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9620).*
- (31) *Moses Circle of Triangle ABC and the Moses Circle of the Euler Anticevian Triangle of the Incenter is the X(1279).*
- (32) *Half-Moses Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(6).*
- (33) *Half-Moses Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9593).*
- (34) *Half-Moses Circle of Triangle ABC and the Half-Moses Circle of the Euler Anticevian Triangle of the Incenter is the X(1279).*
- (35) *Inner Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(7354).*
- (36) *Inner Johnson-Yff Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9613).*
- (37) *Outer Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(11).*
- (38) *Outer Johnson-Yff Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9614).*
- (39) *Lemoine Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1343).*
- (40) *Gallatly Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(1689).*
- (41) *Kenmotu Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(3386).*
- (42) *Kenmotu Circle of Triangle ABC and the Kenmotu Circle of the Euler Anticevian Triangle of the Incenter is the X(238).*
- (43) *Cosine Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(372).*
- (44) *Cosine Circle of Triangle ABC and the Cosine Circle of the Euler Anticevian Triangle of the Incenter is the X(238).*

- (45) *Apollonius Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(386).*
- (46) *Apollonius Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of the Incenter is the X(9549).*

We have discovered 397 internal centers of similitude between circles of the Euler Anticevian triangle. Of these 12 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 3.19. *The External Center of Similitude of the*

- (1) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (2) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(56).*
- (3) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(5450).*
- (4) *Circumcircle of the Euler Anticevian Triangle of the Incenter and the Half-Moses Circle of the Euler Anticevian Triangle of the Incenter is the X(1001).*
- (5) *Incircle of the Euler Anticevian Triangle of the Incenter and the Inner Johnson-Yff Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (6) *Incircle of the Euler Anticevian Triangle of the Incenter and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*
- (7) *Nine-Point Circle of the Euler Anticevian Triangle of the Incenter and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(999).*
- (8) *Nine-Point Circle of the Euler Anticevian Triangle of the Incenter and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter is the X(104).*
- (9) *Antimedial Circle of the Euler Anticevian Triangle of the Incenter and the Circumcircle of the Euler Anticevian Triangle of the Incenter is the X(40).*
- (10) *Antimedial Circle of the Euler Anticevian Triangle of the Incenter and the Nine-Point Circle of the Euler Anticevian Triangle of the Incenter is the X(3576).*
- (11) *Sine-Triple-Angle Circle of the Euler Anticevian Triangle of the Incenter and the Tangential Circle of the Euler Anticevian Triangle of the Incenter is the X(1012).*
- (12) *Inner Johnson-Yff Circle of the Euler Anticevian Triangle of the Incenter and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of the Incenter is the X(1).*

3.9. Inverse Points. We have investigated 342 remarkable points which are inverse images of 12 Kimberling Points wrt circles of the Euler Anticevian Triangle of the Incenter. Of these 9 points are available in the Kimberling's ETC [8] and the rest of 333 points are new points, not available in [8].

Table 2 gives a few of the inverse images wrt circles of the Euler Anticevian triangle of Incenter in terms of the Kimberling points of the reference triangle. The reader may find additional notable points in the Supplementary Material.

In the Table below **T** denotes the Euler Anticevian Triangle of Incenter

Point P	Circle C of T	Inverse of P wrt C
X(1) Incenter	Circumcircle of T	X(36) Inverse of Incenter in the Circumcircle
X(2) Centroid	Circumcircle of T	X(23) Far-Out Point
X(4) Orthocenter	Circumcircle of T	X(186)
X(5) Nine-Point Center	Circumcircle of T	X(2070)
X(6) Symmedian Point	Circumcircle of T	X(187) Schoute Center
X(10) Spieker Center	Circumcircle of T	X(1324)
X(1) Incenter	Nine-Point Circle of T	X(1319) Bevan-Schroder Point
X(3) Circumcenter	Antimedial Circle of T	X(5538)
X(3) Circumcenter	Orthocentroidal Circle of T	X(1385)

TABLE 2.

