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Three Concurrent Circles in Reference to Three Nonconcurrent Cevians

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1. Introduction

There are a handful of well known results regarding concurrent cevians and nonconcurrent cevians, such as Ceva's theorem [1]. Here, we will show that three certain circles related to nonconcurrent cevians are concurrent.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be a point on line AB.
Let E be a point on line AC.
Let F be a point on line BC.
Let lines AF and BE intersect at point G.
Let lines AF and CD intersect at point H.
Let lines BE and CD intersect at point I.
Then circles ABG, ACH, and BCI are concurrent.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
D has coordinates $(x_D, 0)$.
E has coordinates $(x_E, \frac{x_E y_C}{x_C})$.

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F has coordinates $(x_F, (x_F - 1)(\frac{y_C}{x_C - 1}))$.
G has coordinates
$(-(x_E * x_F * (x_C - 1))/(x_C - x_C * x_E - x_C * x_F + x_E * x_F), -(x_E * y_C * (x_F - 1))/(x_C - x_C * x_E - x_C * x_F + x_E * x_F))$.
H has coordinates
$(((x_C - 1) * x_D * x_F)/((x_D - 1) * x_F - x_D + x_C), (x_D * y_C * (x_F - 1))/(x_C - x_D - x_F + x_D * x_F))$.
I has coordinates
$((x_C * x_D + x_C * x_E - x_D * x_E - x_C * x_D * x_E)/(x_C - x_D * x_E), (x_E * y_C - x_D * x_E * y_C)/(x_C - x_D * x_E))$.
Circle ABG is given by the equation
$(x - 1/2)^2 + (y - (-((x_C - 1) * x_F * (1/2 + ((x_C - 1) * x_E * x_F)/(2 * (x_E * x_F - x_C * x_F - x_C * x_E + x_C))))/((x_F - 1) * y_C) - (x_E * (x_F - 1) * y_C)/(2 * (x_E * x_F - x_C * x_F - x_C * x_E + x_C))))^2 - ((1/2)^2 + ((-((x_C - 1) * x_F * (1/2 + ((x_C - 1) * x_E * x_F)/(2 * (x_E * x_F - x_C * x_F - x_C * x_E + x_C))))/((x_F - 1) * y_C) - (x_E * (x_F - 1) * y_C)/(2 * (x_E * x_F - x_C * x_F - x_C * x_E + x_C))))^2) = 0$.
Circle ACH is given by the equation
$(x-(-((x_F-1)*y_C^2+((1-2*x_C)*x_D+x_C^2)*x_F+x_C*x_D-x_C^2)/(2*((x_D-1)*x_F-x_D+x_C))))^2+(y-((((x_D+x_C-1)*x_F-x_D)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_F)/(2*((x_D-1)*x_F-x_D+x_C)*y_C)))^2-(((-((x_F-1)*y_C^2+((1-2*x_C)*x_D+x_C^2)*x_F+x_C*x_D-x_C^2)/(2*((x_D-1)*x_F-x_D+x_C))))^2+(((((x_D+x_C-1)*x_F-x_D)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_F)/(2*((x_D-1)*x_F-x_D+x_C)*y_C)))^2) = 0$.
Circle BCI is given by the equation
$(x-(-(x_E*y_C^2+((-2*x_C-1)*x_D+x_C^2)*x_E+x_C*x_D+x_C)/(2*(x_D*x_E-x_C))))^2+(y-((((x_D+x_C-1)*x_E-x_C)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_E+(x_C-1)*x_C*x_D-x_C^3+x_C^2)/(2*(x_D*x_E-x_C)*y_C)))^2-((1-(-(x_E*y_C^2+((-2*x_C-1)*x_D+x_C^2)*x_E+x_C*x_D+x_C)/(2*(x_D*x_E-x_C))))^2+(((((x_D+x_C-1)*x_E-x_C)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_E+(x_C-1)*x_C*x_D-x_C^3+x_C^2)/(2*(x_D*x_E-x_C)*y_C)))^2) = 0$.
Consider the point J whose coordinates are given by
$x_J = -((x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((((2*x_D+x_C-2)*x_E-x_C*x_D-x_C^2+x_C)*x_F^2-(((x_C+3)*x_D+x_C^2-2*x_C-1)*x_E-x_C*(2*x_D+x_C-1))*x_F+((x_C+1)*x_D-x_C)*x_E-x_C*x_D)*y_C^2+(x_C-1)^2*x_C*(x_E+x_D-x_C-1)*x_F^2+(x_C-1)^2*x_C*(x_D-x_C)*(x_E-1)*x_F))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)$
and
$y_J = (-((x_E-x_C)*x_F-x_C*x_E+x_C)*y_C*(x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((x_F-1)*y_C^2-(x_C-1)*(2*x_D-x_C-1)*x_F+(x_C-1)*x_D-x_C^2+x_C))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*$

