International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 9, 2024, pp. 502-506.
Received November 2, 2024. Published on-line December 31, 2024.
web: http://www.journal-1.eu/
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A New Generalization of the Orthocenter

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Mathematics Subject Classification (2010). 51N20.

1. Introduction

The orthocenter [1] of a triangle is the point of concurrence of the three altitudes. In [2], we constructed some certain points on a triangle and deduced a perspector. Here we will do a similar procedure for a slightly modified construction. As a result, we will obtain a generalization of the orthocenter.

2. Results

Theorem 2.1. *Let A, B, and C be the vertices of a given triangle.*
Let P be the incenter of ABC.
Let α be the point on line AC with $\angle BA\alpha = \pi - \angle BAC$, such that $A\alpha = AB$.
Let β be the point on line AB with $\angle CA\beta = \pi - \angle BAC$, such that $A\beta = AC$.
Let γ be the point on line BC with $\angle \gamma CA = \pi - \angle ACB$, such that $C\gamma = AC$.
Let δ be the point on line AC with $\angle \delta CB = \pi - \angle ACB$, such that $C\delta = CB$.
Let ε be the point on line AB with $\angle \varepsilon BC = \pi - \angle ABC$, such that $\varepsilon B = BC$.
Let ζ be the point on line BC with $\angle \zeta BA = \pi - \angle ABC$, such that $\zeta B = AB$.
Let η be the circumcenter of A, α, and β.
Let θ be the circumcenter of C, γ, and δ.
Let ι be the circumcenter of B, ε, and ζ.
Let κ be a point on line AP.
Let λ be the point on line BP such that $B\lambda : P\lambda = A\kappa : P\kappa$.
Let μ be the point on line CP such that $C\mu : P\mu = A\kappa : P\kappa$.
Then lines $\eta\kappa, \theta\mu$, and $\iota\lambda$ are concurrent.

