International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 6, 2021, pp. 108–136
Received 27 Nov. 2021. Published on-line 31 Dec. 2021
web: http://www.journal-1.eu/

Catalog of Properties of the First Isodynamic Point of a Triangle

Stanley Rabinowitz
545 Elm St Unit 1, Milford, New Hampshire 03055, USA
e-mail: stan.rabinowitz@comcast.net
web: http://www.StanleyRabinowitz.com/

Abstract. The first isodynamic point of a triangle is one of many notable points associated with a triangle. It is named X(15) in the Encyclopedia of Triangle Centers. This paper surveys known results about this point and gives additional properties that were discovered by computer.

Keywords. triangle geometry, first isodynamic point, computer-discovered mathematics, GeometricExplorer.

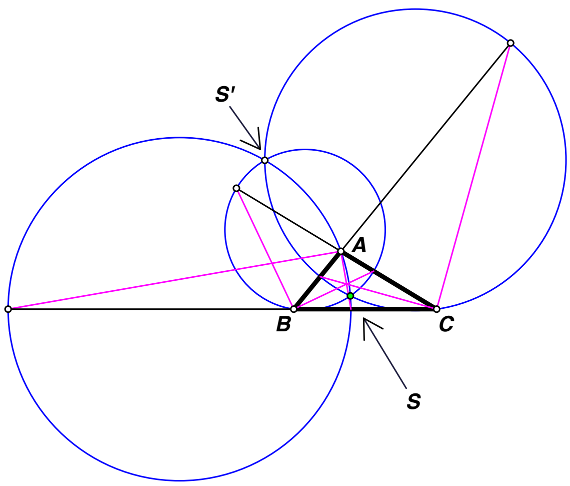
Mathematics Subject Classification (2020). 51M04, 51-08.

Introduction

The first isodynamic point of a triangle is one of many notable points associated with a triangle. It is the point with trilinear coordinates

$$\left(\sin(A+\frac{\pi}{3}) : \sin(B+\frac{\pi}{3}) : \sin(C+\frac{\pi}{3})\right)$$

with respect to the triangle. It is named X_{15} in the Encyclopedia of Triangle Centers [37].



Geometrical definition. The interior and exterior angle bisectors of angle A of $\triangle ABC$ intersects side BC of the triangle (or its extension) in two points, A_1 and A_2. The circle with diameter A_1A_2 is called the A-Apollonius circle and is named C_A. Circles C_B and C_C are defined similarly. The points in which the three Apollonius circles intersect are the isodynamic points of the triangle. The one inside $\odot ABC$ is the 1st isodynamic point and is named S. The other point of intersection is the 2nd isodynamic point and is named S'.

Scope of this catalog. The mathematical literature is vast. We do not attempt to catalog every property involving the 1st isodynamic point that appears somewhere in print or on the internet. We do try to catalog any property that is simple or elegant or that can be obtained from the configuration associated with one of our top-level classifications (Triangle plus S, Triangle with S and other points, Triangle plus lines through S, Quadrilateral plus S, etc.) by applying at most one common geometrical construction (drop a perpendicular, draw an angle bisector, construct a centroid, etc.) When analyzing triangle centers, we only look at the common ones, X_1 through X_{20}.

Classification Scheme

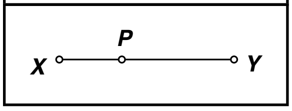
Contents

Introduction 108
1. Triangle plus S 113
1.1. location of S 113
1.2. metric properties 113
1.3. angle properties 113
1.4. geometrical properties 114
2. Triangle plus S and constructions 114
2.1. angle bisectors 114
2.2. circles 114
2.3. Euler lines 115
2.4. equilateral triangles 116
2.5. reflection 116
2.6. sagitta 117
3. Special triangle plus S 117
3.1. isosceles triangle 117
3.2. isosceles right triangle 117
3.3. 30° triangle 117
3.4. 60° triangle 117
3.5. 120° triangle 118
3.6. 150° triangle 119
4. Triangle with S and other points 119
4.1. S and T 119
4.2. S and one notable point 119
4.3. S and T and one notable point 121
4.4. S and T and a point P 121
4.5. S and two notable points 121
4.6. S and three notable points 122
4.7. S and 4 or more notable points 123
5. Triangle plus lines through S 124
5.1. antiparallels 124
5.2. apothems 124
5.3. cevians 125
5.4. circlecevians 125
5.5. circumcevians 125
5.6. inclines 126
5.7. parachords 126
5.8. spokes 127
6. Triangle with S in subtriangles 127
6.1. formed by H 127
6.2. formed by I 128
6.3. formed by K 128
6.4. formed by O 129
6.5. formed by interior point P 129
6.6. formed by X_{13} 129
6.7. formed by similar triangles 129
6.8. formed by excenters 130
6.9. formed by six cevians 130
6.10. formed by a Tucker Hexagon 130
6.11. formed by isogonal cevians 130
6.12. formed by varying vertex A 130
7. Quadrilateral plus S 131
7.1. cyclic quadrilateral 131
7.2. square 132
7.3. trilateral trapezoid 132
8. Pentagon plus S 132
8.1. regular pentagon 132
9. Hexagon plus S 133
9.1. regular hexagon 133
Other Properties 134
References 134

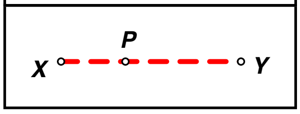
Figures. The figures in this paper are *not* decorative. In order to reduce verbiage and declutter the catalog, given information appearing in the figure is not repeated in words. For example, if a figure shows line segments WX and YZ meeting at a point P, we do not state in words that P is the intersection of WX and YZ. This should make it easier for people who don't read English to use this catalog.

A solid line or circle through multiple points means that it is given that these points lie on the same line or circle. A dashed line or circle through multiple points means that the conclusion of the theorem or result is that these points lie on the same line or circle.

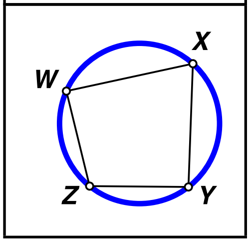
P is a point on segment XY.



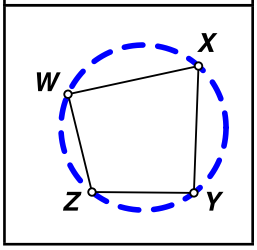
We can conclude that X, P, and Y colline.



It is given that $WXYZ$ is a cyclic quadrilateral.

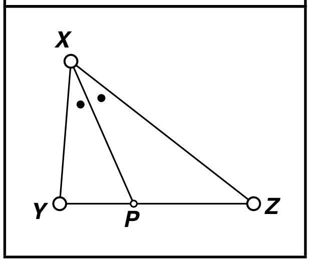


We can conclude that W, X, Y, and Z are concyclic.

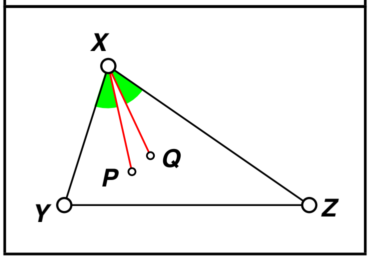


Angles that are given to be equal are marked with the same filled circle. Angles that are concluded to be equal are shaded with the same color.

It is given that $\angle YXP = \angle PXZ$.

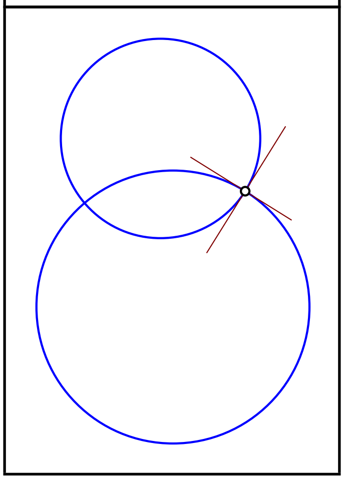


We can conclude that $\angle YXP = \angle QXZ$.



Two perpendicular brown lines at the point of intersection of two circles means that we can conclude that the circles are orthogonal (have perpendicular tangents at that point).

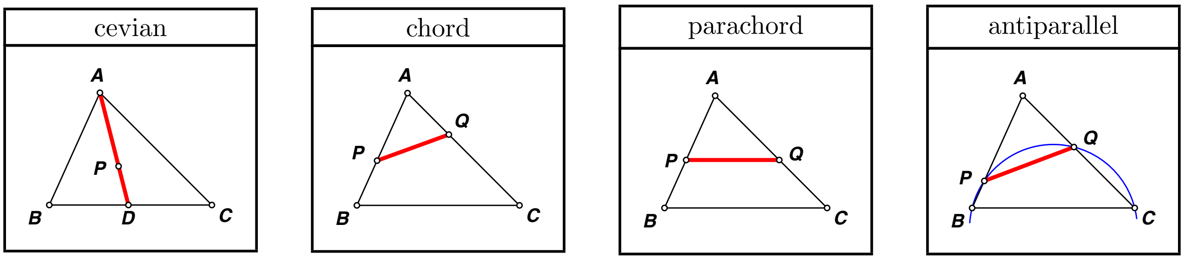
We can conclude that the circles are orthogonal.



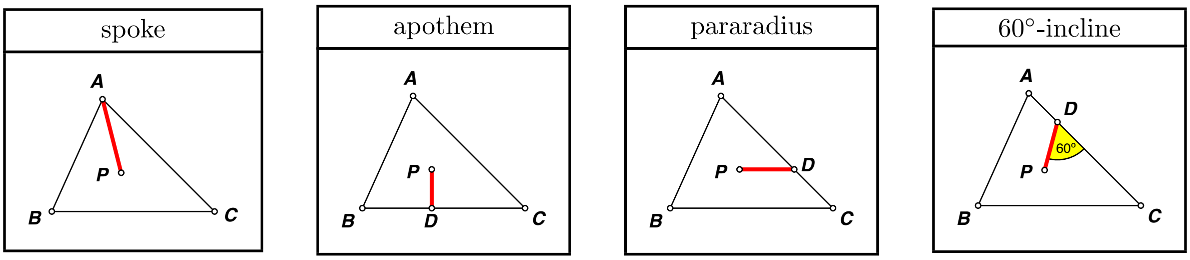
If the title of a section or subsection describes a feature of a figure, then we do not repeat this description if it is obvious from the figure. For example, in a subsection entitled "equilateral triangles", a triangle highlighted in yellow that looks equilateral can be assumed to be an equilateral triangle.

A right-angle marker is used to indicate two lines that are given to be perpendicular. All angles are directed angles. The 1st isodynamic point of $\triangle ABC$ is always colored green. Given information that is not obvious from the associated figure is shown in brown text directly beneath the figure.

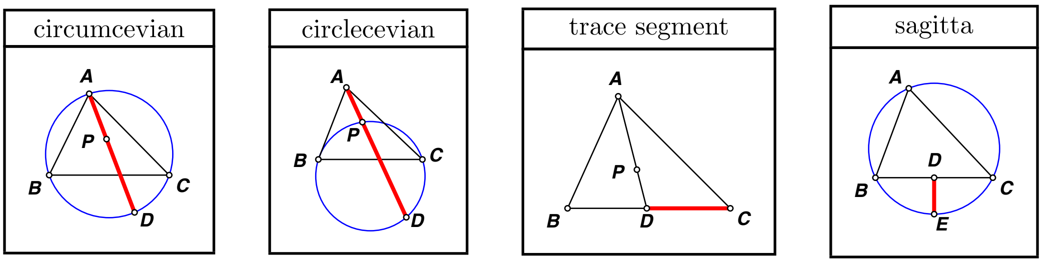
Parts of a triangle. In order to help with the classification process, we give names for various line segments associated with a triangle. A line segment from a vertex of a triangle to a non-vertex point on the opposite side is called a *cevian.* A line segment joining points on two sides of the triangle is called a *chord.* A chord parallel to a side of the triangle is called a *parachord.* If the endpoints of a chord joining points on two sides of a triangle forms a cyclic quadrilateral with the endpoints of the third side, the chord is called an *antiparallel.*



If P is a point inside a triangle, the line segment from P to a vertex is called a *spoke.* The line segment from P to the foot of the perpendicular from P to a side of the triangle is called an *apothem.* A line segment from P parallel to a side of the triangle ending on another side of the triangle is called a *pararadius.* A line segment from P to a side of the triangle that forms an angle of $n°$ with that side is called an $n°$*-incline.*



If P is a point inside a triangle, the line segment from a vertex passing through P and extending to the circumcircle of $\triangle ABC$ is called a *circumcevian.* The line segment from a vertex passing through P and extending to the circumcircle of $\triangle BPC$ is called a *circlecevian.* The endpoint of a cevian through P (other than a vertex) is called a *trace* and the line segment along a side from that trace to a vertex of the triangle is called a *trace segment.* The line segment from the midpoint of the side of a triangle extending outward to the furthest point on the circumcircle is called a *sagitta.*



Discoveries. An asterisk after a property number indicates that the property was discovered by computer, either by using GeometricExplorer, Mathematica, or Geometer's Sketchpad. If a reference is given, this means that the result was posted to an online forum in the hope that some forum member might find a geometrical proof of the property.

