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Generalization of a Result Regarding X(104) as a Perspector

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1. Introduction

In [1], we showed that for a given triangle ABC, there is a certain triangle which is in perspective with ABC at $X(104)$. Here, we will give a generalization of this result.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let D be the point on line AC, with $\angle CAB = \angle DAB$, such that $AD = AB$.
Let E be the point on line AB, with $\angle CAB = \angle CAE$, such that $AE = AC$.
Let F be the point on line BC, with $\angle CBA = \angle FBA$, such that $BF = AB$.
Let G be the point on line AB, with $\angle CBA = \angle CBG$, such that $BG = BC$.
Let H be the point on line BC, with $\angle ACB = \angle ACH$, such that $CH = AC$.
Let I be the point on line AC, with $\angle ACB = \angle ICB$, such that $CI = BC$.
Let M be the circumcenter of A, G, and I.
Let N be the circumcenter of C, D, and F.
Let O be the circumcenter of B, E, and H.
Let α be the incenter of triangle ABC.
Let β be a point on line $A\alpha$.
Let γ be the point on line $B\alpha$ such that $B\gamma : \alpha\gamma = A\beta : \alpha\beta$.
Let δ be the point on line $C\alpha$ such that $C\delta : \alpha\delta = A\beta : \alpha\beta$.
Then lines $M\beta, N\delta$, and $O\gamma$ are concurrent.

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Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0,0)$.
B has coordinates $(1,0)$.
C has coordinates (x_C, y_C).
D has coordinates $(\frac{x_C}{\sqrt{x_C^2+y_C^2}}, \frac{y_C}{\sqrt{x_C^2+y_C^2}})$.
E has coordinates $(\sqrt{x_C^2+y_C^2}, 0)$.
F has coordinates $(\frac{x_C-1}{\sqrt{(x_C-1)^2+y_C^2}}+1, \frac{y_C}{\sqrt{(x_C-1)^2+y_C^2}})$.
G has coordinates $(1-\sqrt{(x_C-1)^2+y_C^2}, 0)$.
H has coordinates $(x_C+\frac{(1-x_C)*\sqrt{x_C^2+y_C^2}}{\sqrt{(x_C-1)^2+y_C^2}}, y_C*(1-\frac{\sqrt{x_C^2+y_C^2}}{\sqrt{(x_C-1)^2+y_C^2}}))$.
I has coordinates $(x_C*(1-\frac{\sqrt{(x_C-1)^2+y_C^2}}{\sqrt{x_C^2+y_C^2}}), y_C*(1-\frac{\sqrt{(x_C-1)^2+y_C^2}}{\sqrt{x_C^2+y_C^2}}))$.
M has coordinates $((1-\sqrt{(x_C-1)^2+y_C^2})/2, -\frac{x_C-x_C^2-y_C^2+(-x_C+\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})}{2y_C})$.
N has coordinates $(\frac{1+\sqrt{x_C^2+y_C^2}-\sqrt{(x_C-1)^2+y_C^2}}{2}, -\frac{(x_C-1)*\sqrt{x_C^2+y_C^2}-x_C*(x_C-1+\sqrt{(x_C-1)^2+y_C^2})-y_C^2}{2y_C})$.
O has coordinates $((1+\sqrt{x_C^2+y_C^2})/2, -\frac{x_C-x_C^2-y_C^2+(\sqrt{x_C^2+y_C^2})*(x_C-1+\sqrt{(x_C-1)^2+y_C^2})}{2y_C})$.
α has coordinates $(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}, \frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})$.
β has coordinates $(x_\beta, \frac{x_\beta*y_C}{x_C+\sqrt{x_C^2+y_C^2}})$.
γ has coordinates $(((y_C^2+x_C^2)^{1/2}+x_\beta*(-(y_C^2+(x_C-1)^2)^{1/2}+x_C-1)+x_C)/((y_C^2+x_C^2)^{1/2}+x_C), \frac{x_\beta*y_C}{x_C+\sqrt{x_C^2+y_C^2}})$.