$x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4).$

We verify that J lies on all three circles since we calculate that

$(-((x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((((2*x_D+x_C-2)*x_E-x_C*x_D-x_C^2+x_C)*x_F^2-(((x_C+3)*x_D+x_C^2-2*x_C-1)*x_E-x_C*(2*x_D+x_C-1))*x_F+((x_C+1)*x_D-x_C)*x_E-x_C*x_D)*y_C^2+(x_C-1)^2*x_C*(x_E+x_D-x_C-1)*x_F^2+(x_C-1)^2*x_C*(x_D-x_C)*(x_E-1)*x_F))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)-1/2)^2+((-((x_E-x_C)*x_F-x_C*x_E+x_C)*y_C*(x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((x_F-1)*y_C^2-(x_C-1)*(2*x_D-x_C-1)*x_F+(x_C-1)*x_D-x_C^2+x_C))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)*(-((x_C-1)*x_F*(1/2+((x_C-1)*x_E*x_F)/(2*(x_E*x_F-x_C*x_F-x_C*x_E+x_C))))/((x_F-1)*y_C)-(x_E*(x_F-1)*y_C)/(2*(x_E*x_F-x_C*x_F-x_C*x_E+x_C))))^2-((1/2)^2+((-((x_C-1)*x_F*(1/2+((x_C-1)*x_E*x_F)/(2*(x_E*x_F-x_C*x_F-x_C*x_E+x_C))))/((x_F-1)*y_C)-(x_E*(x_F-1)*y_C)/(2*(x_E*x_F-x_C*x_F-x_C*x_E+x_C))))^2)=0,$

$(-((x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((((2*x_D+x_C-2)*x_E-x_C*x_D-x_C^2+x_C)*x_F^2-(((x_C+3)*x_D+x_C^2-2*x_C-1)*x_E-x_C*(2*x_D+x_C-1))*x_F+((x_C+1)*x_D-x_C)*x_E-x_C*x_D)*y_C^2+(x_C-1)^2*x_C*(x_E+x_D-x_C-1)*x_F^2+(x_C-1)^2*x_C*(x_D-x_C)*(x_E-1)*x_F))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)-$

$$(-((x_F-1)*y_C^2+((1-2*x_C)*x_D+x_C^2)*x_F+x_C*x_D-x_C^2)/(2*((x_D-1)*x_F-x_D+x_C))))^2+((-((x_E-x_C)*x_F-x_C*x_E+x_C)*y_C*(x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((x_F-1)*y_C^2-(x_C-1)*(2*x_D-x_C-1)*x_F+(x_C-1)*x_D-x_C^2+x_C))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)-((((x_D+x_C-1)*x_F-x_D)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_F)/(2*((x_D-1)*x_F-x_D+x_C)*y_C)))^2-(((-((x_F-1)*y_C^2+((1-2*x_C)*x_D+x_C^2)*x_F+x_C*x_D-x_C^2)/(2*((x_D-1)*x_F-x_D+x_C))))^2+(((((x_D+x_C-1)*x_F-x_D)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_F)/(2*((x_D-1)*x_F-x_D+x_C)*y_C)))^2)=0,$$