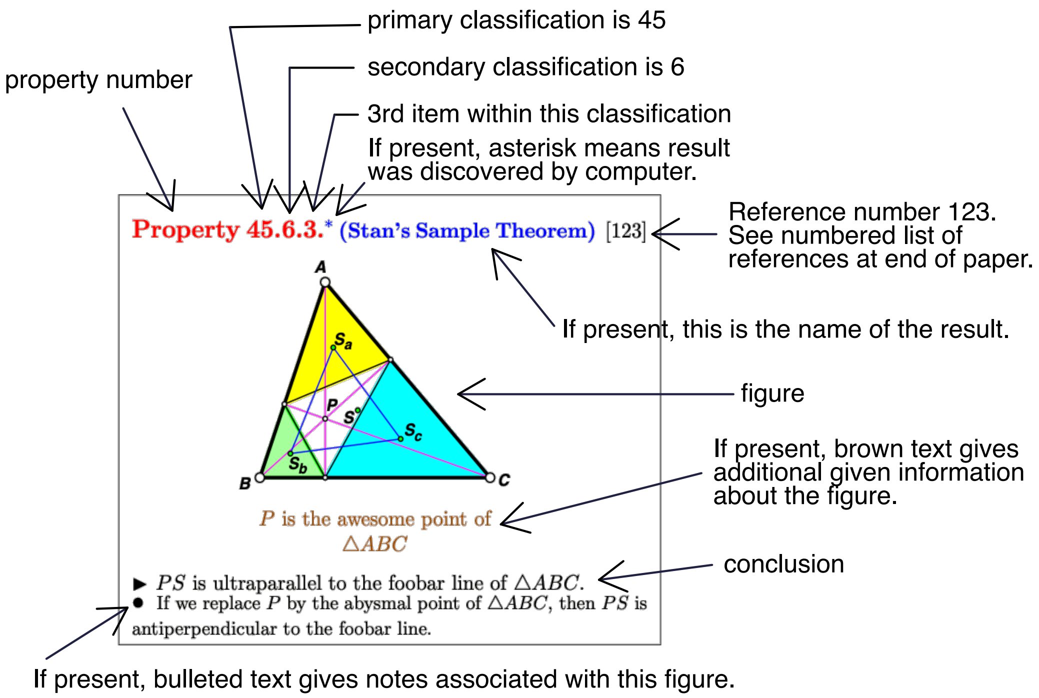
Acknowledgement.

I would like to thank the following people who did thorough reviews of this paper and who provided additional references and results about the isodynamic points of a triangle: Kadir Altıntaş, Dao Thanh Oai, Tran Quang Hung, Kousik Sett.

Notation.

Notation used when describing properties	
Notation	**Description**
$\triangle XYZ$	Triangle XYZ
a, b, c	The lengths of the sides of $\triangle ABC$
s	$(a+b+c)/2$
$\odot XYZ$	The circle through points X, Y, and Z
H	The orthocenter of $\triangle ABC$
K	The symmedian point of $\triangle ABC$
K	When used in an expression, K denotes the area of $\triangle ABC$.
I	The incenter of $\triangle ABC$
M	The centroid of $\triangle ABC$
O	The circumcenter of $\triangle ABC$
r	The inradius of $\triangle ABC$
R	The circumradius of $\triangle ABC$
$R(XYZ)$	The circumradius of $\triangle XYZ$
S	The 1st isodynamic point of $\triangle ABC$
$S(XYZ)$	The 1st isodynamic point of $\triangle XYZ$
S' or T	The 2nd isodynamic point of $\triangle ABC$
Ω	The 1st Brocard point of $\triangle ABC$
Ω'	The 2nd Brocard point of $\triangle ABC$
$[F]$	The area of figure F
ϕ	The golden ratio, $(1+\sqrt{5})/2$
$X - Y - Z$	The points X, Y, and Z colline (i.e. are collinear).
$\angle XYZ$	Directed angle XYZ. This is the angle through which ray $\overrightarrow{YX}$ must be rotated counterclockwise in order to coincide with ray $\overrightarrow{YZ}$.
X_n	The nth Kimberling center of $\triangle ABC$ (see [36])
$X_n(XYZ)$	The nth Kimberling center of $\triangle XYZ$

Key to the property listings.



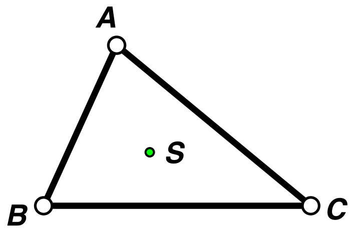
Since there are three shaded triangles, this means that the 1st isodynamic points of these triangles will be named S_a, S_b, and S_c [or S_1, S_2, and S_3]. The 2nd isodynamic points will be named T_a, T_b, and T_c.

1st Isodynamic Point S

1. Triangle plus S

1.1 location of S

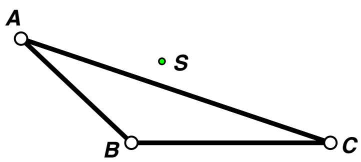
Property 1.1.1.*



Each angle of $\triangle ABC$ has measure less than 120°.

► S lies in the interior of $\triangle ABC$.

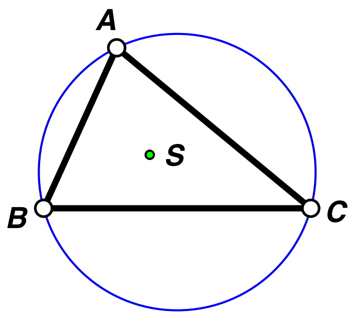
Property 1.1.2.*



$\angle B > 120°$.

► S lies outside $\triangle ABC$.

Property 1.1.3. [86]

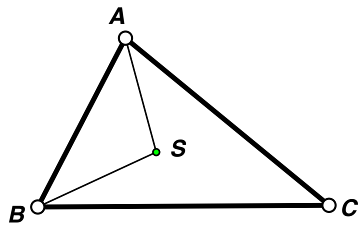


► S lies inside the circumcircle of $\triangle ABC$.

● Equivalently, $SO < R$.

1.2 metric properties

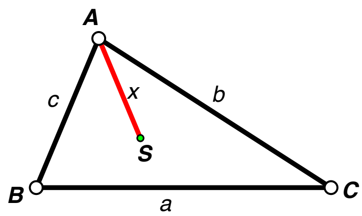
Property 1.2.1. (Tripolar Coordinates) [26, p. 106]



► $\dfrac{AS}{BS} = \dfrac{AC}{BC}$

● A similar property holds for S'. By symmetry, we have $AS \cdot BC = BS \cdot AC = CS \cdot AB$.

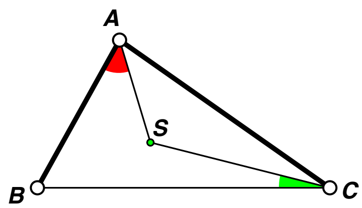
Property 1.2.2. (Spoke Length) [25]



► $x = \dfrac{bc\sqrt{2}}{\sqrt{a^2 + b^2 + c^2 + 4K\sqrt{3}}}$

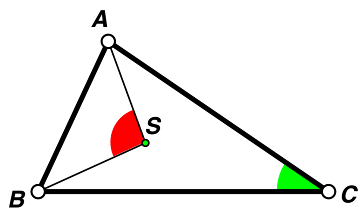
1.3 angle properties

Property 1.3.1. [42]



► $\angle BAS + \angle SCB = 60°$

Property 1.3.2. [35, p. 296]

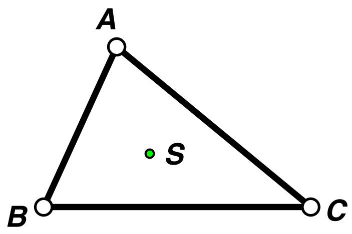


► $\angle ASB = \angle ACB + 60°$

● A similar result holds for S': $\angle ASB = \angle ACB - 60°$

1.4 geometrical properties

Property 1.4.1.* [55]

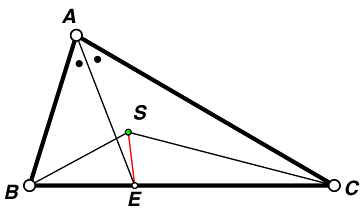


▶ A is the 2nd isodynamic point of $\triangle BCS$.

2. Triangle plus S and constructions

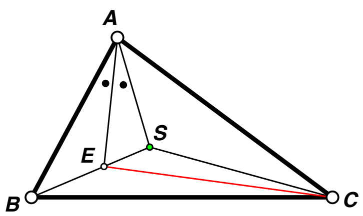
2.1 angle bisectors

Property 2.1.1.* [47]



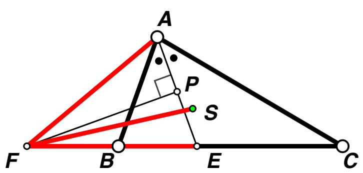
▶ $\angle BSE = \angle ESC$

Property 2.1.2. [47]



▶ $\angle SCE = \angle ECB$

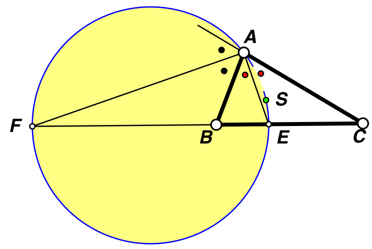
Property 2.1.3.* [51]



$AP = PE$

▶ $E - S - A'$

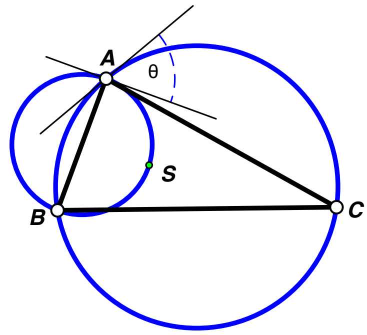
Property 2.1.4. [86]



▶ Circle with diameter EF passes through S.

2.2 circles

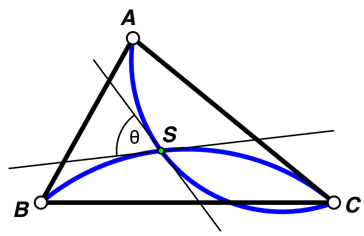
Property 2.2.1. [86]



▶ $\theta = 60°$

● In other words, the circles meet at an angle of 60°.

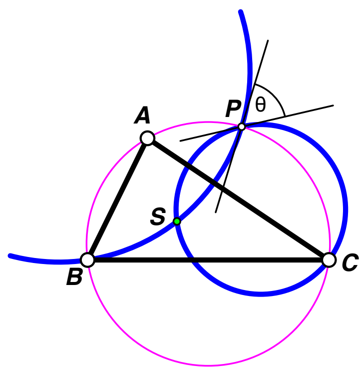
Property 2.2.2. [19]



▶ $\theta = 60°$.

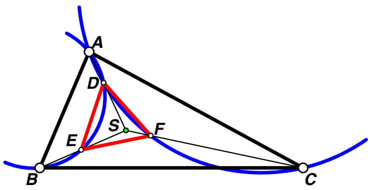
● This is a special case of Property 2.2.3.

Property 2.2.3. [19]



▶ $\theta = 60°$.

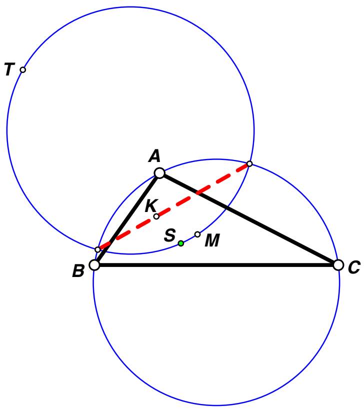
Property 2.2.4. [19]



D is any point on AS.

▶ $\triangle DEF$ is equilateral.
▶ B, E, F, C are concyclic.

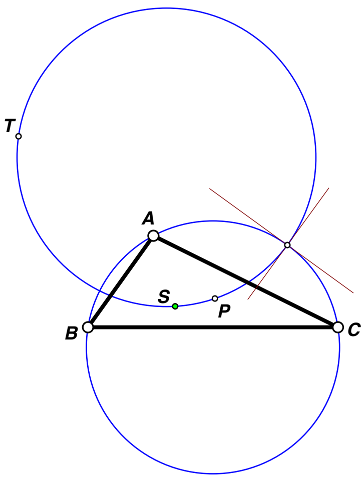
Property 2.2.5.*



P is any point

▶ Common chord of $\odot ABC$ and $\odot PST$ passes through K.

Property 2.2.6. [34]

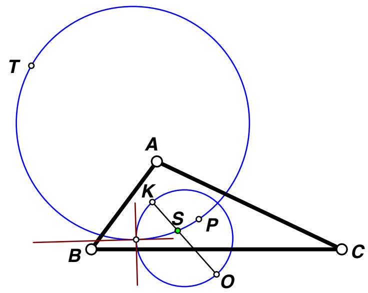


P is any point

▶ $\odot ABC$ and $\odot PST$ are orthogonal.