δ has coordinates
$(-(((x_C-1)*x_\beta-x_C)*(y_C^2+x_C^2)^{1/2}+x_C*x_\beta*(y_C^2+(x_C-1)^2)^{1/2}-x_C^2)/((y_C^2+x_C^2)^{1/2}+x_C), -(y_C*((x_\beta-1)*(y_C^2+x_C^2)^{1/2}+x_\beta*(y_C^2+(x_C-1)^2)^{1/2}-x_C))/((y_C^2+x_C^2)^{1/2}+x_C))$.
Line $M\beta$ is given by the equation
$y = (-\frac{x_C-x_C^2-y_C^2+(-x_C+\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})}{2y_C})$
$+(x-((1-\sqrt{(x_C-1)^2+y_C^2})/2))(\frac{(-\frac{x_C-x_C^2-y_C^2+(-x_C+\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})}{2y_C})-(\frac{x_\beta*y_C}{x_C+\sqrt{x_C^2+y_C^2}})}{((1-\sqrt{(x_C-1)^2+y_C^2})/2)-x_\beta})$.
Line $N\delta$ is given by the equation
$y = (((y_C*((x_\beta-1)*((x_C^2+y_C^2)^{1/2})+x_\beta*(((x_C-1)^2+y_C^2)^{1/2})-x_C))/(((x_C^2+y_C^2)^{1/2})+x_C)-((x_C-1)*((x_C^2+y_C^2)^{1/2})-x_C*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)-y_C^2)/(2*y_C))*(x-(((x_C^2+y_C^2)^{1/2})-(((x_C-1)^2+y_C^2)^{1/2})+1)/2))/((((x_C-1)*x_\beta-x_C)*((x_C^2+y_C^2)^{1/2})+x_C*x_\beta*(((x_C-1)^2+y_C^2)^{1/2})-x_C^2)/(((x_C^2+y_C^2)^{1/2})+x_C)+(((x_C^2+y_C^2)^{1/2})-(((x_C-1)^2+y_C^2)^{1/2})+1)/2)+(-(x_C-1)*((x_C^2+y_C^2)^{1/2})+x_C*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+y_C^2)/(2*y_C)$.
Line $O\gamma$ is given by the equation
$y = ((-(x_\beta*y_C)/(((x_C^2+y_C^2)^{1/2})+x_C)-(((x_C^2+y_C^2)^{1/2})*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)-y_C^2-x_C^2+x_C)/(2*y_C))*(x-(((x_C^2+y_C^2)^{1/2})+1)/2))/((((x_C^2+y_C^2)^{1/2})+1)/2-(((x_C^2+y_C^2)^{1/2})+x_\beta*(-(((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+x_C)/(((x_C^2+y_C^2)^{1/2})+x_C))+(-((x_C^2+y_C^2)^{1/2})*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+y_C^2+x_C^2-x_C)/(2*y_C)$.
Consider the point ε whose coordinates are given by
$(-(y_C^2*(2*x_C^4*x_\beta+x_C^4-12*x_C^3*x_\beta+3*x_C^3-2*x_C^2*x_\beta^2+11*x_C^2*x_\beta-5*x_C^2+4*x_C*x_\beta^2-5*x_C*x_\beta+3*x_C-2*x_\beta^2)-(x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_C-x_C*x_\beta+1)+y_C^2*(-2*x_C^3*x_\beta+4*x_C^3+2*x_C^2*x_\beta-3*x_C^2-5*x_C*x_\beta+3*x_C+2*x_\beta^2)+2*x_C^3-4*x_C^4+$

