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A Note on the Leversha Point

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Abstract. We find the barycentric coordinates and new properties of the Leversha point in the triangle geometry.

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Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

In accordance with Leversha [6] the internal center of similitude of the circumcircle of the Kosnita and tangential triangles is a significant triangle point. We call this point the Leversha point. Note that at present time the Leversha point is not included in the Kimberling's ETC [5], the 11188 points edition of 2016. In this note we find the barycentric coordinates and new properties of the Leversha point.

Figures 1 and 2 illustrate the Leversha point. In figures 1 and 2, ABC is the reference triangle, $OaObOc$ is the Kosnita triangle, c_1 is the circumcircle of the Kosnita triangle, K is the circumcenter of the Kosnita triangle, $T_AT_BT_C$ is the tangential triangle, c_2 is the circumcircle of the tangential triangle, T is the circumcenter of the tangential triangle, and P is the internal center of similitude of circles c_1 and c_2.

We use barycentric coordinates. We refer the reader to [11], [1], [5], [2], [3], [4], [7], [8], [9], [10].

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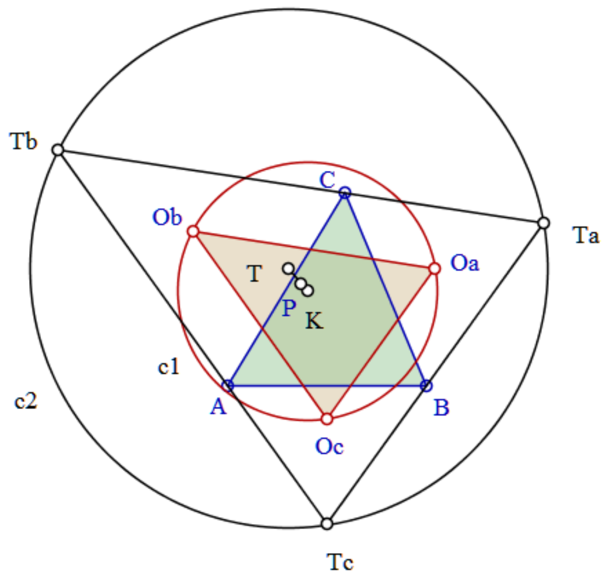

FIGURE 1.

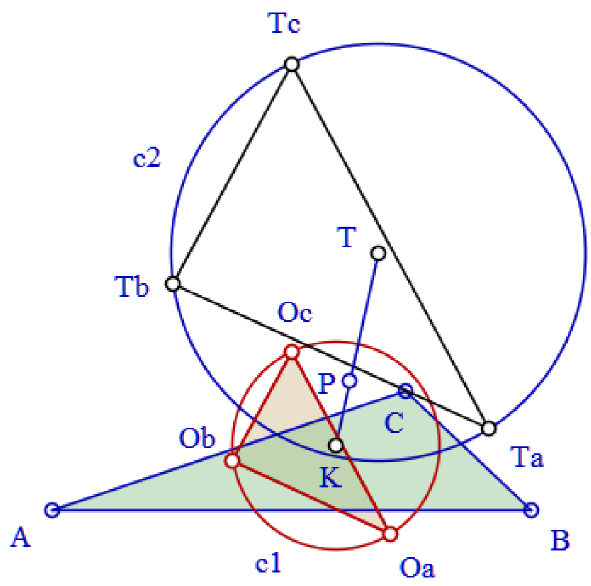

FIGURE 2.

2. KOSNITA TRIANGLE

Here we calculate the barycentric coordinates of the Kosnita triangle. The reader may find the definition of the Kosnita triangle e.g. in [6].

Theorem 2.1. *The barycentric coordinates of the Kosnita triangle OaObOc are as follows:*

$$
\begin{aligned}
Oa = (&a^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2),\\
&-b^2(a^4 + b^4 - 2a^2b^2 - a^2c^2 - b^2c^2),\\
&-c^2(a^4 + c^4 - 2a^2c^2 - a^2b^2 - b^2c^2)).
\end{aligned}
$$

$$Ob = (a^2(a^4 + b^4 - 2a^2b^2 - a^2c^2 - b^2c^2),$$
$$-b^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2),$$
$$c^2(b^4 + c^4 - 2b^2c^2 - a^2b^2 - a^2c^2)).$$
$$Oc = (a^2(a^4 + c^4 - 2a^2c^2 - a^2b^2 - b^2c^2),$$
$$b^2(b^4 + c^4 - 2b^2c^2 - a^2b^2 - a^2c^2),$$
$$-c^2(a^4 + b^4 + c^4 - 2a^2c^2 - 2b^2c^2)).$$

Proof. The Kosnita triangle $OaObOc$ and the tangential triangle $TaTbTc$ are homothetic with center of homothety the circuncenter of triangle ABC and ratio 2 (see e.g Laversha [6], Theorem 12.14). Hence, we can calculate the vertices Oa, Ob and Oc as midpoints of segments OTa, OTb and OTc, by using the midpoint formula (14), [2]. The barycentric coordinate of the Circumcenter O and the Tangential triangle $TaTbTc$ are given in [11], pages 26 and 54, respectively. Another way is we to use the formula for homothety (17), [2], or to use the definition of the Kosnita triangle. □