● In particular, the Parry circle is orthogonal to the circumcircle.

Property 2.2.7. [34]

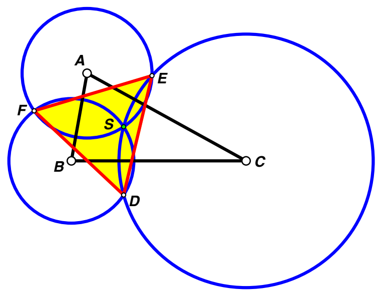


P is any point

▶ Circle with diameter KO is orthogonal to $\odot PST$.

● In particular, the Parry circle is orthogonal to the Brocard circle.

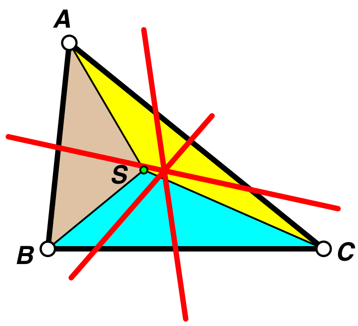
Property 2.2.8. (1st Isodynamic-Dao Triangle) [38]



▶ DEF is an equilateral triangle.

2.3 Euler lines

Property 2.3.1. [69]

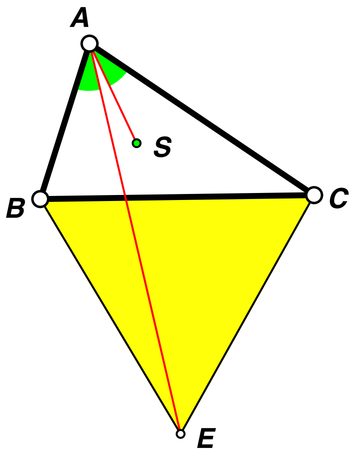


▶ The Euler lines of the three colored triangles concur.

● A computer analysis found that the point of intersection is X_{61}. The three Euler lines will still concur if S is replaced by any point on the circumcircle or on the Neuberg cubic. See [85].

2.4 equilateral triangles

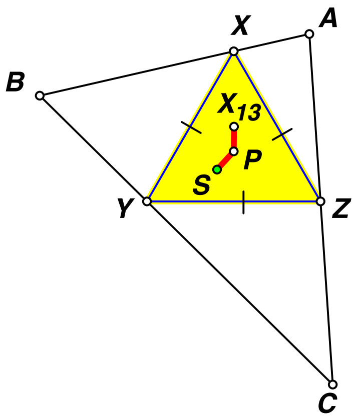
Property 2.4.1. [84]



▶ $\angle BAE = \angle SAC$

- This follows from Property 4.2.10.

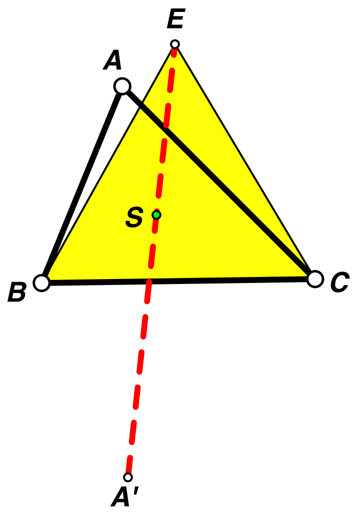
Property 2.4.2. [43]



P is the center of $\triangle XYZ$.

▶ $PS = PX_{13}$

Property 2.4.3. [86]

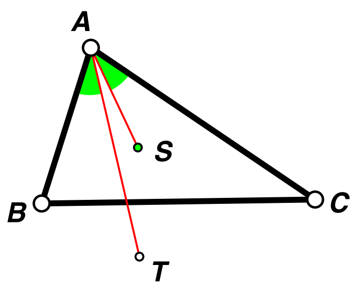


A' is the reflection of A about BC.

▶ $E - S - A'$

2.5 reflection

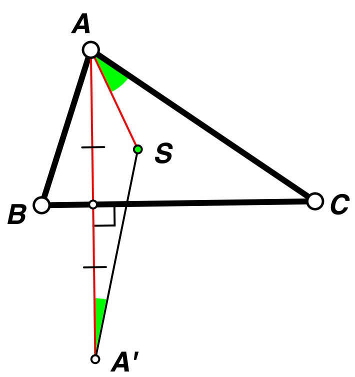
Property 2.5.1.* [56]



T is the reflection of S about BC

▶ $\angle BAT = \angle SAC$

Property 2.5.2. [57]



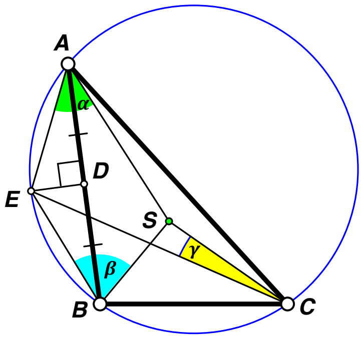
A' is the reflection of A about BC

▶ $\angle SA'A = \angle SAC$

- The property is true if S is replaced by any point on the C-Apollonian circle.

2.6 sagitta

Property 2.6.1.* [60]

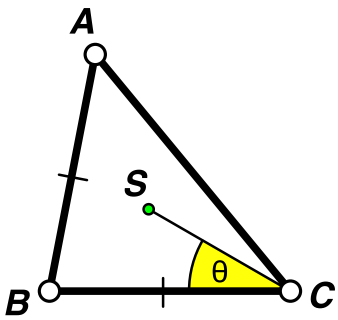


- $\alpha + \gamma = 60°$
- $\beta - \gamma = 60°$

3. Special triangle plus S

3.1 isosceles triangle

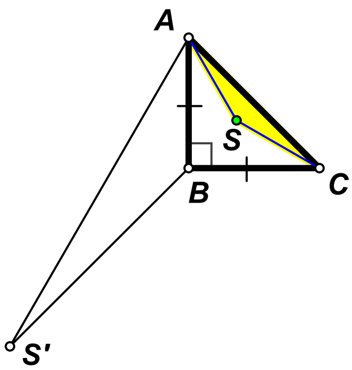
Property 3.1.1.* [42]



- $\theta = 30°$.

3.2 isosceles right triangle

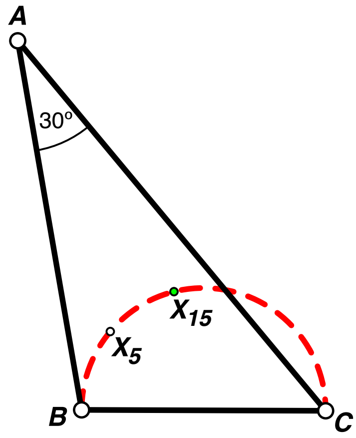
Property 3.2.1.*



- S' is the orthocenter of $\triangle ACS$.
- S is the orthocenter of $\triangle ACS'$.

3.3 30° triangle

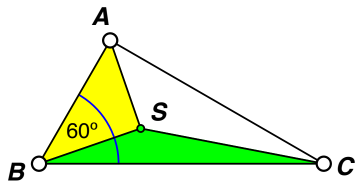
Property 3.3.1.*



- B, X_5, X_{15}, and C are concyclic.

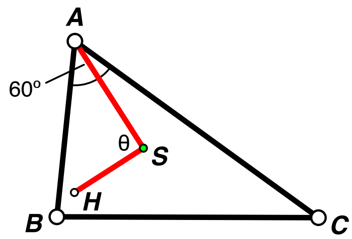
3.4 60° triangle

Property 3.4.1.*



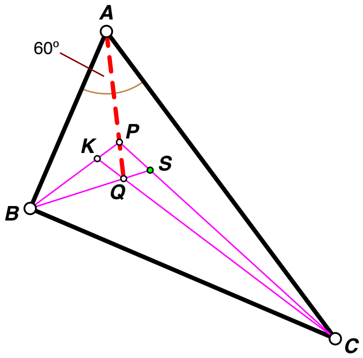
- $[BAS] = [BSC]$.

Property 3.4.2.*



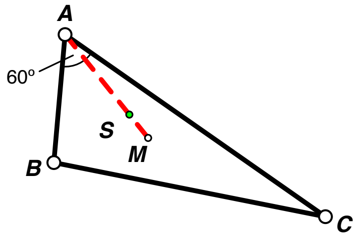
- $\theta = 90°$.

Property 3.4.3.*



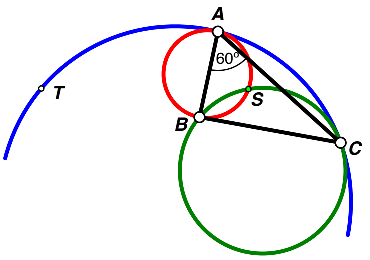
- ▶ $A - P - Q$.
- ▶ $\angle PAC = 30°$.

Property 3.4.4.* [65]



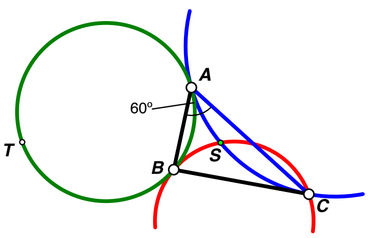
- ▶ $A - S - M$.

Property 3.4.5.*



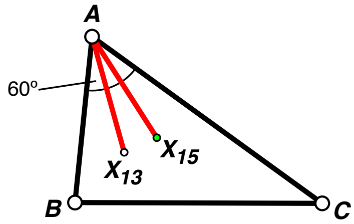
- ▶ $\odot CAT$ and $\odot ABS$ are tangent.
- ▶ $\odot CAT$ and $\odot BSC$ are tangent.

Property 3.4.6.*



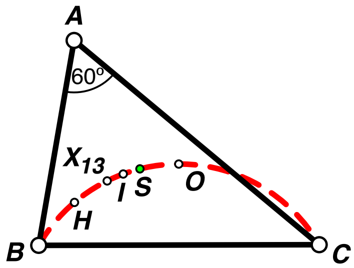
- ▶ $\odot TAB$ and $\odot ASC$ are tangent.
- ▶ $\odot TAB$ and $\odot BSC$ are tangent.

Property 3.4.7.*



- ▶ $AX_{13} = AX_{15}$.

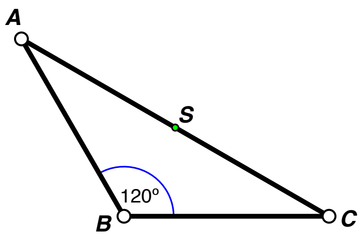
Property 3.4.8. [15]



- ▶ B, H, X_{13}, I, S, O, and C are concyclic.
- ● See also [45]. Other points on this circle (discovered by computer) are X_{399}, X_{616}, and X_{617}.

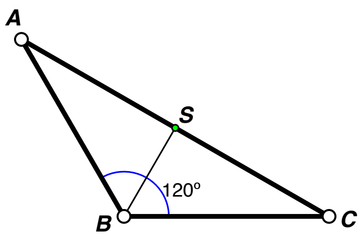
3.5 120° triangle

Property 3.5.1.*



- ▶ S lies on AC.

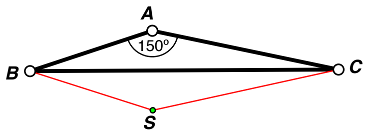
Property 3.5.2.*



- ▶ BS bisects $\angle CBA$.

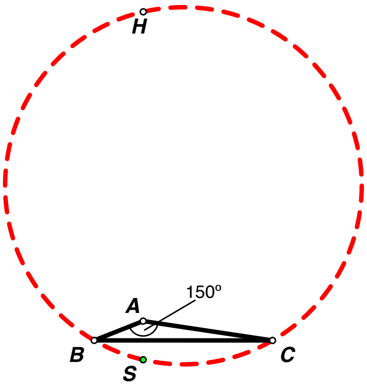
3.6 150° triangle

Property 3.6.1.*



► $ABSC$ is a kite,.

Property 3.6.2.*

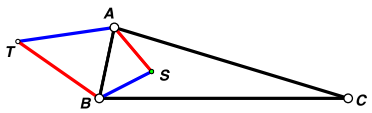


► B, S, C, and H are concyclic.

4. Triangle with S and other points

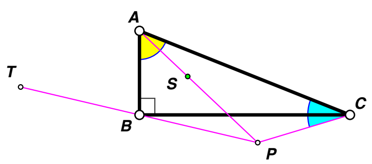
4.1 S and T

Property 4.1.1. [25]



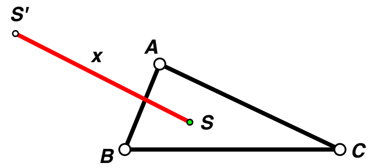
► $\dfrac{SA}{TA} = \dfrac{SB}{TB}$

Property 4.1.2.*



► $\angle BAC - \angle ACP = 30°$

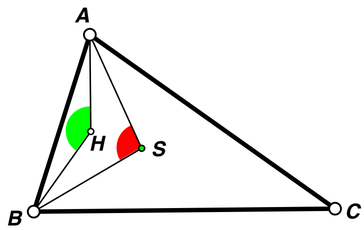
Property 4.1.3. (Distance Between Isodynamic Points) [25]



► $x = \dfrac{2\sqrt{3}abc}{\sqrt{(a^2+b^2+c^2)^2 - 48K^2}}$

4.2 S and one notable point

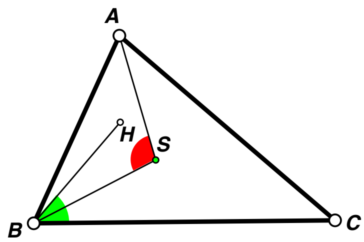
Property 4.2.1.*



S and H lie inside $\triangle ABC$.