$2*x_C^5-(x_C^2+y_C^2)^{1/2}*(y_C^2*(x_C^2*x_\beta-3*x_C^2+x_C-2*x_\beta^2+4*x_\beta-2)-2*x_C^2+4*x_C^3-2*x_C^4-y_C^4+x_C^2*x_\beta*(x_C-1)^2)-x_C^3*x_\beta*(x_C-1)^2)+y_C^4*(x_C^2*x_\beta+2*x_C^2-5*x_C*x_\beta+x_C-2*x_\beta^2+1)+2*x_C^3-4*x_C^4+2*x_C^5+y_C^6+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(x_C^3*x_\beta-5*x_C^2*x_\beta+x_C*y_C^2-3*x_\beta*y_C^2+2*x_C^2+2*y_C^2+x_C*x_\beta*y_C^2)+x_C^3*x_\beta*(x_C-1)^2*(x_C-5))/((x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_\beta+2)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_C-8*x_\beta+2*x_C*x_\beta+4*x_\beta^2+2)+2*x_C*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^2*x_\beta+2*x_C^2-12*x_C*x_\beta+2*x_C+4*x_\beta^2+2*x_\beta)+2*x_C^2*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^3*x_\beta-6*x_C^3+4*x_C^2*x_\beta^2+8*x_C^2-8*x_C*x_\beta^2-2*x_C*x_\beta-4*x_C+4*x_\beta^2+2*x_\beta)+y_C^4*(2*x_\beta-4*x_C+2*x_C*x_\beta+4*x_\beta^2)-2*x_C^3+4*x_C^4-2*x_C^5+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(2*x_C*x_\beta+2*x_C^2*x_\beta+2*x_\beta*y_C^2-2*x_C^2-3*y_C^2)+2*x_C^2*x_\beta*(x_C-1)^2*(x_C+1)),(y_C^4*(x_\beta*(x_C^3+6*x_C^2-7*x_C+3)+5*x_C^2-6*x_C^3+x_\beta^2*(2*x_C-1)*(2*x_C-2)-1)-y_C^6*(-2*x_\beta^2-3*x_\beta+2*x_C+1)+((x_C-1)^2+y_C^2)^{1/2}*(y_C^4*(4*x_C^2-2*x_C+2*x_\beta^2+x_C*x_\beta*(x_C-5)+1)+2*y_C^6+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta^2-4*x_\beta+2*x_C+1)+x_C^2*x_\beta*(x_C-1)^3+x_C*y_C^2*(2*x_C+x_\beta*(x_C+2*x_\beta-4))*(x_C-1))+x_C^3*x_\beta*(x_C-1)^3+x_C*y_C^2*(x_\beta*(2*x_C^2-6*x_C+2*x_\beta+1)+2*x_C^2)*(x_C-1))+2*x_C^4-6*x_C^5+6*x_C^6-2*x_C^7+(x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)*(x_C^4*x_\beta-x_C^2*x_\beta-3*x_C^2*y_C^2+x_C*y_C^2+2*x_C^3-2*x_C^4-2*y_C^4+2*x_C*x_\beta*y_C^2+x_C^2*x_\beta*y_C^2)-x_C*y_C^2*(2*x_C*(3*x_C^2-3*x_C+1)-x_\beta*(2*x_\beta*(x_C-1)^2+(2*x_C-1)*(x_C^2+2*x_C-1)))*(x_C-1)+x_C^3*x_\beta*(x_C-1)^3*(x_C+1))/(y_C*(x_\beta*(2*x_C^5-2*x_C^4-2*x_C^3+2*x_C^2)+y_C^4*(4*x_\beta^2+(2*x_C+2)*x_\beta-4*x_C)+y_C^2*(x_\beta*(4*x_C^3-2*x_C+2)-4*x_C+x_\beta^2*(4*x_C^2-8*x_C+4)+8*x_C^2-6*x_C^3)-2*x_C^3+4*x_C^4-2*x_C^5+(y_C^2*(2*x_C+x_\beta*(4*x_C^2-12*x_C+2)+2*x_C^2+4*x_\beta^2)+(x_C^2+y_C^2)^{1/2}*((4*x_\beta^2+(2*x_C-8)*x_\beta+2*x_C+2)*y_C^2+x_\beta*(2*x_C^3-4*x_C^2+2*x_C))+y_C^4*(2*x_\beta+2)+x_\beta*(2*x_C^4-4*x_C^3+2*x_C^2))*(x_C^2-2*x_C+y_C^2+1)^{1/2}+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta-3)+x_\beta*(2*x_C^4-2*x_C^3-2*x_C^2+2*x_C)-2*x_C^2+4*x_C^3-2*x_C^4+y_C^2*(6*x_C+x_\beta*(4*x_C^2-2*x_C+2)-5*x_C^2-3)))))$.