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Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0,0)$.
B has coordinates $(1,0)$.
C has coordinates (x_C, y_C).
P has coordinates
$(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}, \frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})$.
α has coordinates
$(-x_C/(x_C^2+y_C^2)^{1/2}, -y_C/(x_C^2+y_C^2)^{1/2})$.
β has coordinates
$(-(x_C^2+y_C^2)^{1/2}, 0)$.
γ has coordinates
$(x_C + ((x_C^2+y_C^2)^{1/2} * (x_C-1))/((x_C-1)^2+y_C^2)^{1/2}, (x_C + ((x_C^2+y_C^2)^{1/2} * (x_C - 1))/((x_C-1)^2+y_C^2)^{1/2} - 1) * (y_C/(x_C-1)))$.
δ has coordinates
$((x_C * (((x_C^2+y_C^2) * (x_C^2 - 2*x_C + y_C^2 + 1))^{1/2} + x_C^2 + y_C^2))/(x_C^2+y_C^2), ((x_C * (((x_C^2+y_C^2) * (x_C^2 - 2*x_C + y_C^2 + 1))^{1/2} + x_C^2 + y_C^2))/(x_C^2+y_C^2)) * (y_C/x_C))$.
ε has coordinates
$(1 + ((x_C-1)^2+y_C^2)^{1/2}, 0)$.
ζ has coordinates
$(-((x_C-1) * (((x_C-1)^2+y_C^2)^{1/2} - x_C + 1) - y_C^2)/((x_C-1)^2+y_C^2), (-((x_C-1) * (((x_C-1)^2+y_C^2)^{1/2} - x_C + 1) - y_C^2)/((x_C-1)^2+y_C^2) - 1) * (y_C/(x_C-1)))$.
η has coordinates
$(-(x_C^2+y_C^2)^{1/2}/2, ((x_C-1) * (x_C^2+y_C^2)^{1/2})/(2*y_C))$.
θ has coordinates
$(x_C, y_C + ((x_C^2+y_C^2)^{1/2} * ((x_C-1)^2+y_C^2)^{1/2})/(2*y_C))$.
ι has coordinates
$((2 + ((x_C-1)^2+y_C^2)^{1/2})/2, -(x_C * ((x_C-1)^2+y_C^2)^{1/2})/(2*y_C))$.
κ has coordinates
$(x_k, x_k * (\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})/(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}))$.
λ has coordinates
$(((y_C^2+x_C^2)^{1/2} + x_k * (-(y_C^2 + (x_C-1)^2)^{1/2} + x_C - 1) + x_C)/((y_C^2+x_C^2)^{1/2} + x_C), \frac{x_k * y_C}{x_C+\sqrt{x_C^2+y_C^2}})$.
μ has coordinates
$(-(((x_C-1)*x_k - x_C)*(y_C^2+x_C^2)^{1/2} + x_C*x_k*(y_C^2+(x_C-1)^2)^{1/2} - x_C^2)/((y_C^2+x_C^2)^{1/2} + x_C), -(y_C*((x_k-1)*(y_C^2+x_C^2)^{1/2} + x_k*(y_C^2+(x_C-1)^2)^{1/2} - x_C))/((y_C^2+x_C^2)^{1/2}+x_C))$.
Line $\eta\kappa$ is given by the equation
$y = (x_k*(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})/(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})) + (x-x_k)*(((((x_C - 1)*(x_C^2+y_C^2)^{1/2})/(2*y_C)) - (x_k*(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})/(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})))/((-(x_C^2+y_C^2)^{1/2}/2) - x_k))$.
Line $\theta\mu$ is given by the equation
$y = (y_C + ((x_C^2+y_C^2)^{1/2} * ((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)) + (x-x_C) * (((-(y_C * ((x_k-1) * (y_C^2+x_C^2)^{1/2} + x_k * (y_C^2+(x_C-1)^2)^{1/2} - x_C))/((y_C^2+x_C^2)^{1/2} + x_C)) - (y_C + ((x_C^2+y_C^2)^{1/2} * ((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)))/((-(((x_C-1)*x_k - x_C) * (y_C^2+x_C^2)^{1/2} + x_C * x_k * (y_C^2+(x_C-1)^2)^{1/2} - x_C^2)/((y_C^2+x_C^2)^{1/2} + x_C)) - x_C))$.
Line $\iota\lambda$ is given by the equation
$y = (-(x_C * ((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)) + (x - ((2 + ((x_C-1)^2+y_C^2)^{1/2})/2)) *$