► $\angle AHB + \angle ASB = 240°$

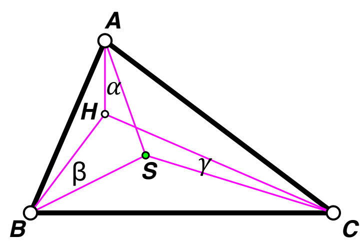
Property 4.2.2.*



S and H lie inside $\triangle ABC$.

► $\angle CBH + \angle ASB = 150°$

Property 4.2.3.*

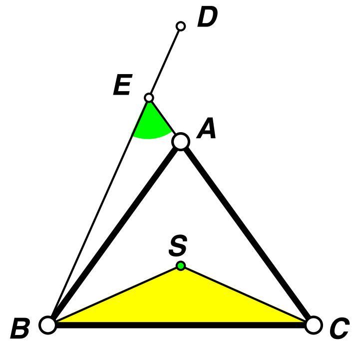


$\angle HAS = \alpha$, $\angle HBS = \beta$, and $\angle HCS = \gamma$

► Using directed angles,

$$\sin\alpha + \sin\beta + \sin\gamma = 0.$$

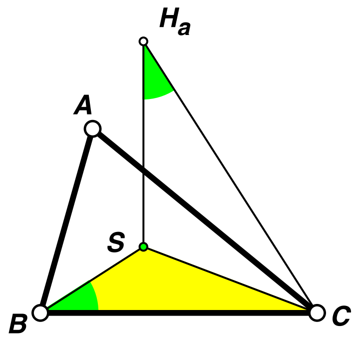
Property 4.2.4.*



D is the orthocenter of $\triangle BCS$.

▶ $\angle BEC = 60°$

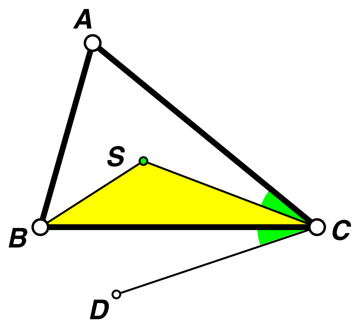
Property 4.2.5.*



$H_a = X_4(BCS)$

▶ $\angle CBS = \angle SH_aC$

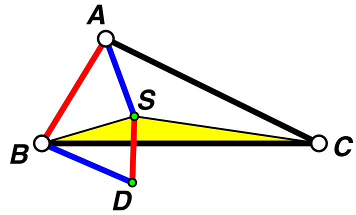
Property 4.2.6.*



$D = X_{14}(BCS)$

▶ $\angle BCD = \angle ACS$

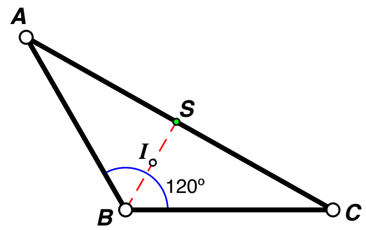
Property 4.2.7.*



$D = X_{15}(BCS)$

▶ $AB \cdot SD = AS \cdot BD$

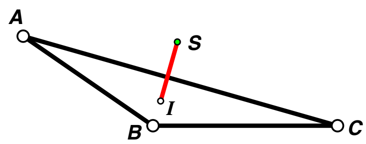
Property 4.2.8.*



▶ $B - I - S$

● This result follows from Property 3.5.2.

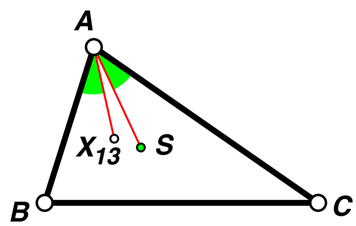
Property 4.2.9.* [77]



▶ $SI < \frac{1}{4}R$.

● The smallest k such that $SI < kR$ is $k \approx 0.2370406267$, where k is the positive root of $4x^6 + 36x^5 + 120x^4 + 288x^3 + 513x^2 - 72x - 16$.

Property 4.2.10. (Isogonal Conjugate) [84]

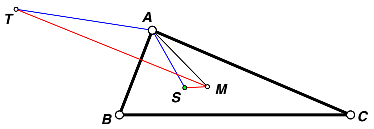


▶ $\angle BAX_{13} = \angle SAC$

● In other words, the isogonal conjugate of X_{15} is X_{13}. See [35, p. 296]. The isotomic conjugate of X_{15} is X_{300}. See[37].

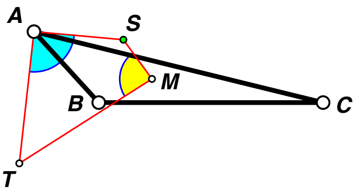
4.3 S and T and one notable point

Property 4.3.1. [25]



▶ $\left(\dfrac{SA}{TA}\right)^3 = \dfrac{SM}{TM}$

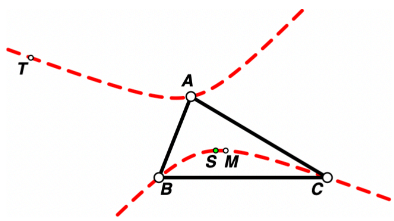
Property 4.3.2.* [75]



$x = \angle TMS$, $\alpha = \angle TAS$

▶ $x = 3\alpha \pmod{2\pi}$

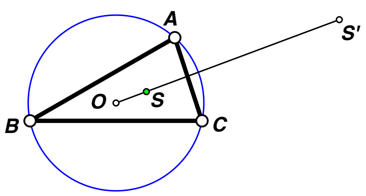
Property 4.3.3.* [74]



▶ A, B, C, S, T, M lie on a hyperbola.

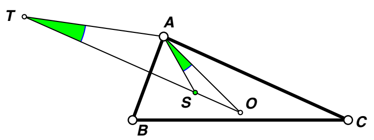
● The center of the hyperbola is X_{18334}.

Property 4.3.4. (Inverse Property) [35, p. 296]



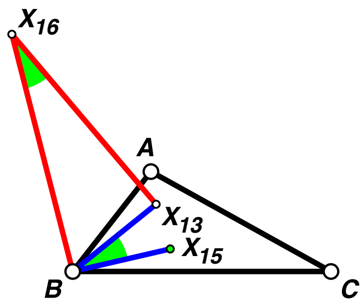
▶ $O - S - S'$
▶ $OS \cdot OS' = R^2$
● Inversion of $\triangle ABC$ with respect to an isodynamic point transforms $\triangle ABC$ into an equilateral triangle.

Property 4.3.5.*



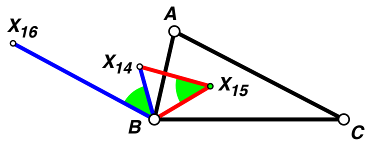
▶ $\triangle OSA \sim \triangle TAO$

Property 4.3.6.* [66]



▶ $\angle BX_{16}X_{13} = \angle X_{15}BX_{13}$

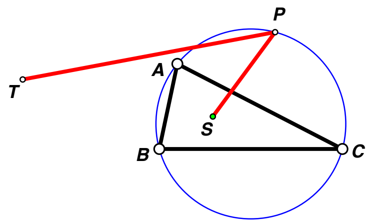
Property 4.3.7.* [67]



▶ $\angle X_{14}BX_{16} = \angle X_{14}X_{15}B$

4.4 S and T and a point P

Property 4.4.1. [25]

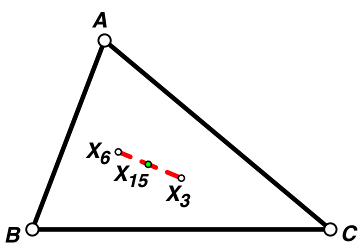


▶ PS/PT remains invariant as P moves along the circumcircle.

● The common ratio is MX_{13}/MX_{14}.

4.5 S and two notable points

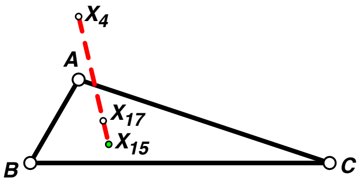
Property 4.5.1. [35, Art. 602]



▶ $X_3 - X_{15} - X_6$
▶ $(X_3X_{15})^2 \cdot X_3X_6 = R^2(X_3X_{15} - X_6X_{15})$.

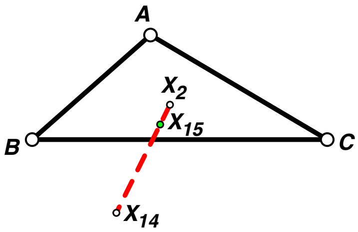
● See also [26, p. 106]. Equation was discovered by computer. Corollary: $X_3X_{15} > X_6X_{15}$.

Property 4.5.2. [21]



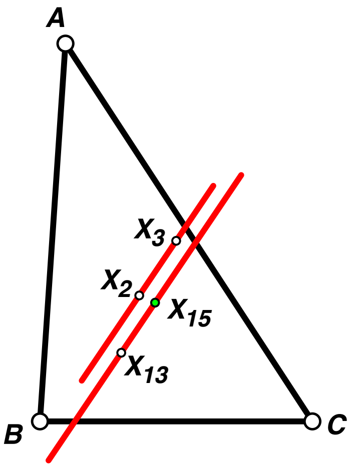
► $X_4 - X_{17} - X_{15}$

Property 4.5.3. [21]



► $X_2 - X_{15} - X_{14}$

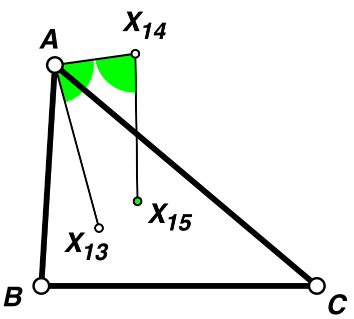
Property 4.5.4. [37]



► $X_{13}X_{15}||X_2X_3$

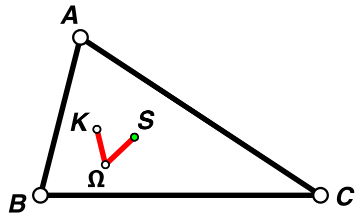
● In other words, $X_{13}X_{15}$ is parallel to the Euler line of $\triangle ABC$.

Property 4.5.5.* [68]



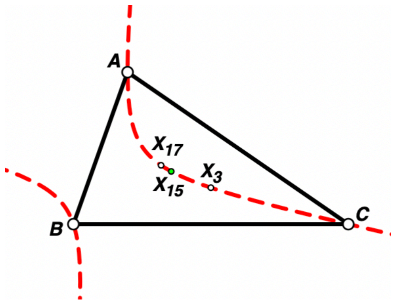
► $\angle X_{14}AX_{14} = \angle AX_{14}X_{15}$

Property 4.5.6.*



► $\angle S\Omega K = 60°$

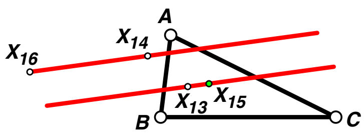
Property 4.5.7.*



► A, B, C, X_3, X_{15}, X_{17} lie on a conic.

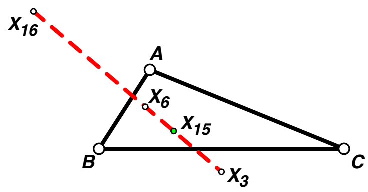
4.6 S and three notable points

Property 4.6.1.*



► $X_{13}X_{15}||X_{14}X_{16}$

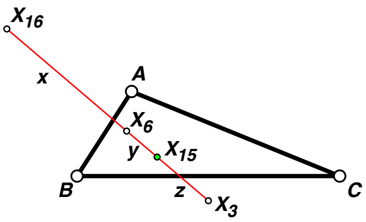
Property 4.6.2. (Brocard Axis) [84]



► $X_3 - X_{15} - X_6 - X_{16}$

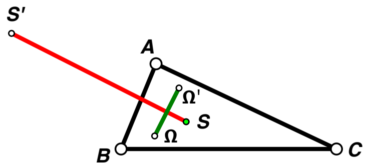
● A few other named points that lie on the Brocard Axis are the 3rd power point, the Brocard midpoint, the Kenmotu point, and the Taylor center. See [82].

Property 4.6.3. (Brocard Axis Metrics)
[84]



- ▶ $y(x+y+z)=xz$
- ▶ $z(x+y+z)=R^2$
- ▶ $\dfrac{1}{z}+\dfrac{1}{x+y+z}=\dfrac{2}{y+z}$
- ▶ $\dfrac{1}{x}+\dfrac{1}{x+y+z}=\dfrac{2}{x+y}$
- ▶ $(x+y)(y+z)=2xz$
- ● See also [26, p. 103] and [21].

