International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 3, 2018, pp.136-139.
Received 31 July 2018. Published on-line 3 August 2018
web: `http://www.journal-1.eu/`

Six Conics Theorem

Nguyen Ngoc Giang[a2] and Dao Thanh Oai[b]
[a] Banking University of Ho Chi Minh City,
36 Ton That Dam street, district 1, Ho Chi Minh City, Vietnam
e-mail: nguyenngocgiang.net@gmail.com
[b] Kien Xuong, Thai Binh, Vietnam
e-mail: daothanhoai@hotmail.com

Abstract. We study a generalization of the Miquel six circles theorem and the Bundle theorem. We call the new theorem the Six Conics Theorem.

Fist, we refer to two famous theorems which are the Miquel six circle theorem and the Bundle theorem as follows:

Theorem 1 (Miquel [1]). *If five circles share four triple-points of intersection then the remaining four points of intersection lie on a sixth circle.*

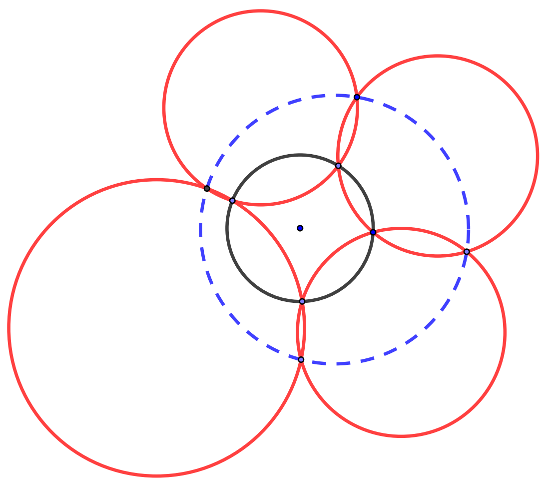

Figure 1.

[2]Corresponding author

Theorem 2 (Bundle-[2])**.** *Given different points* $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$*. If five of the six quadruples* $Q_{ij} := \{A_i, B_i, A_j, B_j\}$*,* $i < j$*, are concyclic (contained in a cycle) on at least four cycles* c_{ij}*, then the 6th quadruple is concyclic, too.*

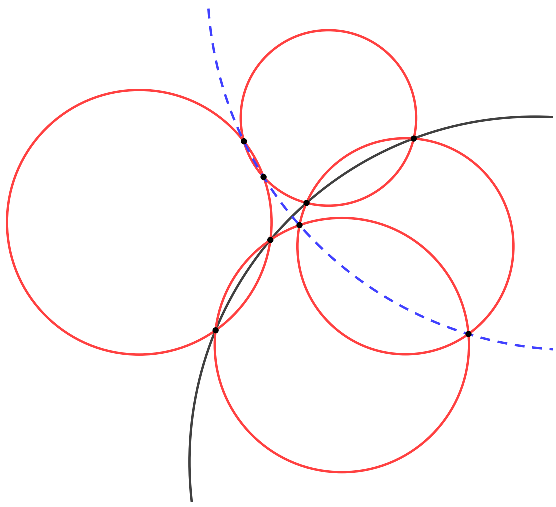

Figure 2.

Theorem 3 (Six Conics Theorem)**.** *Let 16 points* A_i, B_i *for* $i = 1, ..., 8$ *be on the plane so that*

- *Eight points* A_i *for* $i = 1, ..., 8$ *lie on a conic.*
- *Eight points* A_i, B_i *for* $i = 1,\ 2,\ 7,\ 8$ *lie on conic.*
- *Eight points* A_i, B_i *for* $i = 1,\ 4,\ 5,\ 8$ *lie on conic.*
- *Eight points* A_i, B_i *for* $i = 4,\ 5,\ 3,\ 6$ *lie on conic.*
- *Eight points* A_i, B_i *for* $i = 2,\ 3,\ 6,\ 7$ *lie on conic.*

