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Euler Lines that Concur at a Point on a Circle

Benjamin Lee Warren[a] [2]
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

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1. Introduction

We demonstrate here the existence of a circle which is concentric with the second Lozada circle [1].

2. Results

Theorem 2.1. *Let ABC be a triangle and $A'B'C'$ the orthic triangle.*
Let A_b and A_c be the orthogonal projections of A' on b and c respectively.
Define B_a, B_c, C_a, and C_b cyclically.
Then the Euler lines of AA_bA_c, BB_aB_c, and CC_aC_b are concurrent at a point which is concyclic with the centroids of AA_bA_c, BB_aB_c, and CC_aC_b about $X(9730)$.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
The orthocenter has coordinates $(x_C, x_C * (1 - x_C)/y_C)$.
A' has coordinates
$(y_C^2/(y_C^2 + (x_C - 1)^2), -((x_C - 1) * y_C)/(y_C^2 + (x_C - 1)^2))$.
B' has coordinates
$(x_C^2/(y_C^2 + x_C^2), (x_C * y_C)/(y_C^2 + x_C^2))$.
C' has coordinates $(x_C, 0)$.
A_b has coordinates
$((x_C * y_C^2)/((y_C^2 + (x_C - 1)^2) * (y_C^2 + x_C^2)), y_C^3/((y_C^2 + (x_C - 1)^2) * (y_C^2 + x_C^2)))$.

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[2]Corresponding author

A_c has coordinates
$(y_C^2/(y_C^2+(x_C-1)^2),0)$.
B_a has coordinates
$((y_C^4+2*x_C^2*y_C^2-x_C*y_C^2+x_C^4-2*x_C^3+x_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)), y_C^3/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))$.
B_c has coordinates
$(x_C^2/(y_C^2+x_C^2),0)$.
C_a has coordinates
$((y_C^2+(x_C-1)^2*x_C)/(y_C^2+(x_C-1)^2),((x_C-1)^2*y_C)/(y_C^2+(x_C-1)^2))$.
C_b has coordinates
$(x_C^3/(y_C^2+x_C^2),(x_C^2*y_C)/(y_C^2+x_C^2))$.
The centroid of AA_bA_c has coordinates
$(((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3, y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))$.
The circumcenter of AA_bA_c has coordinates
$(y_C^2/(2*(y_C^2+(x_C-1)^2)),-((x_C-1)*y_C)/(2*(y_C^2+(x_C-1)^2)))$.
The Euler line of AA_bA_c is given by the equation
$y=(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))+(x-(((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3))*((y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))-(-((x_C-1)*y_C)/(2*(y_C^2+(x_C-1)^2))))/((((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3)-(y_C^2/(2*(y_C^2+(x_C-1)^2))))$.
The centroid of BB_aB_c has coordinates
$((1+(y_C^4+2*x_C^2*y_C^2-x_C*y_C^2+x_C^4-2*x_C^3+x_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+x_C^2/(y_C^2+x_C^2))/3, y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))$.
The circumcenter of BB_aB_c has coordinates
$((y_C^2+2*x_C^2)/(2*(y_C^2+x_C^2)),(x_C*y_C)/(2*(y_C^2+x_C^2)))$.
The Euler line of BB_aB_c is given by the equation
$y=((x_C*y_C)/(2*(y_C^2+x_C^2)))+(x-((y_C^2+2*x_C^2)/(2*(y_C^2+x_C^2))))*(((x_C*y_C)/(2*(y_C^2+x_C^2)))-(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))/(((y_C^2+2*x_C^2)/(2*(y_C^2+x_C^2)))-((1+(y_C^4+2*x_C^2*y_C^2-x_C*y_C^2+x_C^4-2*x_C^3+x_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+x_C^2/(y_C^2+x_C^2))/3))$.
The centroid of CC_aC_b has coordinates
$((x_C+(y_C^2+(x_C-1)^2*x_C)/(y_C^2+(x_C-1)^2)+x_C^3/(y_C^2+x_C^2))/3,(y_C+((x_C-1)^2*y_C)/(y_C^2+(x_C-1)^2)+(x_C^2*y_C)/(y_C^2+x_C^2))/3)$.
The circumcenter of CC_aC_b has coordinates
$(x_C,y_C/2)$.
The Euler line of CC_aC_b is given by the equation
$y=y_C/2+(x-x_C)*(y_C/2-((y_C+((x_C-1)^2*y_C)/(y_C^2+(x_C-1)^2)+(x_C^2*y_C)/(y_C^2+x_C^2))/3))/(x_C-((x_C+(y_C^2+(x_C-1)^2*x_C)/(y_C^2+(x_C-1)^2)+x_C^3/(y_C^2+x_C^2))/3))$.
Consider the point whose coordinates are given by
$(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2)),((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2)))$.
We verify that this point lies on the Euler lines of AA_bA_c, BB_aB_c, and CC_aC_b since we calculate

$(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))+(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-(((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3))*((y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)))-(-((x_C-1)*y_C)/(2*(y_C^2+(x_C-1)^2))))/((((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3)-(y_C^2/(2*(y_C^2+(x_C-1)^2))))-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2)))=0,$

$((x_C*y_C)/(2*(y_C^2+x_C^2)))+(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2+2*x_C^2)/(2*(y_C^2+x_C^2))))*(((x_C*y_C)/(2*(y_C^2+x_C^2)))-(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))/(((y_C^2+2*x_C^2)/(2*(y_C^2+x_C^2)))-((1+(y_C^4+2*x_C^2*y_C^2-x_C*y_C^2+x_C^4-2*x_C^3+x_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+x_C^2/(y_C^2+x_C^2))/3))-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2)))=0,$

and

$y_C/2+(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-x_C)*(y_C/2-((y_C+((x_C-1)^2*y_C)/(y_C^2+(x_C-1)^2)+(x_C^2*y_C)/(y_C^2+x_C^2))/3))/(x_C-((x_C+(y_C^2+(x_C-1)^2*x_C)/(y_C^2+(x_C-1)^2)+x_C^3/(y_C^2+x_C^2))/3))-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2)))=0$

as desired.