4. EULER ANTICEVIAN TRIANGLE OF CENTROID

4.1. Barycentric Coordinates, area, side lengths.

Theorem 4.1. *The barycentric coordinates of the Euler Anticevian Triangle $EaEbEc$ of Centroid $G = (1, 1, 1)$ are as follows:*

$$(8) \quad Ea = (-1, 2, 2), \quad Eb = (2, -1, 2), \quad Ec = (2, 2, -1).$$

Theorem 4.2. *The area of the Euler Anticevian Triangle $EaEbEc$ of Centroid $G = (1, 1, 1)$ is as follows:*

$$(9) \quad \text{area}(EaEbEc) = \Delta,$$

where Δ is the area of triangle ABC .

Theorem 4.3. *The side lengths of the Euler Anticevian Triangle $EaEbEc$ of $P(u, v, w)$ are as follows:*

$$(10) \quad |EbEc| = a, \quad |EbEc| = b, \quad |EbEc| = c.$$

4.2. Compare with Other Triangles.

Theorem 4.4. *The Euler Anticevian Triangle of Centroid is the:*

- (1) *Triangle of Reflections of the Vertices of Triangle ABC in the Centroid.*
- (2) *Triangle of the Centroids of the Anticevian Corner Triangles of the Centroid.*
- (3) *Triangle of the Orthocenters of the Antipedal Corner Triangles of the Centroid.*
- (4) *Triangle of the Centroids of the Antipedal Corner Triangles of the Orthocenter.*

We have discovered 533 different triangles congruent to the Euler Anticevian triangle of Centroid. A few of congruent triangles are listed below. The reader may find the complete list in the Supplementary Material.

Theorem 4.5. *The Euler Anticevian Triangle of Centroid is congruent to the:*

- (1) *Triangle ABC.*
- (2) *Johnson Triangle.*
- (3) *Circumcevian Triangle of the Circumcenter.*
- (4) *Triangle of Reflections of the Vertices of Triangle ABC in the Incenter.*

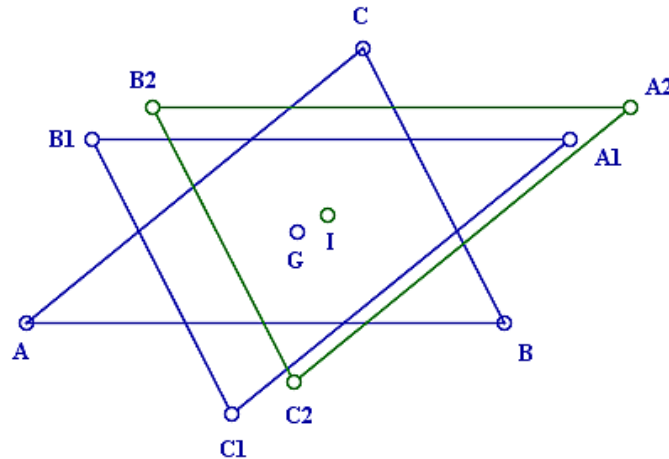


FIGURE 5. Triangle $A_2B_2C_2$ of Reflections of the Vertices of Triangle ABC in the Incenter is congruent to the Euler Anticevian triangle of Centroid $A_1B_1C_1$.

We have discovered 1661 different triangles similar (but different) to the Euler Anticevian triangle of Centroid. A few of similar triangles are listed below. The reader may find the complete list in the Supplementary Material.