We verify that ε lies on all three lines since we calculate that
$(-\frac{x_C-x_C^2-y_C^2+(-x_C+\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})}{2y_C})+(-(y_C^2*(2*x_C^4*x_\beta+x_C^4-12*x_C^3*x_\beta+3*x_C^3-2*x_C^2*x_\beta^2+11*x_C^2*x_\beta-5*x_C^2+4*x_C*x_\beta^2-5*x_C*x_\beta+3*x_C-2*x_\beta^2)-(x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_C-x_C*x_\beta+1)+y_C^2*(-2*x_C^3*x_\beta+4*x_C^3+2*x_C^2*x_\beta-3*x_C^2-5*x_C*x_\beta+3*x_C+2*x_\beta^2)+2*x_C^3-4*x_C^4+2*x_C^5-(x_C^2+y_C^2)^{1/2}*(y_C^2*(x_C^2*x_\beta-3*x_C^2+x_C-2*x_\beta^2+4*x_\beta-2)-2*x_C^2+4*x_C^3-2*x_C^4-y_C^4+x_C^2*x_\beta*(x_C-1)^2)-x_C^3*x_\beta*(x_C-1)^2)+y_C^4*(x_C^2*x_\beta+2*x_C^2-5*x_C*x_\beta+x_C-2*x_\beta^2+1)+2*x_C^3-4*x_C^4+2*x_C^5+y_C^6+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(x_C^3*x_\beta-5*x_C^2*x_\beta+x_C*y_C^2-3*x_\beta*y_C^2+2*x_C^2+2*y_C^2+x_C*x_\beta*y_C^2)+x_C^3*x_\beta*(x_C-1)^2*(x_C-5))/((x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_\beta+2)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_C-8*x_\beta+2*x_C*x_\beta+4*x_\beta^2+2)+2*x_C*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^2*x_\beta+2*x_C^2-12*x_C*x_\beta+2*x_C+4*x_\beta^2+2*x_\beta)+2*x_C^2*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^3*x_\beta-6*x_C^3+4*x_C^2*x_\beta^2+8*x_C^2-8*x_C*x_\beta^2-2*x_C*x_\beta-4*x_C+4*x_\beta^2+2*x_\beta)+y_C^4*(2*x_\beta-4*x_C+2*x_C*x_\beta+4*x_\beta^2)-2*x_C^3+4*x_C^4-2*x_C^5+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(2*x_C*x_\beta+2*x_C^2*x_\beta+2*x_\beta*y_C^2-2*x_C^2-3*y_C^2)+2*x_C^2*x_\beta*(x_C-1)^2*(x_C+1))-((1-\sqrt{(x_C-1)^2+y_C^2})/2))(\frac{(-\frac{x_C-x_C^2-y_C^2+(-x_C+\sqrt{x_C^2+y_C^2})*(\sqrt{(x_C-1)^2+y_C^2})}{2y_C})-(\frac{x_\beta*y_C}{x_C+\sqrt{x_C^2+y_C^2}})}{((1-\sqrt{(x_C-1)^2+y_C^2})/2)-x_\beta})-((y_C^4*(x_\beta*(x_C^3+6*x_C^2-7*x_C+3)+5*x_C^2-6*x_C^3+x_\beta^2*(2*x_C-1)*(2*x_C-2)-1)-y_C^6*(-2*x_\beta^2-3*x_\beta+2*x_C+1)+((x_C-1)^2+y_C^2)^{1/2}*(y_C^4*(4*x_C^2-2*x_C+2*x_\beta^2+x_C*x_\beta*(x_C-5)+1)+2*y_C^6+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta^2-4*x_\beta+2*x_C+1)+x_C^2*x_\beta*(x_C-1)^3+x_C*y_C^2*(2*x_C+x_\beta*(x_C+2*x_\beta-4))*(x_C-1))+x_C^3*x_\beta*(x_C-1)^3+x_C*y_C^2*(x_\beta*(2*x_C^2-6*x_C+2*x_\beta+1)+2*x_C^2)*(x_C-1))+2*x_C^4-6*x_C^5+6*x_C^6-2*x_C^7+(x_C^2+y_C^2)^{1/2}*$