3. Leversha Point

Theorem 3.1. *The barycentric coordinates of the Leversha Point $P = (uP, bP, cP)$ are as follows:*

$$uP = a^2(3a^8 - 6a^6b^2 - 6a^6c^2 + 4b^2c^2a^4 - 2a^2b^2c^4 - 2a^2b^4c^2 + 6a^2b^6 + 6a^2c^6$$
$$-3c^8 - 2b^4c^4 - 3b^8 + 4b^6c^2 + 4b^2c^6),$$
$$vP = b^2(3b^8 - 6b^6c^2 - 6a^2b^6 + 4a^2b^4c^2 - 2b^2c^2a^4 - 2a^2b^2c^4 + 6b^2c^6 + 6a^6b^2$$
$$-3a^8 - 2c^4a^4 - 3c^8 + 4a^2c^6 + 4a^6c^2),$$
$$wP = c^2(3c^8 - 6a^2c^6 - 6b^2c^6 + 4a^2b^2c^4 - 2a^2b^4c^2 - 2b^2c^2a^4 + 6a^6c^2 + 6b^6c^2$$
$$-3b^8 - 2a^4b^4 - 3a^8 + 4a^6b^2 + 4a^2b^6).$$

Proof. We use use the definition of the Leversha point and the barycentric coordinates of Kosnita triangle, given in Theorem 2.1. □

The text two theorems and give alternative ways for finding the barycentric coordinates of the Leversha point. In order to find the barycentric coordinates of the Leversha point, we have to use the homothety formula (17), [2] in Theorem 3.2 and the internal division formula (12), [2] in Theorem 3.3.

Theorem 3.2. *The Leversha Point is the Image of the Center of the Tangential Circle under the Homothety with Center at the Circumcenter and Ratio 2:3.*

Theorem 3.3. *The Leversha Point is the Point Dividing Internally the Directed Segment from the Circumcenter to the Circumcenter of the Tangential Triangle in the Ratio of 2:1.*

Theorem 3.4. *The Laversha Point lies on the Image of the Brocard Circle under the Homothety with Center the Center of the Tangential Circle and Ratio 1:3.*

Theorem 3.5. *The Laversha Point lies on the Image of the Lester Circle under the Homothety with Center the Center of the Tangential Circle and Ratio 1:3.*

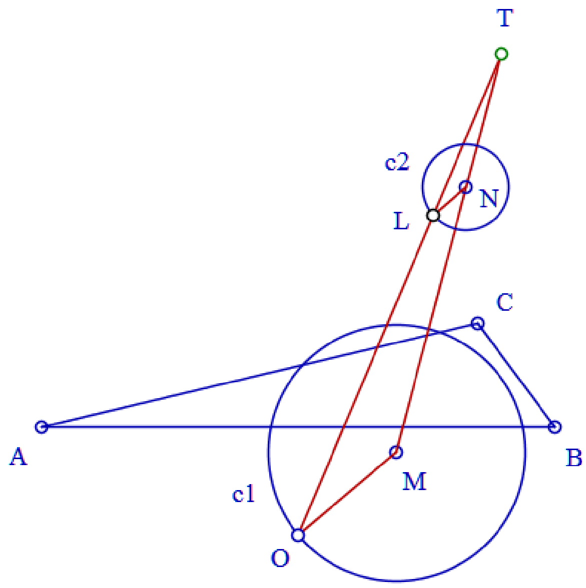

FIGURE 3.

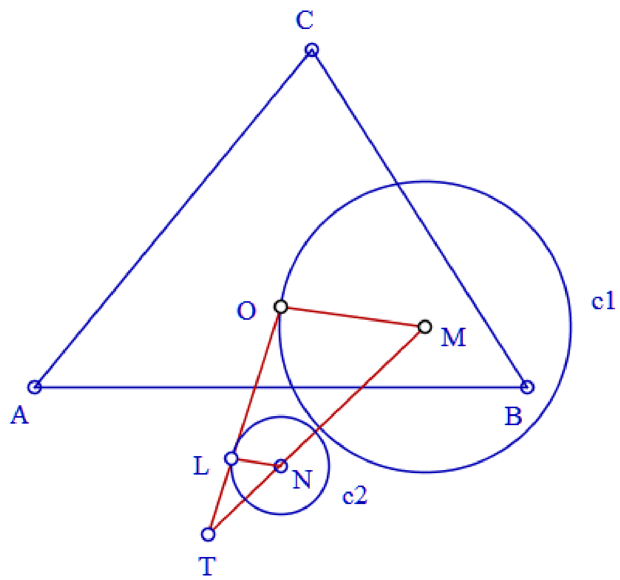

FIGURE 4.

Figure 3 illustrates Theorem 3.4. In figure 3, T is the center of Tangential circle, c_1 is the Brocard circle, M is the center of the Brocard circle, O is the circumcenter of triangle ABC, c_2 is the image of the Brocard circle under homothety with center T and ratio 1:3, N is the center of circle c_1, and L is the Leversha point. Point L lies on circle c_2.

Figure 4 illustrates Theorem 3.5. In figure 4, T is the center of Tangential circle, c_1 is the Lester circle, M is the center of the Lester circle, O is the circumcenter of triangle ABC, c_2 is the image of the Lester circle under homothety with center T and ratio 1:3, N is the center of circle c_1, and L is the Leversha point. Point L lies on circle c_2.

We recommend the reader to generalize theorems 3.4 and 3.5 to the case where the circumcenter O lies on an arbitrary circle.

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