Theorem 4.6. *The Euler Anticevian Triangle of Centroid is similar (but different) to the:*

- (1) *Medial Triangle.*
- (2) *Antimedial Triangle.*
- (3) *Euler Triangle.*
- (4) *Inner Grebe Triangle.*
- (5) *Outer Grebe Triangle.*
- (6) *Inner Yff Triangle.*
- (7) *Outer Yff Triangle.*
- (8) *First Brocard Triangle.*
- (9) *Half-Median Triangle.*
- (10) *Triangle of Reflections of the Orthocenter in the Sidelines of the Cevian Triangle of the Orthocenter.*

Theorem 4.7. *The Euler Anticevian Triangle of Centroid has the same area as the:*

- (1) *Triangle ABC.*
- (2) *Johnson Triangle.*

- (3) *Circumcevian Triangle of the Circumcenter.*
- (4) *Circumcevian Triangle of the First Brocard Point.*
- (5) *Circumcevian Triangle of the Second Brocard Point.*
- (6) *Circumcevian Triangle of the First Beltrami Point.*
- (7) *Circumcevian Triangle of the Second Beltrami Point.*
- (8) *Triangle of Reflections of the Vertices of Triangle ABC in the Incenter.*

4.3. Homothetic Triangles. Below we select a few triangles homothetic with the Euler Anticevian Triangle of Centroid.

Theorem 4.8. *The Homothetic Center of the Euler Anticevian Triangle of Centroid and the*

- (1) *Triangle ABC is the X(2) Centroid.*
- (2) *Euler Triangle is the X(3545).*
- (3) *Inner Yff Triangle is the X(3584).*
- (4) *Outer Yff Triangle is the X(3582).*
- (5) *Pedal Triangle of the Circumcenter is the X(2) Centroid.*
- (6) *Antipedal Triangle of the Orthocenter is the X(2) Centroid.*

4.4. Perspective Triangles with a Set of Different Pectors. We have discovered a set of 886 different pectors (but not homothetic centers) of Euler Anticevian Triangle of Centroid. Of these 31 are Kimberling points. The Kimberling pectors are listed below. The Supplementary material contains the rest of the set of the different pectors which are new points.

Theorem 4.9. *The Pector of the Euler Anticevian Triangle of Centroid and the*

- (1) *Fourth Brocard Triangle is the X(2).*
- (2) *Outer Kiepert-Gallatly Triangle is the X(99).*
- (3) *Pedal Triangle of the Centroid is the X(1992).*
- (4) *Antipedal Triangle of the Centroid is the X(376).*
- (5) *Antipedal Triangle of the Outer Fermat Point is the X(5463).*
- (6) *Antipedal Triangle of the Inner Fermat Point is the X(5464).*
- (7) *Pedal Triangle of the Centroid of the Excentral Triangle is the X(6172).*
- (8) *Triangle of Reflections of the Center of the Orthocentroidal Circle in the Sidelines of the Cevian Triangle of the Orthocenter is the X(381).*
- (9) *Triangle of Reflections of the Circumcenter in the Sidelines of the Antipedal Triangle of the Center of the Orthocentroidal Circle is the X(3).*
- (10) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Incenter in the Centroid is the X(3679).*
- (11) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Nine-Point Center in the Centroid is the X(549).*
- (12) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Symmedian Point in the Centroid is the X(599).*
- (13) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Nagel Point in the Centroid is the X(3241).*
- (14) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Mittenpunkt in the Centroid is the X(6173).*
- (15) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Spieker Center in the Centroid is the X(551).*

- (16) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Feuerbach Point in the Centroid is the X(6174).*
- (17) *Triangle of Reflections of the Vertices of the Cevian Triangle of the de Longchamps Point in the Centroid is the X(3543).*
- (18) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Schiffler Point in the Centroid is the X(6175).*
- (19) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Third Power Point in the Centroid is the X(7818).*
- (20) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Grinberg Point in the Centroid is the X(4688).*
- (21) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Brocard Midpoint in the Centroid is the X(9466).*
- (22) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Third Brocard Point in the Centroid is the X(7757).*
- (23) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Tarry Point in the Centroid is the X(6054).*
- (24) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Steiner Point in the Centroid is the X(671).*
- (25) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Euler Reflection Point in the Centroid is the X(9140).*
- (26) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Kiepert Center in the Centroid is the X(2482).*
- (27) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Equal Parallelians Point in the Centroid is the X(4740).*
- (28) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Center of the Parry Circle in the Centroid is the X(9148).*
- (29) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Weill Point in the Centroid is the X(210).*
- (30) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Center of the Fuhrmann Circle in the Centroid is the X(3655).*
- (31) *Triangle of Reflections of the Vertices of the Cevian Triangle of the Conway Point in the Centroid is the X(7924).*