$$((x_C-1)^2+y_C^2)*(x_C^4*x_\beta-x_C^2*x_\beta-3*x_C^2*y_C^2+x_C*y_C^2+2*x_C^3-2*x_C^4-2*y_C^4+2*x_C*x_\beta*y_C^2+x_C^2*x_\beta*y_C^2)-x_C*y_C^2*(2*x_C*(3*x_C^2-3*x_C+1)-x_\beta*(2*x_\beta*(x_C-1)^2+(2*x_C-1)*(x_C^2+2*x_C-1)))*(x_C-1)+x_C^3*x_\beta*(x_C-1)^3*(x_C+1))/(y_C*(x_\beta*(2*x_C^5-2*x_C^4-2*x_C^3+2*x_C^2)+y_C^4*(4*x_\beta^2+(2*x_C+2)*x_\beta-4*x_C)+y_C^2*(x_\beta*(4*x_C^3-2*x_C+2)-4*x_C+x_\beta^2*(4*x_C^2-8*x_C+4)+8*x_C^2-6*x_C^3)-2*x_C^3+4*x_C^4-2*x_C^5+(y_C^2*(2*x_C+x_\beta*(4*x_C^2-12*x_C+2)+2*x_C^2+4*x_\beta^2)+(x_C^2+y_C^2)^{1/2}*((4*x_\beta^2+(2*x_C-8)*x_\beta+2*x_C+2)*y_C^2+x_\beta*(2*x_C^3-4*x_C^2+2*x_C))+y_C^4*(2*x_\beta+2)+x_\beta*(2*x_C^4-4*x_C^3+2*x_C^2))*(x_C^2-2*x_C+y_C^2+1)^{1/2}+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta-3)+x_\beta*(2*x_C^4-2*x_C^3-2*x_C^2+2*x_C)-2*x_C^2+4*x_C^3-2*x_C^4+y_C^2*(6*x_C+x_\beta*(4*x_C^2-2*x_C+2)-5*x_C^2-3)))))=0,$$

$$(((y_C*((x_\beta-1)*((x_C^2+y_C^2)^{1/2})+x_\beta*(((x_C-1)^2+y_C^2)^{1/2})-x_C))/(((x_C^2+y_C^2)^{1/2})+x_C)-((x_C-1)*((x_C^2+y_C^2)^{1/2})-x_C*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)-y_C^2)/(2*y_C))*(-(y_C^2*(2*x_C^4*x_\beta+x_C^4-12*x_C^3*x_\beta+3*x_C^3-2*x_C^2*x_\beta^2+11*x_C^2*x_\beta-5*x_C^2+4*x_C*x_\beta^2-5*x_C*x_\beta+3*x_C-2*x_\beta^2)-(x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_C-x_C*x_\beta+1)+y_C^2*(-2*x_C^3*x_\beta+4*x_C^3+2*x_C^2*x_\beta-3*x_C^2-5*x_C*x_\beta+3*x_C+2*x_\beta^2)+2*x_C^3-4*x_C^4+2*x_C^5-(x_C^2+y_C^2)^{1/2}*(y_C^2*(x_C^2*x_\beta-3*x_C^2+x_C-2*x_\beta^2+4*x_\beta-2)-2*x_C^2+4*x_C^3-2*x_C^4-y_C^4+x_C^2*x_\beta*(x_C-1)^2)-x_C^3*x_\beta*(x_C-1)^2)+y_C^4*(x_C^2*x_\beta+2*x_C^2-5*x_C*x_\beta+x_C-2*x_\beta^2+1)+2*x_C^3-4*x_C^4+2*x_C^5+y_C^6+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