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Three Circles That Concur at the Incenter

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1. Introduction

In [1], we constructed a collection of things in the context of a triangle and demonstrated the existence of several nontrivial entities. Here, we make the same initial construction and demonstrate that three particular circles in this context concur at the incenter of the triangle.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be the point on line AC, with $\angle CAB = \angle DAB$, such that $AD = AB$.
Let E be the point on line AB, with $\angle CAB = \angle CAE$, such that $AE = AC$.
Let F be the point on line BC, with $\angle CBA = \angle FBA$, such that $BF = AB$.
Let G be the point on line AB, with $\angle CBA = \angle CBG$, such that $BG = BC$.
Let H be the point on line BC, with $\angle ACB = \angle ACH$, such that $CH = AC$.
Let I be the point on line AC, with $\angle ACB = \angle ICB$, such that $CI = BC$.
Then the circles passing through the quadruples $(A, B, H, I), (B, C, D, E)$, and (A, C, F, G) concur at the incenter of ABC.

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Proof. A has coordinates $(0,0)$.
B has coordinates $(1,0)$.
C has coordinates (x_C, y_C).
D has coordinates $(\frac{x_C}{\sqrt{x_C^2+y_C^2}}, \frac{y_C}{\sqrt{x_C^2+y_C^2}})$.
E has coordinates $(\sqrt{x_C^2+y_C^2}, 0)$.
F has coordinates $(\frac{x_C-1}{\sqrt{(x_C-1)^2+y_C^2}}+1, \frac{y_C}{\sqrt{(x_C-1)^2+y_C^2}})$.
G has coordinates $(1-\sqrt{(x_C-1)^2+y_C^2}, 0)$.
H has coordinates $(x_C+\frac{(1-x_C)*\sqrt{x_C^2+y_C^2}}{\sqrt{(x_C-1)^2+y_C^2}}, y_C*(1-\frac{\sqrt{x_C^2+y_C^2}}{\sqrt{(x_C-1)^2+y_C^2}}))$.
I has coordinates $(x_C*(1-\frac{\sqrt{(x_C-1)^2+y_C^2}}{\sqrt{x_C^2+y_C^2}}), y_C*(1-\frac{\sqrt{(x_C-1)^2+y_C^2}}{\sqrt{x_C^2+y_C^2}}))$.
Circle $ABHI$ is given by the equation $(x-1/2)^2+(y+\frac{(\sqrt{(x_C-1)^2+y_C^2})*(\sqrt{x_C^2+y_C^2})-x_C*(x_C-1)-y_C^2}{2y_C})^2-$
$1/4-(\frac{(\sqrt{(x_C-1)^2+y_C^2})*(\sqrt{x_C^2+y_C^2})-x_C*(x_C-1)-y_C^2}{2y_C})^2=0$.
Circle $BCDE$ is given by the equation $(x-\frac{1+\sqrt{x_C^2+y_C^2}}{2})^2+(y-\frac{y_C}{2}+\frac{(x_C-1)*(\sqrt{x_C^2+y_C^2}-x_C)}{2y_C})^2-$
$(1-\frac{1+\sqrt{x_C^2+y_C^2}}{2})^2-(-\frac{y_C}{2}+\frac{(x_C-1)*(\sqrt{x_C^2+y_C^2}-x_C)}{2y_C})^2=0$.
Circle $ACFG$ is given by the equation $(x+\frac{\sqrt{(x_C-1)^2+y_C^2}-1}{2})^2+(y-\frac{x_C*(x_C+\sqrt{(x_C-1)^2+y_C^2}-1)+y_C^2}{2y_C})^2-$
$(\frac{\sqrt{(x_C-1)^2+y_C^2}-1}{2})^2-(\frac{x_C*(x_C+\sqrt{(x_C-1)^2+y_C^2}-1)+y_C^2}{2y_C})^2=0$.
The incenter of ABC has coordinates
$(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}, \frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})$.
We verify that the incenter lies on all three circles since we calculate
$(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}-1/2)^2$
$+(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}+\frac{(\sqrt{(x_C-1)^2+y_C^2})*(\sqrt{x_C^2+y_C^2})-x_C*(x_C-1)-y_C^2}{2y_C})^2-1/4$
$-(\frac{(\sqrt{(x_C-1)^2+y_C^2})*(\sqrt{x_C^2+y_C^2})-x_C*(x_C-1)-y_C^2}{2y_C})^2=0$,
$(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}-\frac{1+\sqrt{x_C^2+y_C^2}}{2})^2$
$+(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}-\frac{y_C}{2}+\frac{(x_C-1)*(\sqrt{x_C^2+y_C^2}-x_C)}{2y_C})^2$
$-(1-\frac{1+\sqrt{x_C^2+y_C^2}}{2})^2-(-\frac{y_C}{2}+\frac{(x_C-1)*(\sqrt{x_C^2+y_C^2}-x_C)}{2y_C})^2=0$,
and
$(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}+\frac{\sqrt{(x_C-1)^2+y_C^2}-1}{2})^2$
$+(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}-\frac{x_C*(x_C+\sqrt{(x_C-1)^2+y_C^2}-1)+y_C^2}{2y_C})^2-(\frac{\sqrt{(x_C-1)^2+y_C^2}-1}{2})^2$
$-(\frac{x_C*(x_C+\sqrt{(x_C-1)^2+y_C^2}-1)+y_C^2}{2y_C})^2=0$ as desired. □