4.5. Kimberling's Points. We have investigated 195 remarkable points of the Euler Anticevian Triangle of the Centroid. Of these 37 points are available in the Kimberling's ETC [8] and the rest of 158 points are new points, not available in [8].

Table 3 gives a few of the centers of the Euler Anticevian triangle of the Centroid in terms of the centers of the reference triangle for Kimberling centers X(n). The reader may find additional notable points of the Euler Anticevian triangle of Centroid in the Supplementary Material.

We can use the properties of the notable points given in Kimberling's ETC [8], but also we can use the Discoverer in order to discover new properties. A few of the new properties are given below.

Theorem 4.10. *The Centroid of the Euler Anticevian Triangle of Centroid is the*

- (1) 1

X(n)	Notable Points of the Euler Anticevian Triangle of Centroid	is the X(n)
X(1)	Incenter	X(3679)
X(2)	Centroid	X(2) Centroid
X(3)	Circumcenter	X(381) Center of the Orthocentroidal Circle
X(4)	Orthocenter	X(376)
X(5)	Nine-Point Center	X(549)
X(6)	Symmedian Point	X(599)
X(7)	Gergonne Point	X(6172)
X(8)	Nagel Point	X(3241)
X(9)	Mittenpunkt	X(6173)
X(10)	Spieker Center	X(551)
X(11)	Feuerbach Point	X(6174)
X(13)	Outer Fermat Point	X(5463)
X(14)	Inner Fermat Point	X(5464)

TABLE 3.

4.6. New Notable Points. By using the "Discoverer" we can find properties of points if the Euler Anticevian Triangle of Centroid which are not Kimberling Points. For example:

Theorem 4.11. *The Feuerbach Perspector of the Euler Anticevian Triangle of Centroid is the*

- (1) *Centroid of the Triangle of the Orthocenters of the Antipedal Corner Triangles of the Feuerbach Perspector.*
- (2) *Midpoint of the Centroid and the Anticomplement of the Feuerbach Perspector.*
- (3) *Reflection of the Feuerbach Perspector in the Centroid.*
- (4) *Reflection of the Centroid in the Complement of the Feuerbach Perspector.*

4.7. Internal Similitude Centers. We have discovered 841 internal centers of similitude between circles of Triangle ABC and circles of the Euler Anticevian triangle of Centroid. Of these 44 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 4.12. *The Internal Center of Similitude of the*

- (1) *Circumcircle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the $X(2)$.*
- (2) *Circumcircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the $X(3524)$.*
- (3) *Circumcircle of Triangle ABC and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the $X(4428)$.*
- (4) *Circumcircle of Triangle ABC and the Orthocentroidal Circle of the Euler Anticevian Triangle of Centroid is the $X(3)$.*
- (5) *Circumcircle of Triangle ABC and the Half-Moses Circle of the Euler Anticevian Triangle of Centroid is the $X(8556)$.*