$(((\frac{x_k*y_C}{x_C+\sqrt{x_C^2+y_C^2}})-(-(x_C*((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)))/((((y_C^2+x_C^2)^{1/2}+x_k*(-(y_C^2+(x_C-1)^2)^{1/2}+x_C-1)+x_C)/((y_C^2+x_C^2)^{1/2}+x_C))-((2+((x_C-1)^2+y_C^2)^{1/2})/2))).$
Consider the point ν whose coordinates are given by
$(-(x_k*(x_C^5-2*x_C^4+x_C^3)-(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_k^2+(2*x_C-x_C^2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_C^3+4*x_k*x_C^2)-x_k*(x_C^4-2*x_C^3+x_C^2))-((x_C-1)^2+y_C^2)^{1/2}*(y_C^2*(x_k*(-x_C^2+3*x_C)+2*x_C^2+2*x_k^2)+x_k*(-x_C^4+5*x_C^3)+2*x_C^4)+y_C^2*(-2*x_k^2+(2*x_C^3-4*x_C^2+x_C)*x_k)-(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-x_k*(x_C-1))+x_k*y_C^4*(x_C-2))/(y_C^2*(4*x_k^2+(4*x_C^2-4*x_C+2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^3+2*x_C^2)+y_C^2*(4*x_k^2+(2*x_C+2)*x_k+2*x_C)+2*x_C^3)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(4*x_k^2+2*x_C*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^2+2*x_C)+2*x_k*y_C^2+x_C^2)+x_k*(2*x_C^3-4*x_C^2+2*x_C))+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+x_k*(2*x_C^4-4*x_C^3+2*x_C^2)), (((x_C-1)^2+y_C^2)^{1/2}*(2*x_C^4-2*x_C^5-x_C*y_C^2*(x_C*(2*x_C-2)-x_k*(3*x_C+x_k*(2*x_C-2)+x_C^2-1))+x_k*y_C^4*(2*x_k+1)+x_C^3*x_k*(x_C-1)*(x_C+1))+(x_C^2+y_C^2)^{1/2}*(2*x_k^2*y_C^4+x_C*((x_C-1)^2+y_C^2)^{1/2}*(x_C^2*x_k-x_C*x_k+3*x_k*y_C^2+x_C^2-x_C^3)+x_C^2*x_k*(x_C-1)^3+x_C*x_k*y_C^2*(x_C-1)*(x_C+2*x_k-2))+x_C^3*x_k*(x_C-1)^3+x_k*y_C^4*(2*x_k+(x_C-1)*(x_C-2))+x_C*x_k*y_C^2*(x_C-1)*(2*x_C^2-4*x_C+2*x_k+1)+x_C*(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-1)*(x_k-1))/(y_C*((x_C^2+y_C^2)^{1/2}*(((x_C-1)^2+y_C^2)^{1/2}*(2*x_k*y_C^2+x_C^2+2*x_C*x_k*(x_C+1))+2*x_k*y_C^2*(x_C+2*x_k)+2*x_C*x_k*(x_C-1)^2)+2*((x_C-1)^2+y_C^2)^{1/2}*(x_C^3*x_k+x_C^3+x_C^2*x_k+x_C*x_k*y_C^2+x_C*y_C^2+2*x_k^2*y_C^2+x_k*y_C^2)+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+2*x_C^2*x_k*(x_C-1)^2+2*x_k*y_C^2*(2*x_C^2-2*x_C+2*x_k+1)))).$
We verify that ν lies on lines $\eta\kappa, \theta\mu$, and $\iota\lambda$ since we calculate that
$(x_k*(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})/(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}}))+(-(x_k*(x_C^5-2*x_C^4+x_C^3)-(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_k^2+(2*x_C-x_C^2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_C^3+4*x_k*x_C^2)-x_k*(x_C^4-2*x_C^3+x_C^2))-((x_C-1)^2+y_C^2)^{1/2}*(y_C^2*(x_k*(-x_C^2+3*x_C)+2*x_C^2+2*x_k^2)+x_k*(-x_C^4+5*x_C^3)+2*x_C^4)+y_C^2*(-2*x_k^2+(2*x_C^3-4*x_C^2+x_C)*x_k)-(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-x_k*(x_C-1))+x_k*y_C^4*(x_C-2))/(y_C^2*(4*x_k^2+(4*x_C^2-4*x_C+2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^3+2*x_C^2)+y_C^2*(4*x_k^2+(2*x_C+2)*x_k+2*x_C)+2*x_C^3)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(4*x_k^2+2*x_C*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^2+2*x_C)+2*x_k*y_C^2+x_C^2)+x_k*(2*x_C^3-4*x_C^2+2*x_C))+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+x_k*(2*x_C^4-4*x_C^3+2*x_C^2))-x_k)*((((((x_C-1)*(x_C^2+y_C^2)^{1/2})/(2*y_C))-(x_k*(\frac{y_C}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})/(\frac{x_C+\sqrt{x_C^2+y_C^2}}{1+\sqrt{x_C^2+y_C^2}+\sqrt{(x_C-1)^2+y_C^2}})))/((-(x_C^2+y_C^2)^{1/2}/2)-x_k))-((((x_C-1)^2+y_C^2)^{1/2}*(2*x_C^4-2*x_C^5-x_C*y_C^2*(x_C*(2*x_C-2)-x_k*(3*x_C+x_k*(2*x_C-2)+x_C^2-1))+x_k*y_C^4*(2*x_k+1)+x_C^3*x_k*(x_C-1)*(x_C+1))+(x_C^2+y_C^2)^{1/2}*(2*x_k^2*y_C^4+x_C*((x_C-1)^2+y_C^2)^{1/2}*(x_C^2*x_k-x_C*x_k+3*x_k*y_C^2+x_C^2-x_C^3)+x_C^2*x_k*(x_C-1)^3+x_C*x_k*y_C^2*(x_C-1)*(x_C+2*x_k-2))+x_C^3*x_k*(x_C-1)^3+x_k*y_C^4*(2*x_k+(x_C-1)*(x_C-2))+x_C*x_k*y_C^2*(x_C-1)*(2*x_C^2-4*x_C+2*x_k+1)+x_C*(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-1)*(x_k-1))/(y_C*((x_C^2+y_C^2)^{1/2}*(((x_C-1)^2+y_C^2)^{1/2}*(2*x_k*y_C^2+x_C^2+2*x_C*x_k*(x_C+1))+2*x_k*y_C^2*(x_C+2*x_k)+2*x_C*x_k*(x_C-1)^2)+2*((x_C-1)^2+y_C^2)^{1/2}*(x_C^3*x_k+x_C^3+x_C^2*x_k+x_C*x_k*y_C^2+x_C*y_C^2+2*x_k^2*y_C^2+x_k*y_C^2)+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+2*x_C^2*x_k*(x_C-1)^2+2*x_k*y_C^2*(2*x_C^2-2*x_C+2*x_k+1)))) = 0,$
$(y_C+((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2})/(2*y_C))+(-(x_k*(x_C^5-2*x_C^4+x_C^3)-(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_k^2+(2*x_C-x_C^2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_C^3+4*x_k*x_C^2)-x_k*(x_C^4-2*x_C^3+x_C^2))-((x_C-1)^2+y_C^2)^{1/2}*(y_C^2*(x_k*(-x_C^2+3*x_C)+2*x_C^2+2*x_k^2)+x_k*(-x_C^4+5*x_C^3)+2*x_C^4)+y_C^2*(-2*x_k^2+(2*x_C^3-4*x_C^2+x_C)*x_k)-(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-x_k*(x_C-1))+x_k*y_C^4*(x_C-2))/(y_C^2*(4*x_k^2+(4*x_C^2-4*$

