International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 10, 2025, pp. 255-259.
Received May 5, 2025. Published on-line July 16, 2025
web: http://www.journal-1.eu/
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The Orthocenter Lies on the Circumcircle of the Circumcevian Triangle Reflected about the Pedal Triangle

Benjamin Lee Warren[a] [2]
e-mail: warrenbenjamin52@yahoo.com
web: https://orcid.org/0000-0002-2829-6953

Mathematics Subject Classification (2010). 51N20.

1. Introduction

We justify the claim made about triangles in Euclid #8407.

2. Results

Theorem 2.1. *Let ABC be a triangle and P a point in the plane with $A'B'C'$ and $A''B''C''$ the pedal and circumcevian triangles respectively.*
Let H be the orthocenter of ABC.
Let A''' be the reflection of A'' about A'.
Define B''' and C''' cyclically.
Let line PA''' intersect the altitude through A at P_A.
Define P_B and P_C cyclically.
Then $H, A''', B''', C''', P_A, P_B$, and P_C are concyclic.

Proof. Without loss of generality, we will work in Cartesian coordinates.
A has coordinates $(0, 0)$.
B has coordinates $(1, 0)$.
C has coordinates (x_C, y_C).
P has coordinates (x_P, y_P).
H has coordinates $(x_C, x_C * (1 - x_C)/y_C)$.
A' has coordinates
$((y_C * ((x_C - 1) * y_P + y_C) + (x_C - 1)^2 * x_P)/(y_C^2 + (x_C - 1)^2), (y_C * (y_C * y_P + (x_C - 1) * x_P - x_C + 1))/(y_C^2 + (x_C - 1)^2))$.

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[2]Corresponding author

B' has coordinates
$((x_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2), (y_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2))$.
C' has coordinates $(x_P, 0)$.
A'' has coordinates
$((x_P * ((y_C^2 + (x_C - 1) * x_C) * y_P + x_P * y_C))/(y_C * (y_P^2 + x_P^2)), (y_P * ((y_C^2 + (x_C - 1) * x_C) * y_P + x_P * y_C))/(y_C * (y_P^2 + x_P^2)))$.
B'' has coordinates
$((y_P * (y_C * (y_P + (x_P - 1) * y_C) + (x_C - 1) * x_C * (x_P - 1)))/(y_C * (y_P^2 + (x_P - 1)^2)), (y_P * ((y_C^2 + (x_C - 1) * x_C) * y_P - (x_P - 1) * y_C))/(y_C * (y_P^2 + (x_P - 1)^2)))$.
C'' has coordinates
$(((x_C * y_P - x_P * y_C) * (y_C * (y_P - y_C) + (x_C - 1) * (x_P - x_C)))/(y_C * ((y_P - y_C)^2 + (x_P - x_C)^2)), (((x_C - 1) * y_P - (x_P - 1) * y_C) * (x_C * y_P - x_P * y_C))/(y_C * ((y_P - y_C)^2 + (x_P - x_C)^2)))$.
A''' has coordinates
$(2*((y_C*((x_C-1)*y_P+y_C)+(x_C-1)^2*x_P)/(y_C^2+(x_C-1)^2))-((x_P*((y_C^2+(x_C-1)*x_C)*y_P+x_P*y_C))/(y_C*(y_P^2+x_P^2))), 2*((y_C*(y_C*y_P+(x_C-1)*x_P-x_C+1))/(y_C^2+(x_C-1)^2))-((y_P*((y_C^2+(x_C-1)*x_C)*y_P+x_P*y_C))/(y_C*(y_P^2+x_P^2))))$.
B''' has coordinates
$(2*((x_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-((y_P*(y_C*(y_P+(x_P-1)*y_C)+(x_C-1)*x_C*(x_P-1)))/(y_C*(y_P^2+(x_P-1)^2))), 2*((y_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-((y_P*((y_C^2+(x_C-1)*x_C)*y_P-(x_P-1)*y_C))/(y_C*(y_P^2+(x_P-1)^2))))$.
C''' has coordinates
$(2*x_P-(((x_C*y_P-x_P*y_C)*(y_C*(y_P-y_C)+(x_C-1)*(x_P-x_C)))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2))), -((((x_C-1)*y_P-(x_P-1)*y_C)*(x_C*y_P-x_P*y_C))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2))))$.
The altitude through A is given by the equation $y = x * (1 - x_C)/y_C$.
The altitude through B is given by the equation $y = (x - 1) * (-x_C)/y_C$.
The altitude through C is given by the equation $x = x_C$.
Line PA''' is given by the equation
$y = y_P + (x - x_P) * (y_P - (2 * ((y_C * (y_C * y_P + (x_C - 1) * x_P - x_C + 1))/(y_C^2 + (x_C-1)^2)) - ((y_P * ((y_C^2 + (x_C - 1) * x_C) * y_P + x_P * y_C))/(y_C * (y_P^2 + x_P^2)))))/(x_P - (2 * ((y_C * ((x_C - 1) * y_P + y_C) + (x_C - 1)^2 * x_P)/(y_C^2 + (x_C - 1)^2)) - ((x_P * ((y_C^2 + (x_C - 1) * x_C) * y_P + x_P * y_C))/(y_C * (y_P^2 + x_P^2)))))$.
Line PB''' is given by the equation
$y = y_P + (x - x_P) * (y_P - (2 * ((y_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2)) - ((y_P * ((y_C^2 + (x_C - 1) * x_C) * y_P - (x_P - 1) * y_C))/(y_C * (y_P^2 + (x_P - 1)^2)))))/(x_P - (2 * ((x_C * (x_C * x_P + y_C * y_P))/(x_C^2 + y_C^2)) - ((y_P * (y_C * (y_P + (x_P - 1) * y_C) + (x_C - 1) * x_C * (x_P - 1)))/(y_C * (y_P^2 + (x_P - 1)^2)))))$.
Line PC''' is given by the equation
$y = y_P + (x - x_P) * (y_P - (-((((x_C - 1) * y_P - (x_P - 1) * y_C) * (x_C * y_P - x_P * y_C))/(y_C * ((y_P - y_C)^2 + (x_P - x_C)^2)))))/(x_P - (2 * x_P - (((x_C * y_P - x_P * y_C) * (y_C * (y_P - y_C) + (x_C - 1) * (x_P - x_C)))/(y_C * ((y_P - y_C)^2 + (x_P - x_C)^2)))))$.
P_A has coordinates
$(-(2*y_C^2*((x_C-1)*y_P-(x_P-1)*y_C)*(y_P^2+x_P^2))/((y_C^2+(x_C-1)^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C)), (2*(x_C-1)*y_C*((x_C-1)*y_P-(x_P-1)*y_C)*(y_P^2+x_P^2))/((y_C^2+(x_C-1)^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C)))$.
P_B has coordinates
$(-(2*x_C*y_C^2*y_P^3-y_C*((2*x_P+1)*y_C^2+x_C^2)*y_P^2+(y_C^4+x_C*(2*x_P^2-4*x_P+2*x_C+1)*y_C^2+x_C^4-x_C^3)*y_P-(x_P-1)*x_P*(2*x_P-1)*y_C^3-x_C^2*(x_P-1)*x_P*y_C)/((y_C^2+x_C^2)*$