Now, consider the circle whose equation is given by

$(x-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2+(y-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2=0.$

We verify that this point and the centroids of AA_bA_c, BB_aB_c, and CC_aC_b lie on this circle since we calculate

$(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+$

$$9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2+(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2=0,$$

$$(((x_C*y_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+y_C^2/(y_C^2+(x_C-1)^2))/3-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2+(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2=0,$$

$$((1+(y_C^4+2*x_C^2*y_C^2-x_C*y_C^2+x_C^4-2*x_C^3+x_C^2)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))+x_C^2/(y_C^2+x_C^2))/3-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2+(y_C^3/(3*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2=0,$$

and

$$((x_C+(y_C^2+(x_C-1)^2*x_C)/(y_C^2+(x_C-1)^2)+x_C^3/(y_C^2+x_C^2))/3-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2+((y_C+((x_C-1)^2*y_C)/(y_C^2+(x_C-1)^2)+(x_C^2*y_C)/(y_C^2+x_C^2))/3-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(-((x_C-1)*y_C^8+(6*x_C^3-15*x_C^2+9*x_C-1)*y_C^6+x_C*(12*x_C^4-45*x_C^3+48*x_C^2-18*x_C+2)*y_C^4+(x_C-1)^2*x_C^2*(10*x_C^3-$$

$29*x_C^2+13*x_C-2)*y_C^2+3*x_C^9-18*x_C^8+42*x_C^7-48*x_C^6+27*x_C^5-6*x_C^4)/((y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^2*(y_C^2+10*x_C^2-10*x_C+1)+9*(x_C-1)^2*x_C^2))-((y_C^2*(3*y_C^2+6*x_C^2-2*x_C+1)+3*(x_C-1)^2*x_C^2)/(6*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2-(((y_C^2+(x_C-1)*x_C)*((3*x_C^2-3*x_C+1)*y_C^6+(15*x_C^4-30*x_C^3+24*x_C^2-9*x_C+1)*y_C^4+3*(x_C-1)^2*x_C^2*(7*x_C^2-7*x_C+2)*y_C^2+9*x_C^8-36*x_C^7+54*x_C^6-36*x_C^5+9*x_C^4))/(y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2)*(y_C^4+(10*x_C^2-10*x_C+1)*y_C^2+9*x_C^4-18*x_C^3+9*x_C^2))-((y_C^6+(5*x_C^2-5*x_C+3)*y_C^4+(x_C-1)*x_C*(7*x_C^2-7*x_C+1)*y_C^2+3*x_C^6-9*x_C^5+9*x_C^4-3*x_C^3)/(6*y_C*(y_C^2+(x_C-1)^2)*(y_C^2+x_C^2))))^2=0$ as desired. □

That the particular Euler lines here are concurrent was first mentioned two weeks ago in Euclid #8078 by Antreas Hatzipolakis, who generalized this notion by remarking that it holds when the orthic triangle is replaced with a triangle which is homothetic with the orthic triangle about the orthocenter.

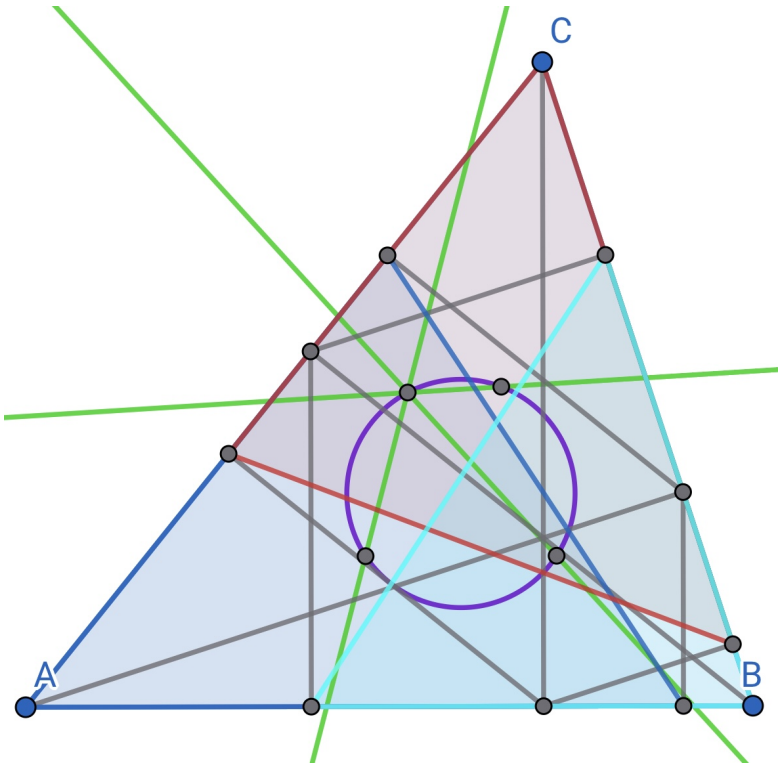

FIGURE 1. Illustration.

REFERENCES

[1] Kimberling, Clark, "X(9730)", *Encyclopedia of Triangle Centers*, 3/21/2025, https://faculty.evansville.edu/ck6/encyclopedia/ETCPart5.htmlX9730.