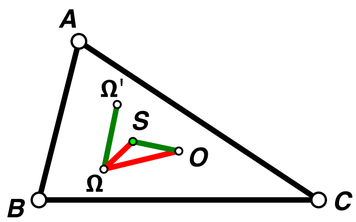
Property 4.6.4. [84]



▶ SS' is the perpendicular bisector of $\Omega\Omega'$.

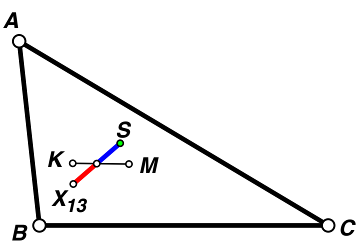
● The point of intersection of the two lines is X_{39}, the Brocard midpoint.

Property 4.6.5.*



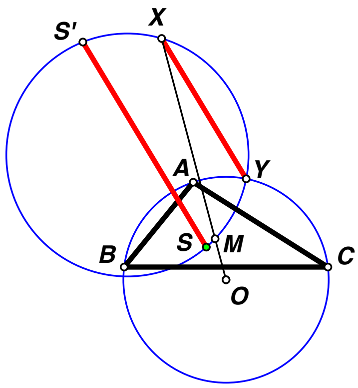
▶ $O\Omega \cdot S\Omega = OS \cdot \Omega\Omega'$

Property 4.6.6. [46]



▶ KM bisects SX_{13}

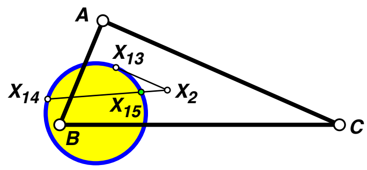
Property 4.6.7. [16]



▶ $SS' \parallel XY$

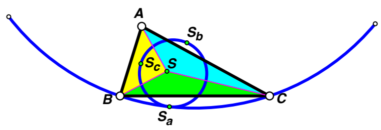
● $Y = X_{110}$.

Property 4.6.8.* [64]



▶ X_2X_{13} is tangent to $\odot X_{13}X_{14}X_{15}$

Property 4.6.9. [4]

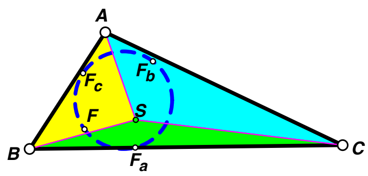


▶ $\odot S_aS_bS_c$ is tangent to $\odot SBC$.

● By symmetry, $\odot S_aS_bS_c$ is also tangent to $\odot SCA$ and $\odot SAB$. The center of $\odot S_aS_bS_c$ is X_{5238}.

4.7 S and 4 or more notable points

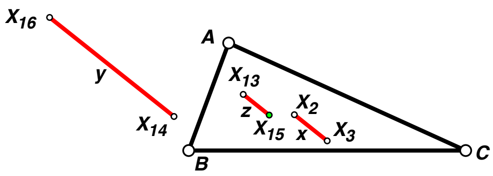
Property 4.7.1. [9]



▶ F_a, F_b, F_c, F concyclic.

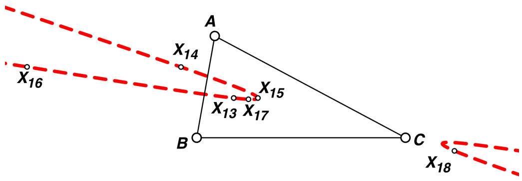
● The result also works for S'. See [30].

Property 4.7.2.*



► $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$

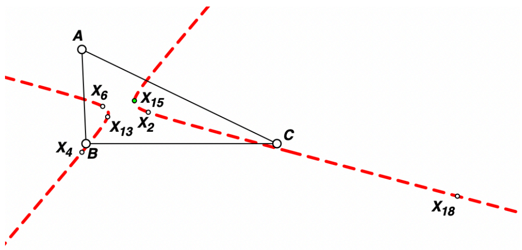
Property 4.7.3. (Evans Conic) [24]



► X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18} lie on a conic.

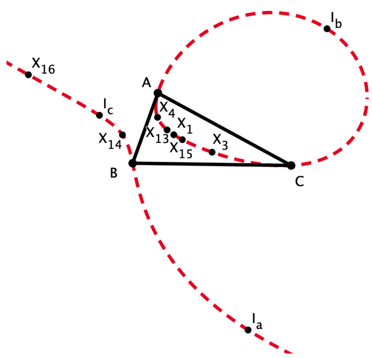
● See also [83].

Property 4.7.4.* [73]



► X_2, X_4, X_6, X_{13}, X_{15}, X_{18} lie on a conic.

Property 4.7.5. (Neuberg Cubic) [85]



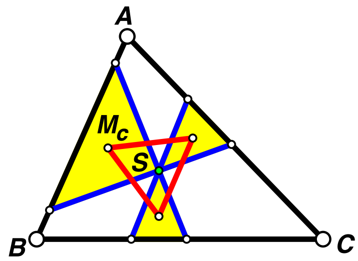
► The following ten points lie on a cubic curve: X_1, X_3, X_4, X_{13}, X_{14}, X_{15}, X_{16}, I_a, I_b, I_c, where I_a is the A-excenter of $\triangle ABC$.

● A few other points that lie on the Neuberg cubic are the reflections of the vertices of the triangle about their opposite sides and the six vertices of equilateral triangles erected on the sides of $\triangle ABC$. See [27].

5. Triangle plus lines through S

5.1 antiparallels

Property 5.1.1. [12]

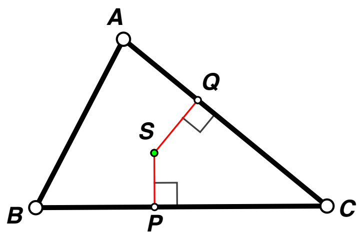


► $\triangle M_a M_b M_c$ is equilateral.

● See also [7].

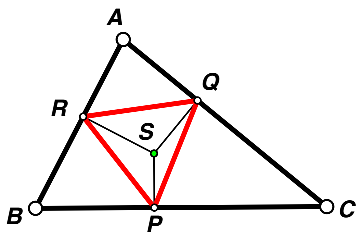
5.2 apothems

Property 5.2.1. (Trilinear Coordinates) [86]



► $\dfrac{SP}{SQ} = \dfrac{\sin(A+60°)}{\sin(B+60°)}$.

Property 5.2.2. (Pedal Triangle) [26, p. 106]



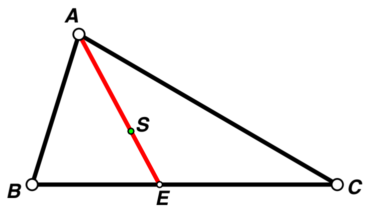
► $\triangle PQR$ is an equilateral triangle.

► $[PQR] = \dfrac{2K^2\sqrt{3}}{a^2+b^2+c^2+4K\sqrt{3}}$

● The center of the equilateral triangle is the midpoint of SX_{13}. See [43] for the area formula.

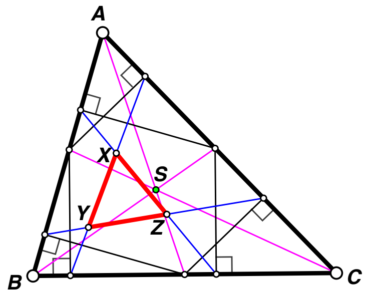
5.3 cevians

Property 5.3.1.* (Cevian Length)



▶ $AE = \dfrac{4\sqrt{2}bcK\sqrt{a^2+b^2+c^2+4K\sqrt{3}}}{(b^2+c^2)(4K+a^2\sqrt{3})-\sqrt{3}(b^2-c^2)^2}$

Property 5.3.2. [10]

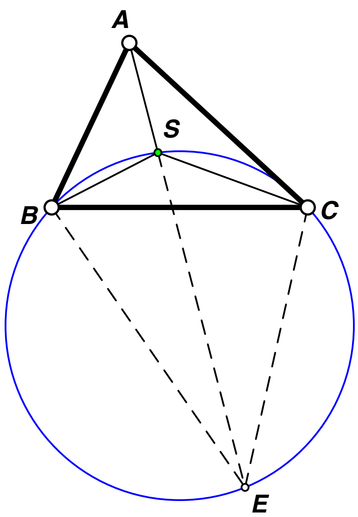


▶ $\triangle XYZ$ is equilateral.

- See also [3].

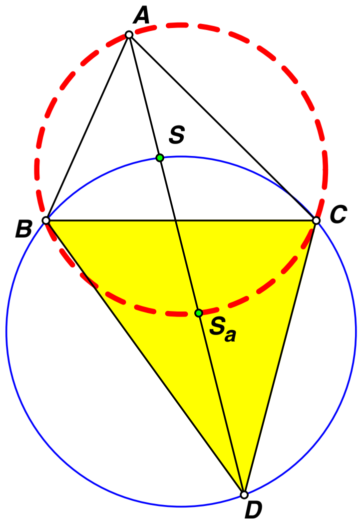
5.4 circlecevians

Property 5.4.1.* [49]



▶ $\angle EBA = \angle ACE = 120°$.

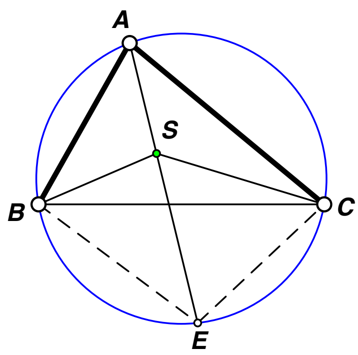
Property 5.4.2. [13]



▶ $A - S - S_a$

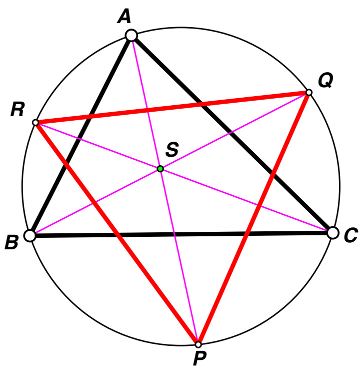
5.5 circumcevians

Property 5.5.1.* [50]



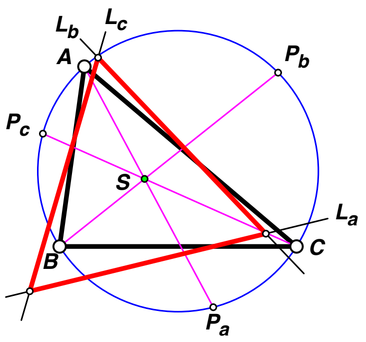
▶ $\angle EBS = \angle SCE = 60°$.

Property 5.5.2. [35, p. 296]



▶ $\triangle PQR$ is an equilateral triangle.

Property 5.5.3. [5]

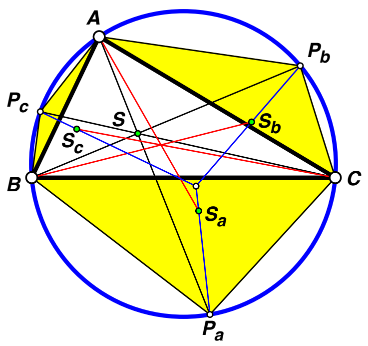


L_a is the Simson line of P_a.

▶ L_a, L_b, L_c bound an equilateral triangle.

● The center of the triangle is the nine-point center of $\triangle ABC$.

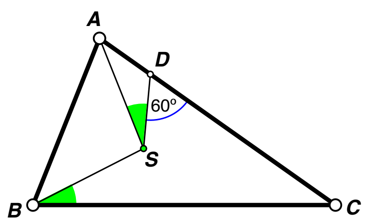
Property 5.5.4. [80, Thm. 8.1.1]



▶ AS_a, BS_b, CS_c concur.
▶ P_aS_a, P_bS_b, P_cS_c concur.

5.6 inclines

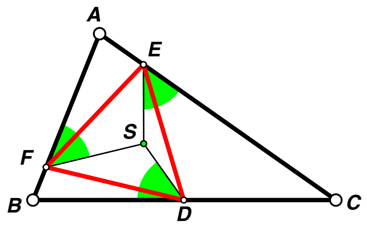
Property 5.6.1.*



▶ $\angle DSA = \angle CBS$.

● This follows from Property 1.3.1.

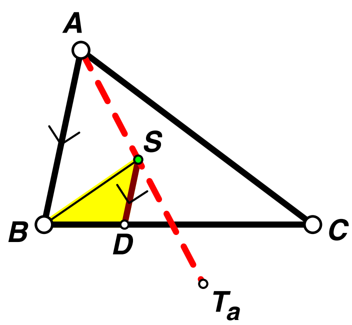
Property 5.6.2.*



▶ $\triangle DEF$ is equilateral.