- (6) *Incircle of Triangle ABC and the Incircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (7) *Incircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(4995).*
- (8) *Incircle of Triangle ABC and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(4654).*
- (9) *Incentral Circle of Triangle ABC and the Incentral Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (10) *Nine-Point Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(3545).*
- (11) *Nine-Point Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (12) *Nine-Point Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (13) *Symmedial Circle of Triangle ABC and the Symmedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (14) *Extouch Circle of Triangle ABC and the Extouch Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (15) *Excentral Circle of Triangle ABC and the Incircle of the Euler Anticevian Triangle of Centroid is the X(3929).*
- (16) *Excentral Circle of Triangle ABC and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (17) *Antimedial Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(3839).*
- (18) *Antimedial Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (19) *Antimedial Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (20) *Tangential Circle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (21) *Spieker Circle of Triangle ABC and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (22) *Stevanovic Circle of Triangle ABC and the Stevanovic Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (23) *Sine-Triple-Angle Circle of Triangle ABC and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (24) *Adams Circle of Triangle ABC and the Adams Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (25) *Conway Circle of Triangle ABC and the Conway Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (26) *Brocard Circle of Triangle ABC and the Brocard Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (27) *Second Brocard Circle of Triangle ABC and the Second Brocard Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (28) *Second Brocard Circle of Triangle ABC and the Orthocentroidal Circle of the Euler Anticevian Triangle of Centroid is the X(3).*
- (29) *Orthocentroidal Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(381).*

- (30) *Orthocentroidal Circle of Triangle ABC and the Second Brocard Circle of the Euler Anticevian Triangle of Centroid is the X(381).*
- (31) *Orthocentroidal Circle of Triangle ABC and the Orthocentroidal Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (32) *Moses Circle of Triangle ABC and the Moses Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (33) *Half-Moses Circle of Triangle ABC and the Half-Moses Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (34) *Parry Circle of Triangle ABC and the Parry Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (35) *Lester Circle of Triangle ABC and the Lester Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (36) *Inner Johnson-Yff Circle of Triangle ABC and the Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (37) *Outer Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(3058).*
- (38) *Outer Johnson-Yff Circle of Triangle ABC and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (39) *Lemoine Circle of Triangle ABC and the Lemoine Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (40) *Gallatly Circle of Triangle ABC and the Gallatly Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (41) *Kenmotu Circle of Triangle ABC and the Kenmotu Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (42) *Cosine Circle of Triangle ABC and the Cosine Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (43) *Apollonius Circle of Triangle ABC and the Apollonius Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (44) *Taylor Circle of Triangle ABC and the Taylor Circle of the Euler Anticevian Triangle of Centroid is the X(2).*

We have discovered 812 internal centers of similitude between circles of the Euler Anticevian triangle of Centroid. Of these 32 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 4.13. *The Internal Center of Similitude of the*

- (1) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (2) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(1699).*
- (3) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (4) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Second Brocard Circle of the Euler Anticevian Triangle of Centroid is the X(381).*
- (5) *Incircle of the Euler Anticevian Triangle of Centroid and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (6) *Incircle of the Euler Anticevian Triangle of Centroid and the Adams Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*

- (7) *Incircle of the Euler Anticevian Triangle of Centroid and the Conway Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (8) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (9) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Sine-Triple-Angle Circle of the Euler Anticevian Triangle of Centroid is the X(9140).*
- (10) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Apollonius Circle of the Euler Anticevian Triangle of Centroid is the X(551).*
- (11) *Excentral Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(1699).*
- (12) *Excentral Circle of the Euler Anticevian Triangle of Centroid and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(551).*
- (13) *Excentral Circle of the Euler Anticevian Triangle of Centroid and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the X(6173).*
- (14) *Antimedial Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (15) *Antimedial Circle of the Euler Anticevian Triangle of Centroid and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(551).*
- (16) *Spieker Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (17) *Spieker Circle of the Euler Anticevian Triangle of Centroid and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(6173).*
- (18) *Sine-Triple-Angle Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(9140).*
- (19) *Adams Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (20) *Adams Circle of the Euler Anticevian Triangle of Centroid and the Conway Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (21) *Conway Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (22) *Conway Circle of the Euler Anticevian Triangle of Centroid and the Adams Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (23) *Second Brocard Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(381).*
- (24) *Moses Circle of the Euler Anticevian Triangle of Centroid and the Half-Moses Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (25) *Moses Circle of the Euler Anticevian Triangle of Centroid and the Gallatly Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (26) *Half-Moses Circle of the Euler Anticevian Triangle of Centroid and the Moses Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (27) *Half-Moses Circle of the Euler Anticevian Triangle of Centroid and the Gallatly Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (28) *Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*