(x_C^3*x_\beta-5*x_C^2*x_\beta+x_C*y_C^2-3*x_\beta*y_C^2+2*x_C^2+2*y_C^2+x_C*x_\beta*y_C^2)+x_C^3*x_\beta*(x_C-1)^2*(x_C-5))/((x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_\beta+2)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_C-8*x_\beta+2*x_C*x_\beta+4*x_\beta^2+2)+2*x_C*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^2*x_\beta+2*x_C^2-12*x_C*x_\beta+2*x_C+4*x_\beta^2+2*x_\beta)+2*x_C^2*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^3*x_\beta-6*x_C^3+4*x_C^2*x_\beta^2+8*x_C^2-8*x_C*x_\beta^2-2*x_C*x_\beta-4*x_C+4*x_\beta^2+2*x_\beta)+y_C^4*(2*x_\beta-4*x_C+2*x_C*x_\beta+4*x_\beta^2)-2*x_C^3+4*x_C^4-2*x_C^5+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(2*x_C*x_\beta+2*x_C^2*x_\beta+2*x_\beta*y_C^2-2*x_C^2-3*y_C^2)+2*x_C^2*x_\beta*(x_C-1)^2*(x_C+1))-(((x_C^2+y_C^2)^{1/2})-(((x_C-1)^2+y_C^2)^{1/2})+1)/2))/((((x_C-1)*x_\beta-x_C)*((x_C^2+y_C^2)^{1/2})+x_C*x_\beta*(((x_C-1)^2+y_C^2)^{1/2})-x_C^2)/(((x_C^2+y_C^2)^{1/2})+x_C)+(((x_C^2+y_C^2)^{1/2})-(((x_C-1)^2+y_C^2)^{1/2})+1)/2)+(-(x_C-1)*((x_C^2+y_C^2)^{1/2})+x_C*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+y_C^2)/(2*y_C)-((y_C^4*(x_\beta*(x_C^3+6*x_C^2-7*x_C+3)+5*x_C^2-6*x_C^3+x_\beta^2*(2*x_C-1)*(2*x_C-2)-1)-y_C^6*(-2*x_\beta^2-3*x_\beta+2*x_C+1)+((x_C-1)^2+y_C^2)^{1/2}*(y_C^4*(4*x_C^2-2*x_C+2*x_\beta^2+x_C*x_\beta*(x_C-5)+1)+2*y_C^6+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta^2-4*x_\beta+2*x_C+1)+x_C^2*x_\beta*(x_C-1)^3+x_C*y_C^2*(2*x_C+x_\beta*(x_C+2*x_\beta-4))*(x_C-1))+x_C^3*x_\beta*(x_C-1)^3+x_C*y_C^2*(x_\beta*(2*x_C^2-6*x_C+2*x_\beta+1)+2*x_C^2)*(x_C-1))+2*x_C^4-6*x_C^5+6*x_C^6-2*x_C^7+(x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)*(x_C^4*x_\beta-x_C^2*x_\beta-3*x_C^2*y_C^2+x_C*y_C^2+2*x_C^3-2*x_C^4-2*y_C^4+2*x_C*x_\beta*y_C^2+x_C^2*x_\beta*y_C^2)-x_C*y_C^2*(2*x_C*(3*x_C^2-3*x_C+1)-x_\beta*(2*x_\beta*(x_C-1)^2+(2*x_C-1)*(x_C^2+2*x_C-1)))*(x_C-1)+x_C^3*x_\beta*(x_C-1)^3*(x_C+1))/(y_C*(x_\beta*(2*x_C^5-2*x_C^4-2*x_C^3+2*x_C^2)+y_C^4*(4*x_\beta^2+(2*x_C+2)*x_