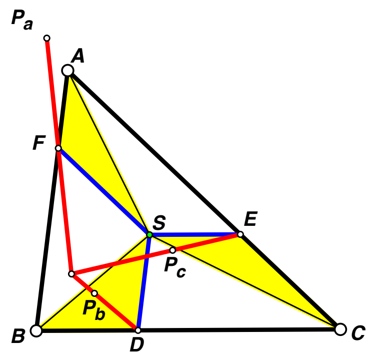
5.7 parachords

Property 5.7.1.* [62]



▶ $A - S - T_a$

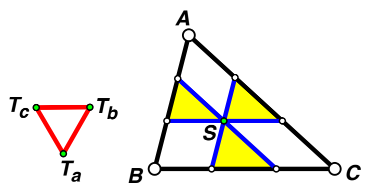
Property 5.7.2.* [72]



P_a is the 2nd Napoleon point of $\triangle ASF$.

▶ DP_b, EP_c, and FP_a are concurrent.

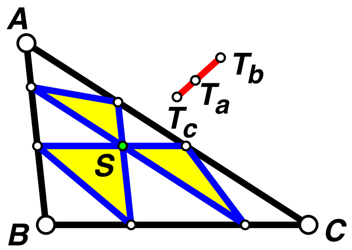
Property 5.7.3.* [58]



▶ $\triangle T_aT_bT_c$ is equilateral.

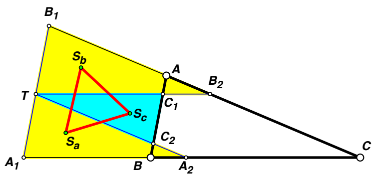
● The center of the equilateral triangle is X_{39555}.

Property 5.7.4.* [78]



▶ $T_c - T_a - T_b$.

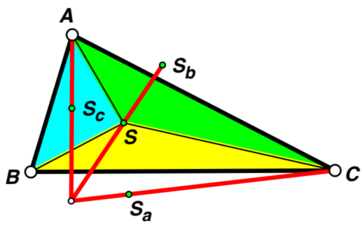
Property 5.7.5.* [61]



▶ $\triangle S_aS_bS_c$ is equilateral.

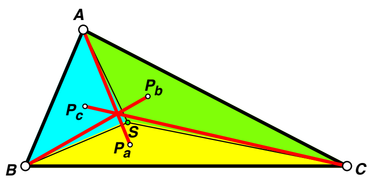
5.8 spokes

Property 5.8.1.* [70]



▶ AS_c, CS_a, and SS_b are concurrent.

Property 5.8.2.* [71]



P_a is the incenter of $\triangle SBC$.

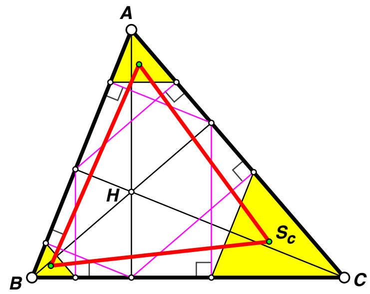
▶ AP_a, BP_b, and CP_c are concurrent.

● The result remains true if P_a is replaced by $X_n(SBC)$, for $n = 2, 3, 6, 13, 15, 31, 32, 36, 39,$ or 50.

6. Triangle with S in subtriangles

6.1 formed by H

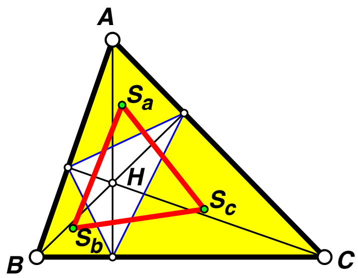
Property 6.1.1. [6]



▶ $\triangle S_aS_bS_c$ is equilateral.

● $\triangle T_aT_bT_c$ is equilateral and $\triangle T_aT_bT_c \cong \triangle S_aS_bS_c$.

Property 6.1.2. (3rd isodynamic-Dao triangle) [39]

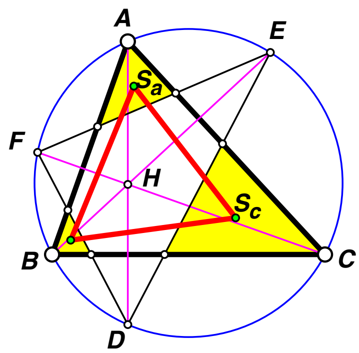


▶ $\triangle S_aS_bS_c$ is equilateral.

● Result is also true when using S' instead of S.

● See also [2].

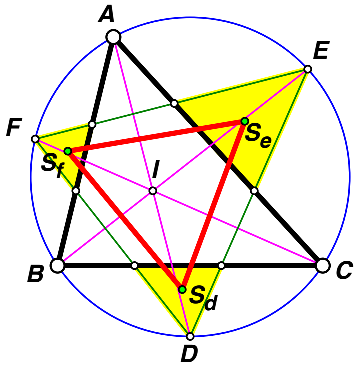
Property 6.1.3. [19]



▶ $\triangle S_aS_bS_c$ is equilateral.

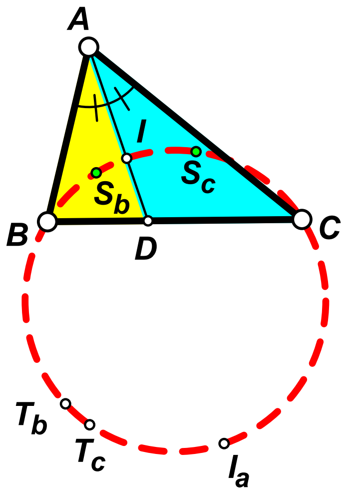
6.2 formed by I

Property 6.2.1. [19]



► $\triangle S_dS_eS_f$ is equilateral.

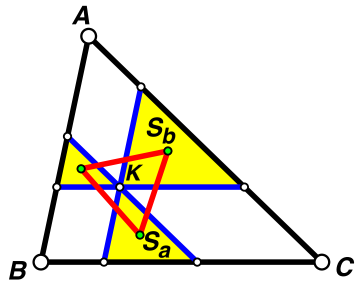
Property 6.2.2.* [54]



► B, C, S_b, S_c, T_b, T_c, I, I_a concyclic

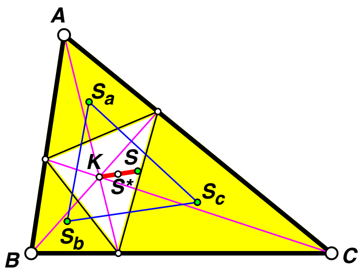
6.3 formed by K

Property 6.3.1. [2]



► $\triangle S_aS_bS_c$ is equilateral.

Property 6.3.2. [32]

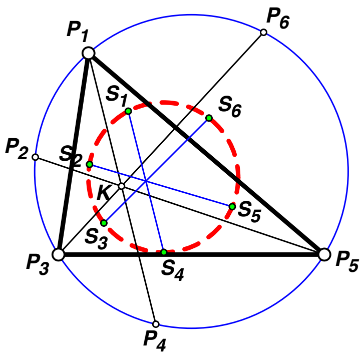


$S^* = S(S_a, S_b, S_c)$

► $K - S^* - S$.

● $S^* = X_{61}$.

Property 6.3.3. [1]



$K = K(P_1, P_3, P_5)$
$S_i = S(P_{i-1}P_iP_{i+1})$

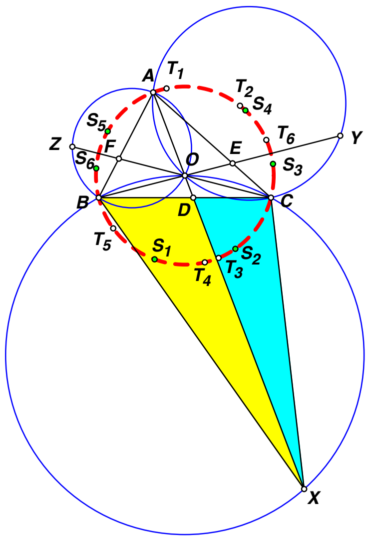
► S_1, S_2, S_3, S_4, S_5, S_6 are concyclic.

► P_1P_4, P_2P_5, P_3P_6 concur.

● Result is also true using S'.

6.4 formed by O

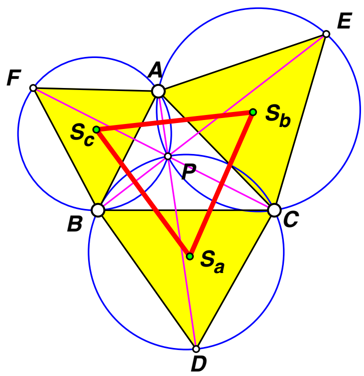
Property 6.4.1. [11]



▶ $\{S_1, S_2, \ldots S_6, T_1, T_2, \ldots T_6\}$ lie on $\odot ABC$.

6.5 formed by interior point P

Property 6.5.1. [8]

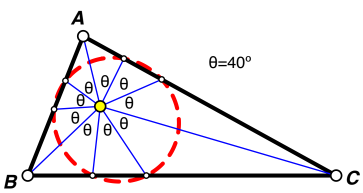


▶ $\triangle S_aS_bS_c$ is equilateral.

● $\triangle T_aT_bT_c$ is equilateral and $\triangle T_aT_bT_c \cong \triangle S_aS_bS_c$. See [22].

6.6 formed by X_{13}

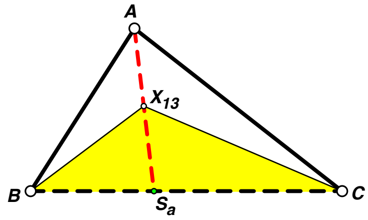
Property 6.6.1. [31]



Yellow point = X_{13}

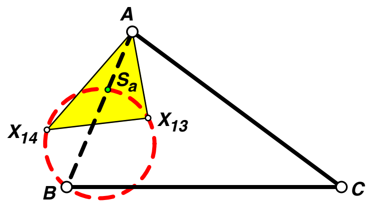
▶ S lies on the axis of an ellipse that passes through the six points on the perimeter of $\triangle ABC$.

Property 6.6.2. [80, Thm. 7.8.1]



▶ $A - X_{13} - S_a$
▶ $B - S_a - C$

Property 6.6.3. [80, Thm. 7.8.2]

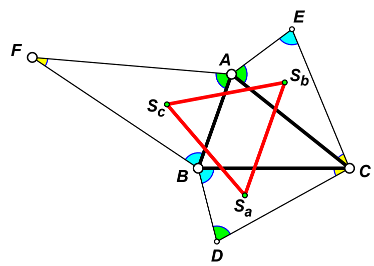


$\triangle AX_{14}X_{13}$ has same orientation as $\triangle ABC$

▶ X_{13}, S_a, X_{14}, B are concyclic.

6.7 formed by similar triangles

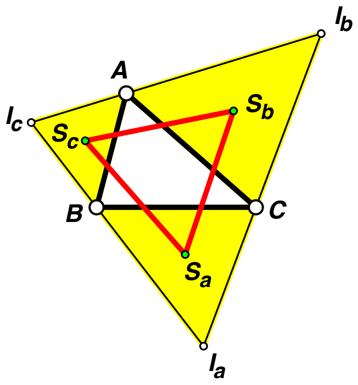
Property 6.7.1. [76]



▶ $S_aS_bS_c$ form an equilateral triangle.

6.8 formed by excenters

Property 6.8.1. [23]

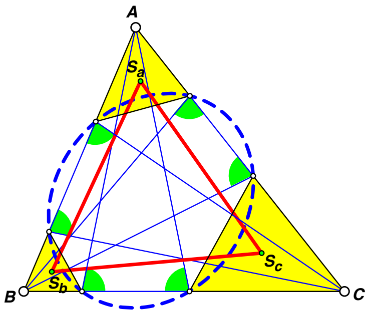


▶ $S_aS_bS_c$ form an equilateral triangle.

● Result is also true using S'.

6.9 formed by six cevians

Property 6.9.1. [20]



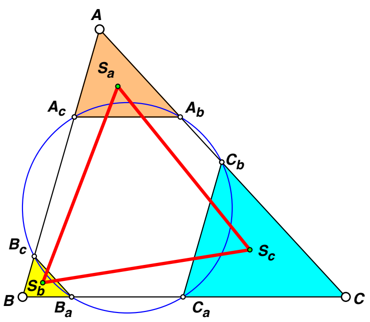
▶ $S_aS_bS_c$ is an equilateral triangle.

▶ Vertices of green angles lie on an ellipse.

● Result is also true using S'.

6.10 formed by a Tucker Hexagon

Property 6.10.1. (Tucker Hexagon) [18]



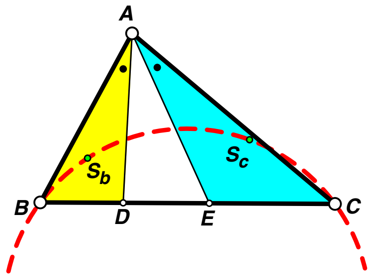
$C_aC_b \parallel AB$, $A_bA_c \parallel BC$, $B_cB_a \parallel CA$

▶ $\triangle S_aS_bS_c$ is equilateral.