REFERENCES

[1] Warren, B.L., "Three Congruent Circles, Six Parallel Lines, X(104), and a New Construction of the Intouch Triangle", *International Journal of Computer Discovered Mathematics*, Under Review.

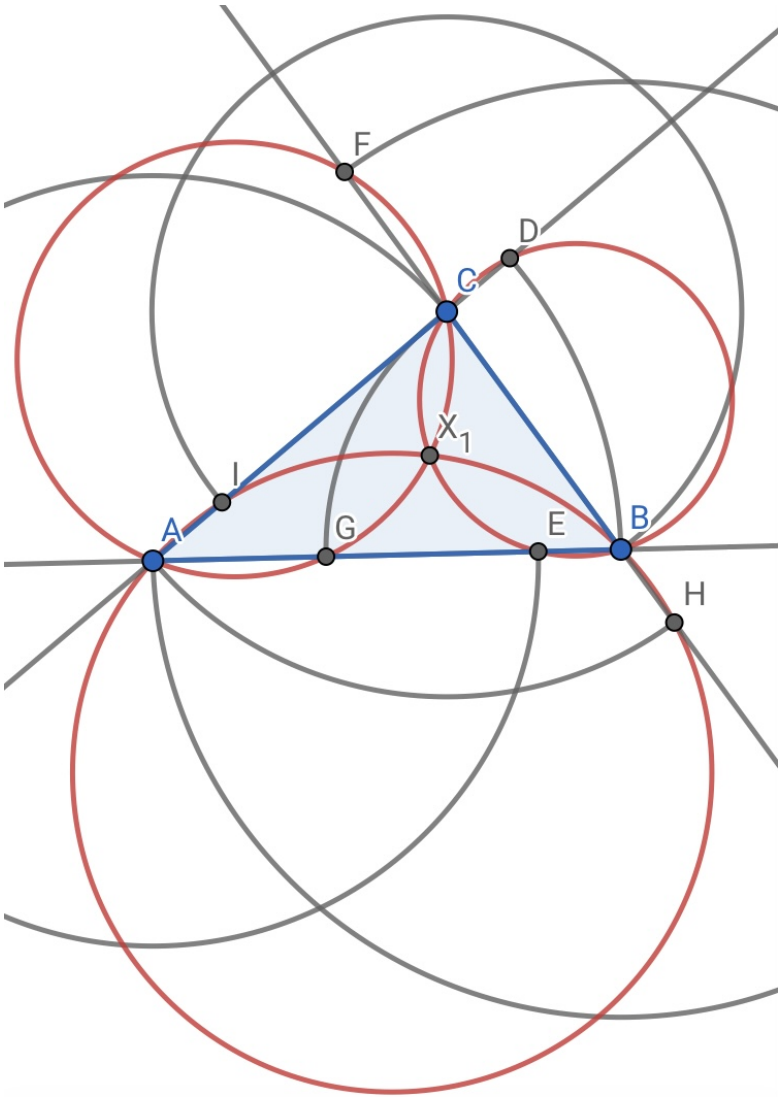

Figure 1. Illustration.