- (29) *Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (30) *Gallatly Circle of the Euler Anticevian Triangle of Centroid and the Moses Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (31) *Gallatly Circle of the Euler Anticevian Triangle of Centroid and the Half-Moses Circle of the Euler Anticevian Triangle of Centroid is the X(9466).*
- (32) *Apollonius Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(551).*

4.8. External Similitude Centers. We have discovered 806 internal centers of similitude between circles of Triangle ABC and circles of the Euler Anticevian triangle of Centroid. Of these 11 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 4.14. *The External Center of Similitude of the*

- (1) *Circumcircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (2) *Circumcircle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (3) *Incircle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(5298).*
- (4) *Incircle of Triangle ABC and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (5) *Nine-Point Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (6) *Nine-Point Circle of Triangle ABC and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(3839).*
- (7) *Nine-Point Circle of Triangle ABC and the Tangential Circle of the Euler Anticevian Triangle of Centroid is the X(5064).*
- (8) *Antimedial Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (9) *Spieker Circle of Triangle ABC and the Incircle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (10) *Half-Moses Circle of Triangle ABC and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(5306).*
- (11) *Inner Johnson-Yff Circle of Triangle ABC and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(5434).*

We have discovered 394 external centers of similitude between circles of the Euler Anticevian triangle of Centroid. Of these 30 are Kimberling points. They are listed in the theorem below. The new points are listed in the Supplementary material.

Theorem 4.15. *The External Center of Similitude of the*

- (1) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (2) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Eccentral Circle of the Euler Anticevian Triangle of Centroid is the X(3679).*

- (3) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(3543).*
- (4) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Moses Circle of the Euler Anticevian Triangle of Centroid is the X(7818).*
- (5) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Half-Moses Circle of the Euler Anticevian Triangle of Centroid is the X(599).*
- (6) *Circumcircle of the Euler Anticevian Triangle of Centroid and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(6174).*
- (7) *Incircle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(6174).*
- (8) *Incircle of the Euler Anticevian Triangle of Centroid and the Spieker Circle of the Euler Anticevian Triangle of Centroid is the X(3241).*
- (9) *Incircle of the Euler Anticevian Triangle of Centroid and the Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (10) *Incircle of the Euler Anticevian Triangle of Centroid and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (11) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (12) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(6174).*
- (13) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Excentral Circle of the Euler Anticevian Triangle of Centroid is the X(165).*
- (14) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Antimedial Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (15) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Tangential Circle of the Euler Anticevian Triangle of Centroid is the X(381).*
- (16) *Nine-Point Circle of the Euler Anticevian Triangle of Centroid and the Moses Circle of the Euler Anticevian Triangle of Centroid is the X(2482).*
- (17) *Excentral Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(3679).*
- (18) *Excentral Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(165).*
- (19) *Antimedial Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(3543).*
- (20) *Antimedial Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2).*
- (21) *Tangential Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(381).*
- (22) *Spieker Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(3241).*
- (23) *Moses Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(7818).*
- (24) *Moses Circle of the Euler Anticevian Triangle of Centroid and the Nine-Point Circle of the Euler Anticevian Triangle of Centroid is the X(2482).*
- (25) *Half-Moses Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(599).*

- (26) *Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (27) *Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (28) *Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Circumcircle of the Euler Anticevian Triangle of Centroid is the X(6174).*
- (29) *Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Incircle of the Euler Anticevian Triangle of Centroid is the X(376).*
- (30) *Outer Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid and the Inner Johnson-Yff Circle of the Euler Anticevian Triangle of Centroid is the X(376).*

4.9. Inverse Points. We have investigated 345 remarkable points which are inverse images of 12 Kimberling Points wrt circles of the Euler Anticevian Triangle of Centroid. Of these 4 points are available in the Kimberling's ETC [8] and the rest of 331 points are new points, not available in [8].