● The result is true if S is replaced by S'. The two equilateral triangles are congruent.

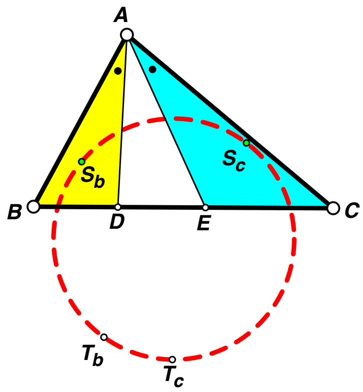
6.11 formed by isogonal cevians

Property 6.11.1. [33]



▶ B, C, S_b, S_c concyclic

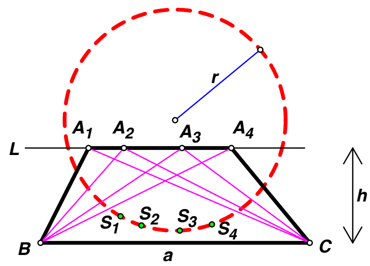
Property 6.11.2. [33]



▶ S_b, S_c, T_b, T_c concyclic

6.12 formed by varying vertex A

Property 6.12.1. [79]

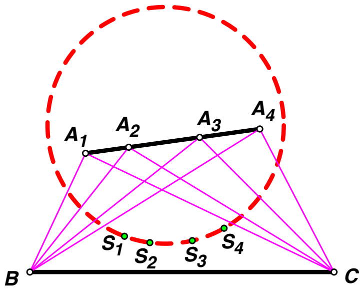


$L \parallel BC$, $S_i = S(A_iBC)$

▶ S_1, S_2, S_3, S_4 are concyclic.

▶ $r = \dfrac{a^2}{2h + a\sqrt{3}}$

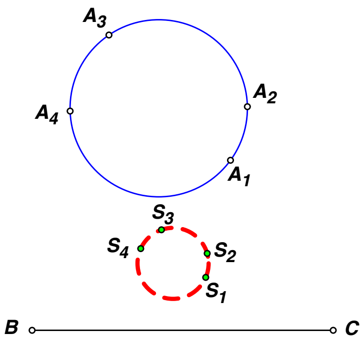
Property 6.12.2. [28]



$S_i = S(A_iBC)$

► S_1, S_2, S_3, S_4 are concyclic.

Property 6.12.3. [81]



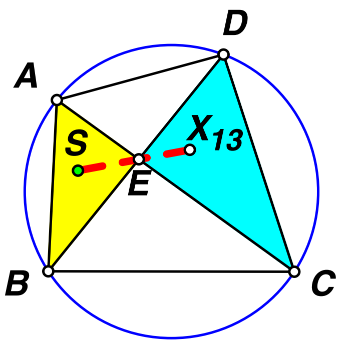
$S_i = S(A_iBC)$

► S_1, S_2, S_3, S_4 are concyclic.

7. Quadrilateral plus S

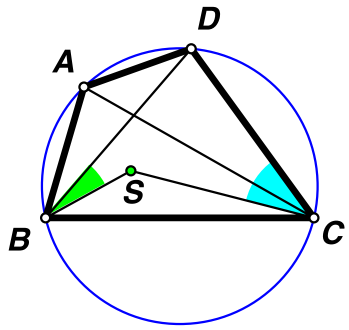
7.1 cyclic quadrilateral

Property 7.1.1. [17]



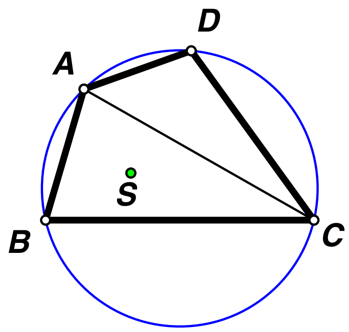
► $S - E - X_{13}$

Property 7.1.2.*



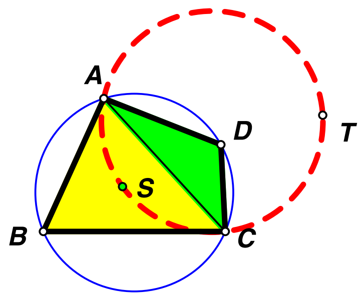
► $\angle SBD + \angle DCS = 60°$

Property 7.1.3.*



► $$\frac{1}{R(BDS)} + \frac{1}{R(CDS)} = \frac{1}{R(ADS)}$$

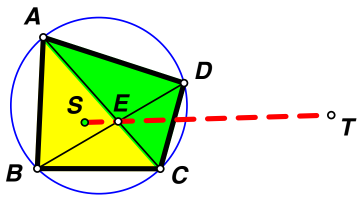
Property 7.1.4.*



$T = X_{16}(ACD)$

► A, S, C, T concyclic

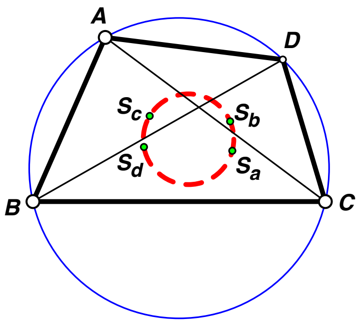
Property 7.1.5.*



$T = X_{16}(ACD)$

- ▶ $S - E - T$

Property 7.1.6. [2]

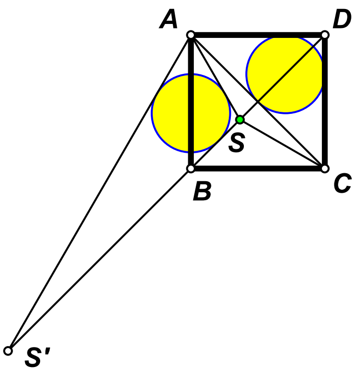


$S_a = S(BCD)$

- ▶ S_a, S_b, S_c, S_d are concyclic.

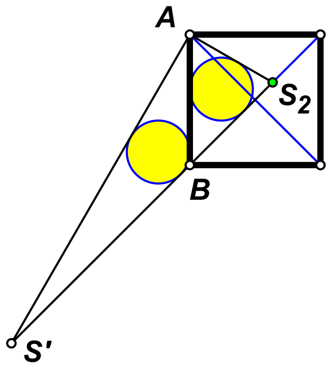
7.2 square

Property 7.2.1.* [52]



- ▶ Yellow incircles are congruent.

Property 7.2.2.* [53]

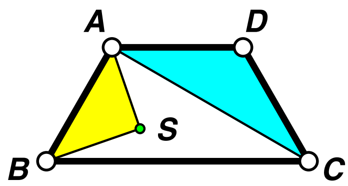


S_2 is the 1st isodynamic point of $\triangle ACD$.

- ▶ Yellow incircles are congruent.

7.3 trilateral trapezoid

Property 7.3.1.*

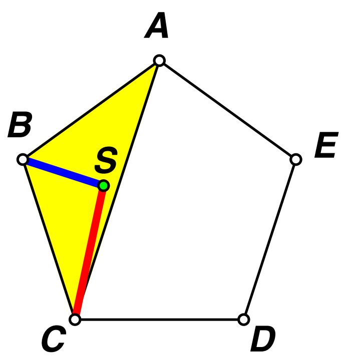


- ▶ $[ADC] = \frac{7}{4}[BAS]$
- ▶ $[CDS] = 2[BCS]$
- ▶ $[ACS] = 3[BDS]$
- ▶ $[ABC] = 7[BDS]$

8. Pentagon plus S

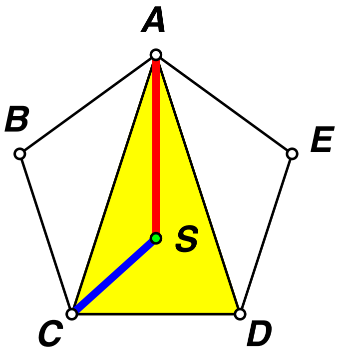
8.1 regular pentagon

Property 8.1.1. [41]



- ▶ $CS/BS = \phi$
- ▶ $SD/CS = \sqrt{2}$

Property 8.1.2. [41]

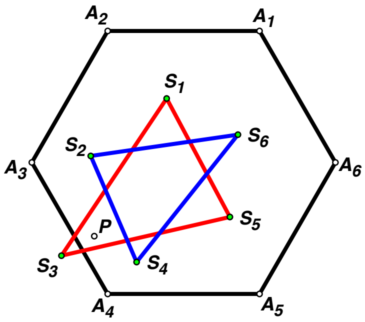


- ▶ $AS/CS = \phi$
- ▶ $SE/CS = \sqrt{2}$

9. Hexagon plus S

9.1 regular hexagon

Property 9.1.1. [63]



$P_i = S(PA_iA_{i+1})$

- ▶ $\triangle S_1S_3S_5 \sim \triangle S_4S_6S_2$

Other Properties

Other properties of the first isodynamic point (discovered by computer) were found by Dekov [21]. A typical result is: the first isodynamic point is the isogonal conjugate of the inner Fermat point of the anticevian triangle of the outer Fermat point.

Many properties of the first isodynamic point can be found in [37]. A typical result is: X_{15} is the isogonal conjugate of the isotomic conjugate of X_{298}. Also, lists are given for many of the lines, circles, conics, and cubics that X_{15} lies on.

Many properties of the isodynamic points can be found in the dissertation [44].

Many properties of the isodynamic points can be found in [80].

References

[1] Kadir Altıntaş, *Post in Plane Geometry Research Facebook group*. Nov. 6, 2019.
https://www.facebook.com/groups/2008519989391030/posts/2446896342220057/

[2] Kadir Altıntaş, *More on Isodynamic points*. Romantics of Geometry Facebook group. Apr. 3, 2020.
https://www.facebook.com/groups/parmenides52/posts/2873619306085090/

[3] Kadir Altıntaş, *Problem 8664*. Romantics of Geometry Facebook group. July 26, 2021.
https://www.facebook.com/groups/parmenides52/posts/4199802460133428/

[4] Kadir Altıntaş, *Problem 8673*. Romantics of Geometry Facebook group. July 27, 2021.
https://www.facebook.com/groups/parmenides52/posts/4202723889841285

[5] Kadir Altıntaş, *Problem 8754*. Romantics of Geometry Facebook group. Aug. 12, 2021.
https://www.facebook.com/groups/parmenides52/posts/4250823725031301/

[6] Kadir Altıntaş, *Problem 8762*. Romantics of Geometry Facebook group. Aug. 14, 2021.
https://www.facebook.com/groups/parmenides52/posts/4255249784588695/

[7] Kadir Altıntaş, *Problem 8828*. Romantics of Geometry Facebook group. Aug. 30, 2021.
https://www.facebook.com/groups/parmenides52/posts/4303979189715754/

[8] Kadir Altıntaş, *Problem 1574*. Geometri Günlüğü. Nov. 28, 2020.
https://geometry-diary.blogspot.com/2020/11/1574.html

[9] Kadir Altıntaş, *Problem 1625: A Circle Through Feuerbach Point*. Geometri Günlüğü. Jan. 8, 2021.
https://geometry-diary.blogspot.com/2021/01/1625-concyclic-points.html

[10] Kadir Altıntaş, *Problem 1684: P-Cevian Orthiac Triangle of Isodynamic Points*. Geometri Günlüğü. Apr. 8, 2021.
https://geometry-diary.blogspot.com/2021/04/1984-p-cevian-orthiac-triangle-of.html

[11] Kadir Altıntaş, *Problem 1696: Isodynamic Points on Circumcircle*. Geometri Günlüğü. May 10, 2021.
https://geometry-diary.blogspot.com/2021/05/1696-isodynamic-points-on-circumcircle.html

[12] Kadir Altıntaş, *Problem 1746: An Equilateral Triangle from Antiparallels*. Geometri Günlüğü. Aug. 29, 2021.
https://geometry-diary.blogspot.com/2021/08/1746-equilateral-triangle-from.html

[13] Elias M. Hagos, *Special case of Euclid message 1455: Concurrency On Circumcircle*. Feb. 15, 2021.
https://groups.io/g/euclid/message/1455

[14] Nathan Altshiller-Court, *College Geometry*, 2nd edition, Barnes & Noble, New York, 1952.
http://books.google.com/books?id=GeBUAAAAYAAJ

[15] Titu Andreescu, Dorin Andrica, and Catalin Barbu, *Problem O196*, Mathematical Reflections 7.3(2011).

