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Proof of the Circle Centered at X(68325)

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1. Results

Theorem 1.1. *Let ABC be a triangle.*
Let A_b be the reflection of A about the median through B.
Define A_c, B_a, B_c, C_a, and C_b cyclically.
Let P_A be the circumcenter of A, B_c, and C_b.
Define P_B and P_C cyclically.
Then the centroids of ABC, AP_BP_C, BP_AP_C, and CP_AP_B are concyclic about $X(68325)$.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
The median through A is given by the equation $y = (y_C * x)/(x_C + 1)$.
The median through B is given by the equation $y = (y_C * (x - 1))/(x_C - 2)$.
The median through C is given by the equation $y = (y_C * (2 * x - 1))/(2 * x_C - 1)$.
A_b has coordinates
$((2 * y_C^2)/(y_C^2 + (x_C - 2)^2), -(2 * (x_C - 2) * y_C)/(y_C^2 + (x_C - 2)^2))$.
A_c has coordinates
$((4 * y_C^2)/(4 * y_C^2 + (2 * x_C - 1)^2), -(2 * (2 * x_C - 1) * y_C)/(4 * y_C^2 + (2 * x_C - 1)^2))$.
B_a has coordinates
$(2 * (x_C + 1)^2/(x_C^2 + 2 * x_C + y_C^2 + 1) - 1, (2 * (x_C + 1) * y_C)/(y_C^2 + (x_C + 1)^2))$.
B_c has coordinates
$((2 * x_C - 1)^2/(4 * y_C^2 + (2 * x_C - 1)^2), (2 * (2 * x_C - 1) * y_C)/(4 * y_C^2 + (2 * x_C - 1)^2))$.
C_a has coordinates

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$(((x_C+2)*y_C^2+x_C*(x_C+1)^2)/(y_C^2+(x_C+1)^2), (y_C*(y_C^2+x_C^2-1))/(y_C^2+(x_C+1)^2))$.
C_b has coordinates
$(((x_C-2)*(y_C^2+(x_C-2)*x_C))/(y_C^2+(x_C-2)^2), (y_C*(y_C^2+(x_C-2)*x_C))/(y_C^2+(x_C-2)^2))$.
P_A has coordinates
$((-2*y_C^2-2*x_C^2-1)/6+x_C, ((2*x_C-1)*(y_C^2+(x_C-2)*(x_C-1)))/(6*y_C))$.
P_B has coordinates
$((2*y_C^2+2*x_C^2+2*x_C+3)/6, -((2*x_C-1)*(y_C^2+x_C^2+x_C))/(6*y_C))$.
P_C has coordinates
$((4*x_C+1)/6, (3*y_C^2-x_C^2+x_C+2)/(6*y_C))$.
The centroid of ABC has coordinates $((x_C+1)/3, y_C/3)$.
The centroid of AP_BP_C has coordinates
$((y_C^2+(x_C+1)*(x_C+2))/9, -((x_C-2)*y_C^2+(x_C-1)*(x_C+1)^2)/(9*y_C))$.
The centroid of BP_AP_C has coordinates
$(-(y_C^2+x_C^2-5*x_C-3)/9, ((x_C+1)*y_C^2+(x_C-2)^2*x_C)/(9*y_C))$.
The centroid of CP_AP_B has coordinates
$((7*x_C+1)/9, y_C/3-(2*x_C-1)^2/(9*y_C))$.
Consider the circle centered at $X(68325)$ which passes through the centroid of ABC, whose equation is given by
$(x-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2+(y-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2-((x_C+1)/3-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2-(y_C/3-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2=0$.
We verify that the centroids of AP_BP_C, BP_AP_C, and CP_AP_B lie on this circle since we calculate
$((y_C^2+(x_C+1)*(x_C+2))/9-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2+(-((x_C-2)*y_C^2+(x_C-1)*(x_C+1)^2)/(9*y_C)-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2-((x_C+1)/3-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2-(y_C/3-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2=0$,
$(-(y_C^2+x_C^2-5*x_C-3)/9-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2+(((x_C+1)*y_C^2+(x_C-2)^2*x_C)/(9*y_C)-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2-((x_C+1)/3-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2-(y_C/3-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2=0$,
and
$((7*x_C+1)/9-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2+(y_C/3-(2*x_C-1)^2/(9*y_C)-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2-((x_C+1)/3-(-((2*x_C-1)*y_C^2+2*(x_C+1)^2*(2*x_C-7))/54-((x_C-2)*(x_C+1)*(2*x_C-1)*(x_C^2-x_C+1))/(54*y_C^2)))^2-(y_C/3-(-(2*(y_C^2*(y_C^2+2*x_C^2-2*x_C-7)+(x_C-1)*x_C*(x_C^2-x_C+5))-1)/(54*y_C)))^2=0$
as desired. □

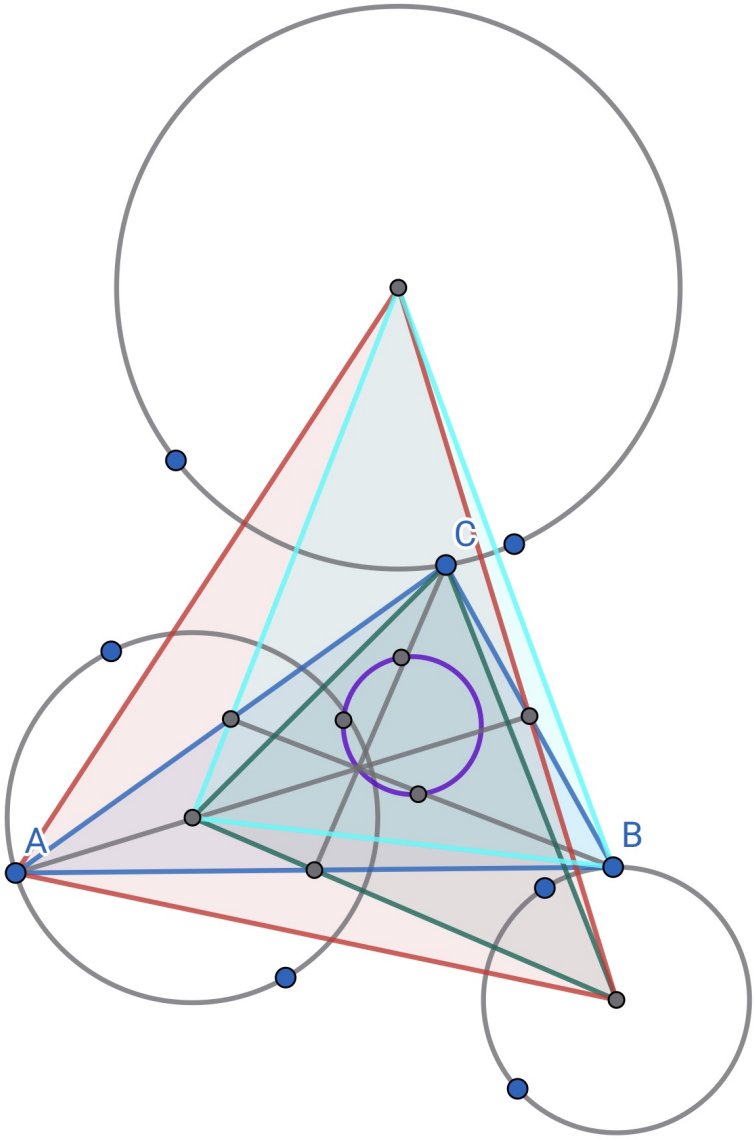

Figure 1. Illustration.

References

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