Table 4 gives a few of the inverse images wrt circles of the Euler Anticevian triangle of Centroid in terms of the Kimberling points of the reference triangle. The reader may find additional notable points in the Supplementary Material.

In the Table below **T** denotes the Euler Anticevian Triangle of Centroid.

Point P	Circle C of T	Inverse of P wrt C
X(2) Centroid	Nine-Point Circle	X(7426)
X(2) Centroid	Brocard Circle	X(9140)
X(4) Orthocenter	Orthocentroidal Circle	X(3524)
X(5) Nine-Point Center	Orthocentroidal Circle	X(5054)

TABLE 4.

SUPPLEMENTARY MATERIAL

The enclosed supplementary material contains theorems related to the topic.

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REFERENCES

- [1] P. Douillet, *Translation of the Kimberling's Glossary into barycentrics*, 2012, <http://www.ddekov.eu/e2/htm/links/Douillet.pdf>.
- [2] Francisco Javier García Capitán. *Barycentric Coordinates*, International Journal of Computer Discovered Mathematics, 2015, vol. 0, no 0, 32-48. <http://www.journal-1.eu/2015/01/Francisco-Javier-Barycentric-Coordinates-pp.32-48.pdf>.

- [3] S. Grozdev and D. Dekov, *A Survey of Mathematics Discovered by Computers*, International Journal of Computer Discovered Mathematics, 2015, vol.0, no.0, 3-20. <http://www.journal-1.eu/2015/01/Grozdev-Dekov-A-Survey-pp.3-20.pdf>.
- [4] S. Grozdev and D. Dekov, *Computer Discovered Mathematics: Euler Triangles*, International Journal of Computer Discovered Mathematics, 2016, vol.1, no.1, 1-10. <http://www.journal-1.eu/2016-1/Grozdev-Dekov-Euler-Triangles-pp.1-10.pdf>.
- [5] S. Grozdev and D. Dekov, *Barycentric Coordinates: Formula Sheet*, International Journal of Computer Discovered Mathematics, vol.1, 2016, no 2, 75-82. <http://www.journal-1.eu/2016-2/Grozdev-Dekov-Barycentric-Coordinates-pp.75-82.pdf>.
- [6] S. Grozdev and V. Nenkov, *Three Remarkable Points on the Medians of a Triangle* (Bulgarian), Sofia, Archimedes, 2012.
- [7] S. Grozdev and V. Nenkov, *On the Orthocenter in the Plane and in the Space* (Bulgarian), Sofia, Archimedes, 2012.
- [8] C. Kimberling, *Encyclopedia of Triangle Centers - ETC*, <http://faculty.evansville.edu/ck6/encyclopedia/ETC.html>.
- [9] Gerry Leversha, *The Geometry of the Triangle*, The United Kingdom Mathematical Trust, The Pathways Series no.2, 2013.
- [10] G. Paskalev and I. Tchobanov, *Remarkable Points in the Triangle* (in Bulgarian), Sofia, Narodna Prosveta, 1985.
- [11] G. Paskalev, *With coordinates in Geometry* (in Bulgarian), Sofia, Modul-96, 2000.
- [12] M. Schindler and K.Cheny, *Barycentric Coordinates in Olympiad Geometry*, 2012, <http://www.mit.edu/~evanchen/handouts/bary/bary-full.pdf>.
- [13] P. Yiu, *Introduction to the Geometry of the Triangle*, 2001, new version of 2013, <http://math.fau.edu/Yiu/YIUIntroductionToTriangleGeometry130411.pdf>.