$x_C+2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^3+2*x_C^2)+y_C^2*(4*x_k^2+(2*x_C+2)*x_k+2*x_C)+2*x_C^3)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(4*x_k^2+2*x_C*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^2+2*x_C)+2*x_k*y_C^2+x_C^2)+x_k*(2*x_C^3-4*x_C^2+2*x_C))+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+x_k*(2*x_C^4-4*x_C^3+2*x_C^2))-x_C)*(((-(y_C*((x_k-1)*(y_C^2+x_C^2)^{1/2}+x_k*(y_C^2+(x_C-1)^2)^{1/2}-x_C))/((y_C^2+x_C^2)^{1/2}+x_C))-(y_C+((x_C^2+y_C^2)^{1/2}*((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)))/((-(((x_C-1)*x_k-x_C)*(y_C^2+x_C^2)^{1/2}+x_C*x_k*(y_C^2+(x_C-1)^2)^{1/2}-x_C^2)/((y_C^2+x_C^2)^{1/2}+x_C))-x_C))-((((x_C-1)^2+y_C^2)^{1/2}*(2*x_C^4-2*x_C^5-x_C*y_C^2*(x_C*(2*x_C-2)-x_k*(3*x_C+x_k*(2*x_C-2)+x_C^2-1))+x_k*y_C^4*(2*x_k+1)+x_C^3*x_k*(x_C-1)*(x_C+1))+(x_C^2+y_C^2)^{1/2}*(2*x_k^2*y_C^4+x_C*((x_C-1)^2+y_C^2)^{1/2}*(x_C^2*x_k-x_C*x_k+3*x_k*y_C^2+x_C^2-x_C^3)+x_C^2*x_k*(x_C-1)^3+x_C*x_k*y_C^2*(x_C-1)*(x_C+2*x_k-2))+x_C^3*x_k*(x_C-1)^3+x_k*y_C^4*(2*x_k+(x_C-1)*(x_C-2))+x_C*x_k*y_C^2*(x_C-1)*(2*x_C^2-4*x_C+2*x_k+1)+x_C*(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-1)*(x_k-1))/(y_C*((x_C^2+y_C^2)^{1/2}*(((x_C-1)^2+y_C^2)^{1/2}*(2*x_k*y_C^2+x_C^2+2*x_C*x_k*(x_C+1))+2*x_k*y_C^2*(x_C+2*x_k)+2*x_C*x_k*(x_C-1)^2)+2*((x_C-1)^2+y_C^2)^{1/2}*(x_C^3*x_k+x_C^3+x_C^2*x_k+x_C*x_k*y_C^2+x_C*y_C^2+2*x_k^2*y_C^2+x_k*y_C^2)+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+2*x_C^2*x_k*(x_C-1)^2+2*x_k*y_C^2*(2*x_C^2-2*x_C+2*x_k+1)))) = 0,$