and

$$(-((x_E*y_C^2-x_C*(2*x_D-x_C)*x_E+x_C*x_D)*((((2*x_D+x_C-2)*x_E-x_C*x_D-x_C^2+x_C)*x_F^2-(((x_C+3)*x_D+x_C^2-2*x_C-1)*x_E-x_C*(2*x_D+x_C-1))*x_F+((x_C+1)*x_D-x_C)*x_E-x_C*x_D)*y_C^2+(x_C-1)^2*x_C*(x_E+x_D-x_C-1)*x_F^2+(x_C-1)^2*x_C*(x_D-x_C)*(x_E-1)*x_F))/((x_E*x_F-x_C*x_F-x_C*x_E+x_C)^2*y_C^4+((((2*x_D-1)^2+2*(x_C-1)*x_C)*x_E^2-2*x_C*(2*x_D^2-x_C*x_D+2*x_C^2-x_C-1)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^2)*x_F^2-2*(2*x_C*x_D^2*x_E^2+2*x_D^2*x_E^2-x_C^2*x_D*x_E^2-2*x_C*x_D*x_E^2-x_D*x_E^2+2*x_C^3*x_E^2-2*x_C^2*x_E^2+x_C*x_E^2-x_C^2*x_D^2*x_E-3*x_C*x_D^2*x_E+2*x_C^3*x_D*x_E-2*x_C^4*x_E-x_C^3*x_E+3*x_C^2*x_E+x_C^2*x_D^2-2*x_C^3*x_D+2*x_C^2*x_D+2*x_C^4-x_C^3-x_C^2)*x_F+((x_C+1)^2*x_D^2-2*x_C*(x_C^2+1)*x_D+2*x_C^4-2*x_C^3+x_C^2)*x_E^2-2*x_C*(x_C*x_D^2+x_D^2-2*x_C^2*x_D+x_C*x_D+2*x_C^3-2*x_C^2)*x_E+x_C^2*x_D^2-2*(x_C-1)*x_C^2*x_D+2*x_C^4-2*x_C^3)*y_C^2+(x_C-1)^2*x_C^2*(x_E+x_D-x_C-1)^2*x_F^2+2*(x_C-1)^2*x_C^2*(x_D-x_C)*(x_E-1)*(x_E+x_D-x_C-1)*x_F+(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E^2-2*(x_C-1)^2*x_C^2*(x_D-x_C)^2*x_E+(x_C-1)^2*x_C^2*x_D^2-2*(x_C-1)^2*x_C^3*x_D+x_C^6-2*x_C^5+x_C^4)-((((x_D+x_C-1)*x_E-x_C)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_E+(x_C-1)*x_C*x_D-x_C^3+x_C^2)/(2*(x_D*x_E-x_C)*y_C)))^2-((1-(-(x_E*y_C^2+((-2*x_C-1)*x_D+x_C^2)*x_E+x_C*x_D+x_C)/(2*(x_D*x_E-x_C))))^2+(((((x_D+x_C-1)*x_E-x_C)*y_C^2-(x_C-1)*x_C*(x_D-x_C)*x_E+(x_C-1)*x_C*x_D-x_C^3+x_C^2)/(2*(x_D*x_E-x_C)*y_C)))^2)=0$$

as desired. □

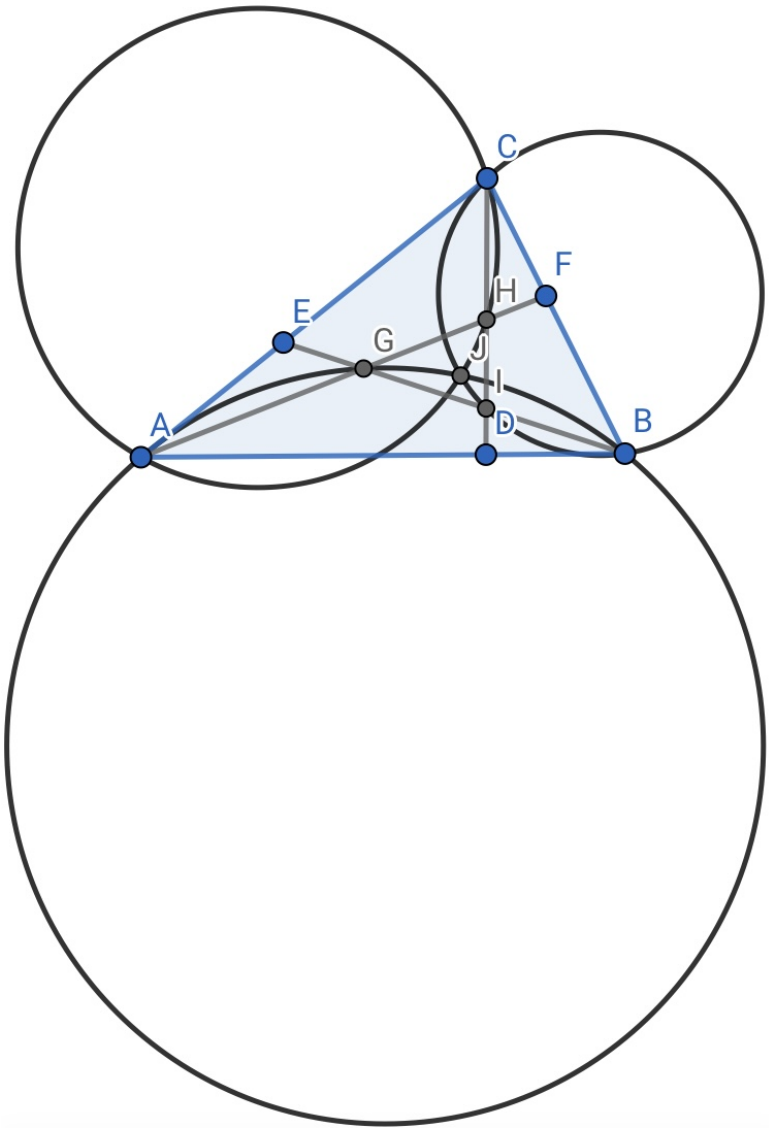

Figure 1. Illustration.

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