[16] Jean-Louis-Ayme, *Post 5285 in ADGEOM: A Conjecture*, June 6, 2019.
http://adgeom.epizy.com/message.php?msg=5285

[17] Dao Thanh Oai, *Problem 8828*. Romantics of Geometry Facebook group. May 5, 2018.
https://www.facebook.com/groups/parmenides52/posts/1688095664637466/

[18] Dao Thanh Oai, *Problem 9047*. Romantics of Geometry Facebook group. Nov. 13, 2021.
https://www.facebook.com/groups/parmenides52/posts/4537733999673604/

[19] Dao Thanh Oai and Kadir Altıntaş, Nov. 14. 2021 post, Plane Geometry Research Facebook group.
https://www.facebook.com/groups/2008519989391030/posts/3027154977527521/

[20] Dao Thanh Oai, *Euclid Message 3140: Any triangle center $X(n)$ of three triangle form an equilateral triangle*. Nov. 13, 2021.
https://groups.io/g/euclid/topic/87040226

[21] Deko Dekov, *First Isodynamic Point*. Journal of Computer-Generated Euclidean Geometry, 3.1(2008)1–15.
http://www.ddekov.eu/j/2008/JCGEG200801.pdf

[22] Ngo Quang Duong, *Post 5334 in ADGEOM: Generalization of the Napoleon theorem*. July 18, 2019.
http://adgeom.epizy.com/message.php?msg=5334

[23] Ngo Quang Duong, *Post 5336 in ADGEOM: Generalization of the Napoleon theorem*. July 20, 2019.
http://adgeom.epizy.com/message.php?msg=5336

[24] Lawrence S. Evans, *A Conic Through Six Triangle Centers*. Forum Geometricorum. 2(2002)89–92.
https://forumgeom.fau.edu/FG2002volume2/FG200211.pdf

[25] H. E. Fettis, The Fermat and Hessian Points of a Triangle, The American Mathematical Monthly, 53(1946)74–78.
http://math.fau.edu/yiu/PSRM2015/yiu/New%20Folder%20(4)/Downloaded%20Papers/TriangleGeometry/AMMFermatandHessianpoints.pdf

[26] William Gallatly, *The Modern Geometry of the Triangle, 2nd ed.*. Hodgson, London: 1913.
http://www.archive.org/details/moderngeometryof00gallrich

[27] Bernard Gibert, *Cubics in the Triangle Plane: Neuberg Cubic.*
https://bernard-gibert.pagesperso-orange.fr/Exemples/k001.html

[28] Elias M. Hagos, *Comment on Euclid message 3253: Isodynamic Points Circles.* Nov. 19, 2021.
https://groups.io/g/euclid/message/3072

[29] Andreas Hatzipolakis, *Euclid message 3072: An Ellipse.* Nov. 10, 2021.
https://groups.io/g/euclid/message/3072

[30] Tran Quang Hung, *Problem 3673.* Romantics of Geometry Facebook group. Oct. 8, 2019.
https://www.facebook.com/groups/parmenides52/posts/2489301777850180/

[31] Tran Quang Hung, *Post 4750 in ADGEOM: Six points are on Ellipse* June 23, 2018.
http://adgeom.epizy.com/message.php?msg=4750

[32] Tran Quang Hung, *Post 4871 in ADGEOM: 1st Isodynamic and Symmedian point* Aug. 13, 2018.
http://adgeom.epizy.com/message.php?msg=4871

[33] Tran Quang Hung, *Problem 6830.* Romantics of Geometry Facebook group. Nov. 30, 2020.
https://www.facebook.com/groups/parmenides52/posts/3531488570298157/

[34] Er Jkh, pesonal correspondence. Nov. 20, 2021.

[35] Roger Arthur Johnson, *Modern Geometry: An Elementary Treatise on the Geometry of the Triangle and the Circle.* Houghton Miflin, Boston: 1929.
http://books.google.com/books?id=KVdtAAAAMAAJ

[36] Clark Kimberling, *Encyclopedia of Triangle Centers.*
https://faculty.evansville.edu/ck6/encyclopedia/ETC.html

[37] Clark Kimberling, *Encyclopedia of Triangle Centers, entry for X(15), The 1st Isodynamic Point.*
https://faculty.evansville.edu/ck6/encyclopedia/ETC.html#X15

[38] Clark Kimberling, *Encyclopedia of Triangle Centers, entry for X(16802).*
https://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16802

[39] Clark Kimberling, *Encyclopedia of Triangle Centers, Preamble to X(31683), 3rd and 4th isodynamic-Dao triangles and related centers.*
https://faculty.evansville.edu/ck6/encyclopedia/ETCPart16.html#X31683

[40] Seiichi Kirikami, *Problem 8163.* Romantics of Geometry Facebook group. May 12, 2021.
https://www.facebook.com/groups/parmenides52/permalink/3983853791728297/

[41] Seiichi Kirikami, *Problem 8232.* Romantics of Geometry Facebook group. May 27, 2021.
https://www.facebook.com/groups/parmenides52/permalink/4029777707135905/

[42] Konstantin Knop, *Problem 4850.* Romantics of Geometry Facebook group. March 28, 2020.
https://www.facebook.com/groups/parmenides52/permalink/2860382184075469/

[43] Trịnh Xuân Minh, *Problem 3786.* Romantics of Geometry Facebook group. Oct. 26, 2019.
https://www.facebook.com/groups/parmenides52/permalink/2526137244166633/

[44] M. Juliana Moll, *The isodynamic points of a triangle.* Dissertation, Catholic University of America, 1942.

[45] Tarik Adnan Moon, *The Apollonian Circles and Isodynamic Points*, Mathematical Reflections 6(2010)1–14.
https://www.academia.edu/2044876/The_Apollonian_Circles_and_Isodynamic_Points

[46] nassarawa, *Post 5310 in ADGEOM: A point on GK.* June 27, 2019.
http://adgeom.epizy.com/message.php?msg=5310

[47] Stanley Rabinowitz, *Problem 4846.* Romantics of Geometry Facebook group. March 28, 2020.
https://www.facebook.com/groups/parmenides52/posts/2860279000752454

[48] Stanley Rabinowitz, *Problem 4847.* Romantics of Geometry Facebook group. March 28, 2020.
https://www.facebook.com/groups/parmenides52/posts/2860312284082459

[49] Stanley Rabinowitz, *Problem 4848.* Romantics of Geometry Facebook group. March 28, 2020.
https://www.facebook.com/groups/parmenides52/posts/2860334940746860/

[50] Stanley Rabinowitz, *Problem 4849.* Romantics of Geometry Facebook group. March 28, 2020.
https://www.facebook.com/groups/parmenides52/posts/2860325794081108/

[51] Stanley Rabinowitz, *Problem 4904.* Romantics of Geometry Facebook group. April 3, 2020.
https://www.facebook.com/groups/parmenides52/posts/2873343289446025/

[52] Stanley Rabinowitz, *Problem 6526.* Romantics of Geometry Facebook group. Oct. 22, 2020.
https://www.facebook.com/groups/parmenides52/posts/3424463207667361/

[53] Stanley Rabinowitz, *Problem 6527.* Romantics of Geometry Facebook group. Oct. 22, 2020.
https://www.facebook.com/groups/parmenides52/posts/3424464207667261/

[54] Stanley Rabinowitz, *Problem 6825.* Romantics of Geometry Facebook group. Nov. 28, 2020.
https://www.facebook.com/groups/parmenides52/posts/3525298000917214/

[55] Stanley Rabinowitz, *Problem 7294.* Romantics of Geometry Facebook group. April 3, 2020.
https://www.facebook.com/groups/parmenides52/posts/3700750573371955/

[56] Stanley Rabinowitz, *Problem 8847.* Romantics of Geometry Facebook group. Sept. 2, 2021.
https://www.facebook.com/groups/parmenides52/posts/4313141355466204/

[57] Stanley Rabinowitz, *Problem 8848.* Romantics of Geometry Facebook group. Sept. 2, 2021.
https://www.facebook.com/groups/parmenides52/posts/4313140115466328/

[58] Stanley Rabinowitz, *Problem 8850.* Romantics of Geometry Facebook group. Sept. 2, 2021.
https://www.facebook.com/groups/parmenides52/posts/4313422142104792/

[59] Stanley Rabinowitz, *Problem 9037.* Romantics of Geometry Facebook group. Nov. 10, 2021.
https://www.facebook.com/groups/parmenides52/posts/4528342423946095/

[60] Stanley Rabinowitz, *Problem 9044.* Romantics of Geometry Facebook group. Nov. 12, 2021.
https://www.facebook.com/groups/parmenides52/posts/4536317349815269/

[61] Stanley Rabinowitz, *Problem 9063.* Romantics of Geometry Facebook group. Nov. 16, 2021.
https://www.facebook.com/groups/parmenides52/posts/4547828058664198/

[62] Stanley Rabinowitz, *Problem 9075*. Romantics of Geometry Facebook group. Nov. 17, 2021.
https://www.facebook.com/groups/parmenides52/posts/4551981188248885/

[63] Stanley Rabinowitz, *Problem 9077*. Romantics of Geometry Facebook group. Nov. 19, 2021.
https://www.facebook.com/groups/parmenides52/posts/4557280784385592/

[64] Stanley Rabinowitz, *Problem 9091*. Romantics of Geometry Facebook group. Nov. 24, 2021.
https://www.facebook.com/groups/parmenides52/posts/4573160769464260/

[65] Stanley Rabinowitz, *Problem 9107*. Romantics of Geometry Facebook group. Nov. 25, 2021.
https://www.facebook.com/groups/parmenides52/posts/4577174032396267/

[66] Stanley Rabinowitz, *Problem 9101*. Romantics of Geometry Facebook group. Nov. 27, 2021.
https://www.facebook.com/groups/parmenides52/posts/4584246715022332/

[67] Stanley Rabinowitz, *Problem 9102*. Romantics of Geometry Facebook group. Nov. 27, 2021.
https://www.facebook.com/groups/parmenides52/posts/4584268345020169/

[68] Stanley Rabinowitz, *Problem 9103*. Romantics of Geometry Facebook group. Nov. 27, 2021.
https://www.facebook.com/groups/parmenides52/posts/4584348048345532/

[69] Stanley Rabinowitz, *Problem SR40*, Plane Geometry Research Facebook group. Nov. 9, 2021.
https://www.facebook.com/groups/2008519989391030/posts/3023672237875795/

[70] Stanley Rabinowitz, *Problem SR41*, Plane Geometry Research Facebook group. Nov. 17, 2021.
https://www.facebook.com/groups/2008519989391030/posts/3029562903953395/

[71] Stanley Rabinowitz, *Problem SR42*, Plane Geometry Research Facebook group. Nov. 17, 2021.
https://www.facebook.com/groups/2008519989391030/posts/3029553853954300/

[72] Stanley Rabinowitz, *Problem SR43*, Plane Geometry Research Facebook group. Nov. 17, 2021.
https://www.facebook.com/groups/2008519989391030/posts/3029706590605693/

[73] Stanley Rabinowitz, *Euclid message 3337: A new conic?*. Nov. 23, 2021.
https://groups.io/g/euclid/message/3337

[74] Stanley Rabinowitz, *Euclid message 3356: Another new conic?*. Nov. 27, 2021.
https://groups.io/g/euclid/message/3356

[75] Stanley Rabinowitz, *Euclid message 3368: Triple angle property*. Nov. 28, 2021.
https://groups.io/g/euclid/message/3368

[76] Stanley Rabinowitz and Ercole Suppa, *Equilateral Triangles formed by the Centers of Erected Triangles*, International Journal of Computer Discovered Mathematics. 6(2021)43–67.
http://www.journal-1.eu/2021/Stanley%20Rabinowitz.%20Equilateral%20Triangles%20formed%20by%20the%20Centers%20of%20Erected%20Triangles,%20pp.%2043-67.pdf

[77] Stanley Rabinowitz, Location of Triangle Centers Relative to the Incircle and Circumcircle, International Journal of Computer Discovered Mathematics, in preparation.

[78] Kousik Sett, *Problem 9064*. Romantics of Geometry Facebook group. Nov. 16, 2021.
https://www.facebook.com/groups/parmenides52/posts/4547541495359521/

[79] Kousik Sett, *Problem 9079*. Romantics of Geometry Facebook group. Nov. 18, 2021.
https://www.facebook.com/groups/parmenides52/posts/4555892671191070/

[80] Alexander Skutin, Tran Quang Hung, Antreas Hatzipolakis, and Kadir Altıntaş. *Cosmology of Plane Geometry*. arXiv preprint arXiv:1908.05084 (2021).
https://arxiv.org/abs/1908.05084

[81] Vu Thanh Tung, *Comment on Euclid message 3253: Isodynamic Points Circles*. Nov. 20, 2021.
https://groups.io/g/euclid/message/3072

[82] Eric W. Weisstein, *Brocard Axis*, From MathWorld–A Wolfram Web Resource.
https://mathworld.wolfram.com/BrocardAxis.html

[83] Eric W. Weisstein, *Evans Conic*, From MathWorld–A Wolfram Web Resource.
https://mathworld.wolfram.com/EvansConic.html

[84] Eric W. Weisstein, *Isodynamic Points*, From MathWorld–A Wolfram Web Resource.
https://mathworld.wolfram.com/IsodynamicPoints.html

[85] Eric W. Weisstein, *Neuberg Cubic*, From MathWorld–A Wolfram Web Resource.
https://mathworld.wolfram.com/NeubergCubic.html

[86] Wikipedia contributors, *Isodynamic point*, Wikipedia, The Free Encyclopedia. Retrieved October 30, 2021.
https://en.wikipedia.org/w/index.php?title=Isodynamic_point&oldid=1013993110