and

$(-(x_C*((x_C-1)^2+y_C^2)^{1/2})/(2*y_C))+(-(x_k*(x_C^5-2*x_C^4+x_C^3)-(x_C^2+y_C^2)^{1/2}*(y_C^2*(2*x_k^2+(2*x_C-x_C^2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_C^3+4*x_k*x_C^2)-x_k*(x_C^4-2*x_C^3+x_C^2))-((x_C-1)^2+y_C^2)^{1/2}*(y_C^2*(x_k*(-x_C^2+3*x_C)+2*x_C^2+2*x_k^2)+x_k*(-x_C^4+5*x_C^3)+2*x_C^4)+y_C^2*(-2*x_k^2+(2*x_C^3-4*x_C^2+x_C)*x_k)-(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-x_k*(x_C-1))+x_k*y_C^4*(x_C-2))/(y_C^2*(4*x_k^2+(4*x_C^2-4*x_C+2)*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^3+2*x_C^2)+y_C^2*(4*x_k^2+(2*x_C+2)*x_k+2*x_C)+2*x_C^3)+(x_C^2+y_C^2)^{1/2}*(y_C^2*(4*x_k^2+2*x_C*x_k)+((x_C-1)^2+y_C^2)^{1/2}*(x_k*(2*x_C^2+2*x_C)+2*x_k*y_C^2+x_C^2)+x_k*(2*x_C^3-4*x_C^2+2*x_C))+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+x_k*(2*x_C^4-4*x_C^3+2*x_C^2))-((2+((x_C-1)^2+y_C^2)^{1/2})/2))*(((\frac{x_k*y_C}{x_C+\sqrt{x_C^2+y_C^2}})-(-(x_C*((x_C-1)^2+y_C^2)^{1/2})/(2*y_C)))/((((y_C^2+x_C^2)^{1/2}+x_k*(-(y_C^2+(x_C-1)^2)^{1/2}+x_C-1)+x_C)/((y_C^2+x_C^2)^{1/2}+x_C))-((2+((x_C-1)^2+y_C^2)^{1/2})/2)))-((((x_C-1)^2+y_C^2)^{1/2}*(2*x_C^4-2*x_C^5-x_C*y_C^2*(x_C*(2*x_C-2)-x_k*(3*x_C+x_k*(2*x_C-2)+x_C^2-1))+x_k*y_C^4*(2*x_k+1)+x_C^3*x_k*(x_C-1)*(x_C+1))+(x_C^2+y_C^2)^{1/2}*(2*x_k^2*y_C^4+x_C*((x_C-1)^2+y_C^2)^{1/2}*(x_C^2*x_k-x_C*x_k+3*x_k*y_C^2+x_C^2-x_C^3)+x_C^2*x_k*(x_C-1)^3+x_C*x_k*y_C^2*(x_C-1)*(x_C+2*x_k-2))+x_C^3*x_k*(x_C-1)^3+x_k*y_C^4*(2*x_k+(x_C-1)*(x_C-2))+x_C*x_k*y_C^2*(x_C-1)*(2*x_C^2-4*x_C+2*x_k+1)+x_C*(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}*(x_C-1)*(x_k-1))/(y_C*((x_C^2+y_C^2)^{1/2}*(((x_C-1)^2+y_C^2)^{1/2}*(2*x_k*y_C^2+x_C^2+2*x_C*x_k*(x_C+1))+2*x_k*y_C^2*(x_C+2*x_k)+2*x_C*x_k*(x_C-1)^2)+2*((x_C-1)^2+y_C^2)^{1/2}*(x_C^3*x_k+x_C^3+x_C^2*x_k+x_C*x_k*y_C^2+x_C*y_C^2+2*x_k^2*y_C^2+x_k*y_C^2)+(x_C^2+y_C^2)^{3/2}*((x_C-1)^2+y_C^2)^{1/2}+2*x_k*y_C^4+2*x_C^2*x_k*(x_C-1)^2+2*x_k*y_C^2*(2*x_C^2-2*x_C+2*x_k+1)))) = 0$