\beta-4*x_C)+y_C^2*(x_\beta*(4*x_C^3-2*x_C+2)-4*x_C+x_\beta^2*(4*x_C^2-8*x_C+4)+8*x_C^2-6*x_C^3)-2*x_C^3+4*x_C^4-2*x_C^5+(y_C^2*(2*x_C+x_\beta*(4*x_C^2-12*x_C+2)+2*x_C^2+4*x_\beta^2)+(x_C^2+y_C^2)^{1/2}*((4*x_\beta^2+(2*x_C-8)*x_\beta+2*x_C+2)*y_C^2+x_\beta*(2*x_C^3-4*x_C^2+2*x_C))+y_C^4*(2*x_\beta+2)+x_\beta*(2*x_C^4-4*x_C^3+2*x_C^2))*(x_C^2-2*x_C+y_C^2+1)^{1/2}+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta-3)+x_\beta*(2*x_C^4-2*x_C^3-2*x_C^2+2*x_C)-2*x_C^2+4*x_C^3-2*x_C^4+y_C^2*(6*x_C+x_\beta*(4*x_C^2-2*x_C+2)-5*x_C^2-3)))))=0,$$

and

$$((-(x_\beta*y_C)/(((x_C^2+y_C^2)^{1/2})+x_C)-(((x_C^2+y_C^2)^{1/2})*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)-y_C^2-x_C^2+x_C)/(2*y_C))*(-(y_C^2*(2*x_C^4*x_\beta+x_C^4-12*x_C^3*x_\beta+3*x_C^3-2*x_C^2*x_\beta^2+11*x_C^2*x_\beta-5*x_C^2+4*x_C*x_\beta^2-5*x_C*x_\beta+3*x_C-2*x_\beta^2)-(x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_C-x_C*x_\beta+1)+y_C^2*(-2*x_C^3*x_\beta+4*x_C^3+2*x_C^2*x_\beta-3*x_C^2-5*x_C*x_\beta+3*x_C+2*x_\beta^2)+2*x_C^3-4*x_C^4+2*x_C^5-(x_C^2+y_C^2)^{1/2}*(y_C^2*(x_C^2*x_\beta-3*x_C^2+x_C-2*$$

$x_\beta^2+4*x_\beta-2)-2*x_C^2+4*x_C^3-2*x_C^4-y_C^4+x_C^2*x_\beta*(x_C-1)^2)-x_C^3*x_\beta*(x_C-1)^2)+y_C^4*(x_C^2*x_\beta+2*x_C^2-5*x_C*x_\beta+x_C-2*x_\beta^2+1)+2*x_C^3-4*x_C^4+2*x_C^5+y_C^6+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(x_C^3*x_\beta-5*x_C^2*x_\beta+x_C*y_C^2-3*x_\beta*y_C^2+2*x_C^2+2*y_C^2+x_C*x_\beta*y_C^2)+x_C^3*x_\beta*(x_C-1)^2*(x_C-5))/((x_C^2-2*x_C+y_C^2+1)^{1/2}*(y_C^4*(2*x_\beta+2)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_C-8*x_\beta+2*x_C*x_\beta+4*x_\beta^2+2)+2*x_C*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^2*x_\beta+2*x_C^2-12*x_C*x_\beta+2*x_C+4*x_\beta^2+2*x_\beta)+2*x_C^2*x_\beta*(x_C-1)^2)+y_C^2*(4*x_C^3*x_\beta-6*x_C^3+4*x_C^2*x_\beta^2+8*x_C^2-8*x_C*x_\beta^2-2*x_C*x_\beta-4*x_C+4*x_\beta^2+2*x_\beta)+y_C^4*(2*x_\beta-4*x_C+2*x_C*x_\beta+4*x_\beta^2)-2*x_C^3+4*x_C^4-2*x_C^5+(x_C^2+y_C^2)^{1/2}*(x_C^2-2*x_C+y_C^2+1)*(2*x_C*x_\beta+2*x_C^2*x_\beta+