Then eight points B_i $i = 1,...,\ 8$ *lie on a conic.*

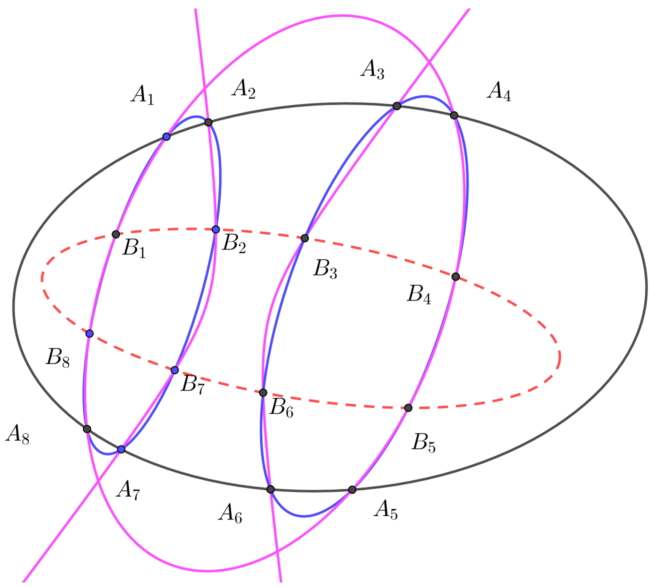

Figure 3.

We use a theorem as follows to prove the theorem above:

Theorem 4 ([3])**.** *Given a curve (K) of degree m and three curves (C_i) of degree n, for $i = 1, 2, 3$. Let (C_1) meets (K) at mn points. Let (C_2) meets (K) at mn points. Let (C_3) meet (K) at mn points.*

Let (C_1), (C_2) and K have d common points. Let points P_1, P_2,..., P_{mn-d} lie on (C_1) and (K) but do not lie on (C_2).

Let (C_2), (C_3) and K have $mn - d$ common points. Let points Q_1, Q_2, Q_3,...,Q_d lie on C_3 and (K) but don't lie on C_2.

Then the mn points P_1, P_2,...,P_{mn-d}, Q_1, Q_2, Q_3,...,Q_d lie on a curve of degree n, where $mn - d < \frac{n^2+3n}{2}$ and $d < \frac{n^2+3n}{2}$.

The first figure 1, make with $m = 3$, $n = 2$ and $d = 2, mn - d = 4$

Theorem 4 is a propblem posed by us at Mathoverflow forum. A proof by Alex Degtyarev, you can see in [3].

Another application of Theorem 4 with many votes in [4].

Note that Mathoverflow is high Math forum, with Euclidean Geometry problem posted at there with 14 votes. So the application [4] is nice question. But we think Theorem 3 is nice than the problem [4].

Theorem 3 has more signification and is symmetric and is a high version Miquel Six Circles Theorem and Bundle theorem. Therefore we consider Theorem 3 as a problem which is proposed to AMM.

Back to proof of Theorem 1.

First we namely four five conics as follows:

Eight points A_i, B_i for $i = 1, 4, 5, 8$ lie on conic (C_1)

Eight points A_i for $i = 1, ..., 8$ lie on a conic (C_2).

Eight points A_i, B_i for $i = 2, 3, 6, 7$ lie on conic (C_3)

Eight points A_i, B_i for $i = 1, 2, 7, 8$ lie on conic (C_4)

Eight points A_i, B_i for $i = 4, 5, 3, 6$ lie on conic (C_5)

We consider two conics (C_4) and $(C5)$ are quartic (K) degree 4; Application of Theorem 2 with $m = 4$, $n = 2$, $d = 2$, $mn - d = 4$.

(K) meets (C_1) at eight points A_i, B_i for $i =$ 1, 4, 5, 8 (K) meets (C_2) at eight points A_i for $i = 1, ..., 8$ (K) meets (C_3) at eight points A_i, B_i for $i =$ 2, 3, 6, 7 (K) , (C_1) and (C_2) have four common points A_1, A_4, A_5, A_8. Four points B_1, B_4, B_5, B_8 lie on (C_1) and (K) but don not lie on (C_2).

(K) , (C_2) and (C_3) have four common points A_2, A_3, A_6, A_7.

Four points B_2, B_3, B_6, B_7 lie on (C_3) and (K) but do not lie on (C_2).

Therefore eight points B_1, B_4, B_5, B_8, B_2, B_3, B_6, B_7 lie on a curve degree 2 which is a conic named the 6th conic. So we call theorem 1 being The six Conic Theorem.

REFERENCES

[1] Pedoe, Dan (1988) [1970], Geometry / A Comprehensive Course, Dover, ISBN 0-486-65812-0, p. 424

[2] Hartmann, Erich. Planar Circle Geometries, an Introduction to Möbius-, Laguerre- and Minkowski Planes. (PDF; 891 kB) Department of Mathematics, Darmstadt University of Technology

[3] A problem of four curves, available at https://mathoverflow.net/questions/232430/a-problem-of-four-curves
[4] https://mathoverflow.net/questions/232342/a-cubic-and-six-conics-problem