as desired. □

If we take $\kappa = A$, then ν becomes the orthocenter of ABC.

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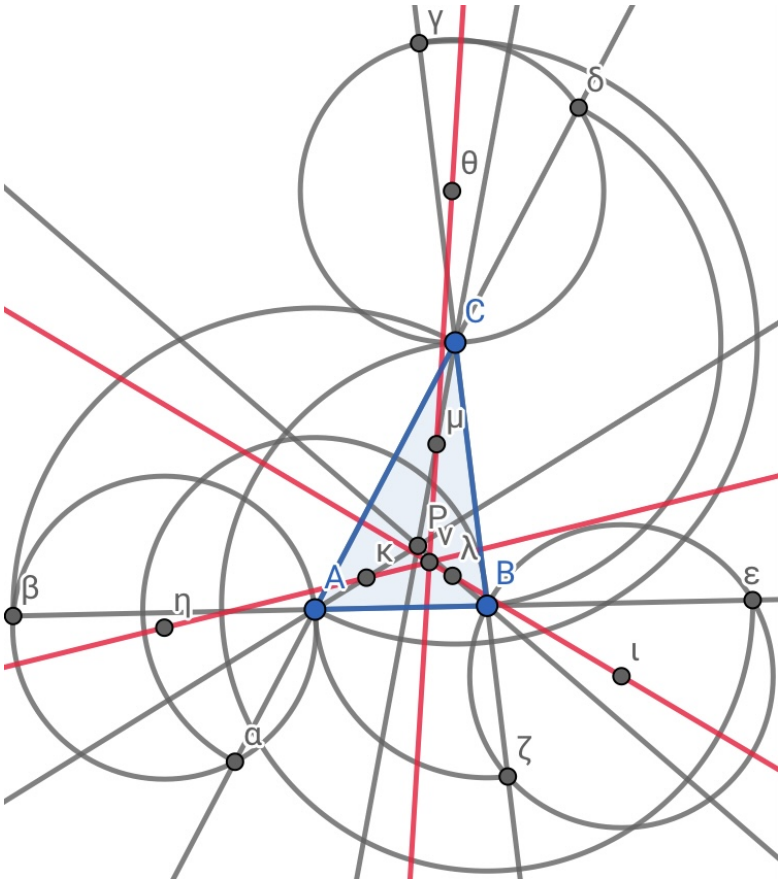

Figure 1. Illustration.