2*x_\beta*y_C^2-2*x_C^2-3*y_C^2)+2*x_C^2*x_\beta*(x_C-1)^2*(x_C+1))-(((x_C^2+y_C^2)^{1/2})+1)/2))/((((x_C^2+y_C^2)^{1/2})+1)/2-(((x_C^2+y_C^2)^{1/2})+x_\beta*(-(((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+x_C)/(((x_C^2+y_C^2)^{1/2})+x_C))+(-((x_C^2+y_C^2)^{1/2})*((((x_C-1)^2+y_C^2)^{1/2})+x_C-1)+y_C^2+x_C^2-x_C)/(2*y_C)-((y_C^4*(x_\beta*(x_C^3+6*x_C^2-7*x_C+3)+5*x_C^2-6*x_C^3+x_\beta^2*(2*x_C-1)*(2*x_C-2)-1)-y_C^6*(-2*x_\beta^2-3*x_\beta+2*x_C+1)+((x_C-1)^2+y_C^2)^{1/2}*(y_C^4*(4*x_C^2-2*x_C+2*x_\beta^2+x_C*x_\beta*(x_C-5)+1)+2*y_C^6+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta^2-4*x_\beta+2*x_C+1)+x_C^2*x_\beta*(x_C-1)^3+x_C*y_C^2*(2*x_C+x_\beta*(x_C+2*x_\beta-4))*(x_C-1))+x_C^3*x_\beta*(x_C-1)^3+x_C*y_C^2*(x_\beta*(2*x_C^2-6*x_C+2*x_\beta+1)+2*x_C^2)*(x_C-1))+2*x_C^4-6*x_C^5+6*x_C^6-2*x_C^7+(x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)*(x_C^4*x_\beta-x_C^2*x_\beta-3*x_C^2*y_C^2+x_C*y_C^2+2*x_C^3-2*x_C^4-2*y_C^4+2*x_C*x_\beta*y_C^2+x_C^2*x_\beta*y_C^2)-x_C*y_C^2*(2*x_C*(3*x_C^2-3*x_C+1)-x_\beta*(2*x_\beta*(x_C-1)^2+(2*x_C-1)*(x_C^2+2*x_C-1)))*(x_C-1)+x_C^3*x_\beta*(x_C-1)^3*(x_C+1))/(y_C*(x_\beta*(2*x_C^5-2*x_C^4-2*x_C^3+2*x_C^2)+y_C^4*(4*x_\beta^2+(2*x_C+2)*x_\beta-4*x_C)+y_C^2*(x_\beta*(4*x_C^3-2*x_C+2)-4*x_C+x_\beta^2*(4*x_C^2-8*x_C+4)+8*x_C^2-6*x_C^3)-2*x_C^3+4*x_C^4-2*x_C^5+(y_C^2*(2*x_C+x_\beta*(4*x_C^2-12*x_C+2)+2*x_C^2+4*x_\beta^2)+(x_C^2+y_C^2)^{1/2}*((4*x_\beta^2+(2*x_C-8)*x_\beta+2*x_C+2)*y_C^2+x_\beta*(2*x_C^3-4*x_C^2+2*x_C))+y_C^4*(2*x_\beta+2)+x_\beta*(2*x_C^4-4*x_C^3+2*x_C^2))*(x_C^2-2*x_C+y_C^2+1)^{1/2}+(x_C^2+y_C^2)^{1/2}*(y_C^4*(2*x_\beta-3)+x_\beta*(2*x_C^4-2*x_C^3-2*x_C^2+2*x_C)-2*x_C^2+4*x_C^3-2*x_C^4+y_C^2*(6*x_C+x_\beta*(4*x_C^2-2*x_C+2)-5*x_C^2-3)))))=0$
as desired. □

References

[1] Warren, B.L., "Three Congruent Circles, Six Parallel Lines, X(104), and a New Construction of the Intouch Triangle", *International Journal of Computer Discovered Mathematics*, Under Review.

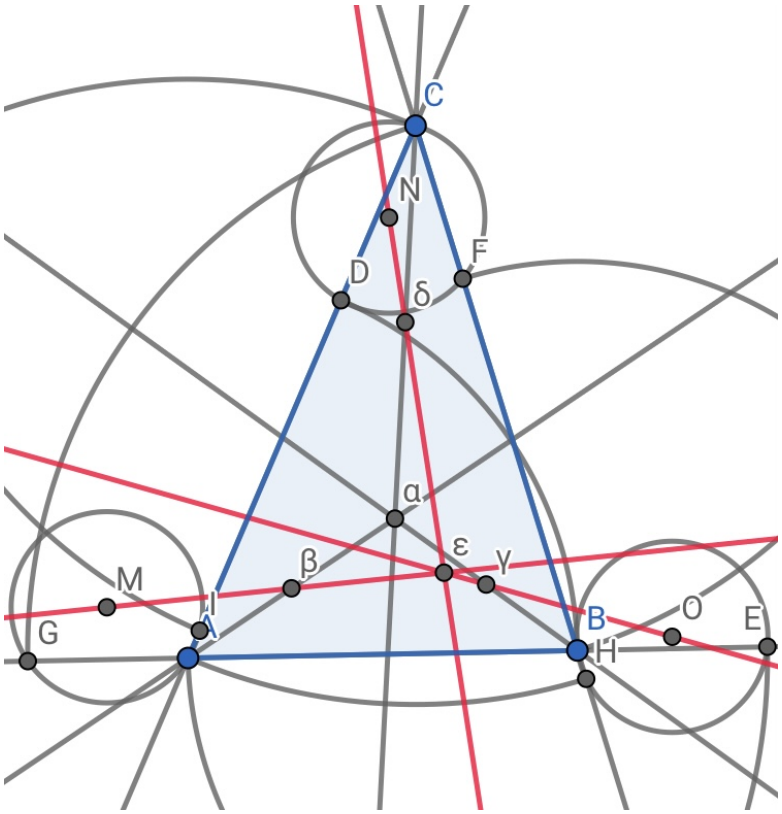

FIGURE 1. Illustration.