$(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C)), (2*x_C*y_C*(x_C*y_P-x_P*y_C)*(y_P^2+(x_P-1)^2))/((y_C^2+x_C^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C))).$
P_C has coordinates
$(x_C, y_P+(x_C-x_P)*(y_P-(-((((x_C-1)*y_P-(x_P-1)*y_C)*(x_C*y_P-x_P*y_C))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2))))))/(x_P-(2*x_P-(((x_C*y_P-x_P*y_C)*(y_C*(y_P-y_C)+(x_C-1)*(x_P-x_C)))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2))))))).$
Consider the circle whose equation is given by
$(x-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(y-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0.$
We verify that $H, A''', B''', C''', P_A, P_B$, and P_C lie on this circle since we calculate
$(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$
$(2*((y_C*((x_C-1)*y_P+y_C)+(x_C-1)^2*x_P)/(y_C^2+(x_C-1)^2))-((x_P*((y_C^2+(x_C-1)*x_C)*y_P+x_P*y_C))/(y_C*(y_P^2+x_P^2)))-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(2*((y_C*(y_C*y_P+(x_C-1)*x_P-x_C+1))/(y_C^2+(x_C-1)^2))-((y_P*((y_C^2+(x_C-1)*x_C)*y_P+x_P*y_C))/(y_C*(y_P^2+x_P^2)))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$
$(2*((x_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-((y_P*(y_C*(y_P+(x_P-1)*y_C)+(x_C-1)*x_C*(x_P-1)))/(y_C*(y_P^2+(x_P-1)^2)))-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(2*((y_C*(x_C*x_P+y_C*y_P))/(x_C^2+y_C^2))-((y_P*((y_C^2+(x_C-1)*x_C)*y_P-(x_P-1)*y_C))/(y_C*(y_P^2+(x_P-1)^2)))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+$

$$(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$$

$$(2*x_P-(((x_C*y_P-x_P*y_C)*(y_C*(y_P-y_C)+(x_C-1)*(x_P-x_C)))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2)))-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(-((((x_C-1)*y_P-(x_P-1)*y_C)*(x_C*y_P-x_P*y_C))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2)))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$$

$$(-(2*y_C^2*((x_C-1)*y_P-(x_P-1)*y_C)*(y_P^2+x_P^2))/((y_C^2+(x_C-1)^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C))-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+((2*(x_C-1)*y_C*((x_C-1)*y_P-(x_P-1)*y_C)*(y_P^2+x_P^2))/((y_C^2+(x_C-1)^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$$

$$(-(2*x_C*y_C^2*y_P^3-y_C*((2*x_P+1)*y_C^2+x_C^2)*y_P^2+(y_C^4+x_C*(2*x_P^2-4*x_P+2*x_C+1)*y_C^2+x_C^4-x_C^3)*y_P-(x_P-1)*x_P*(2*x_P-1)*y_C^3-x_C^2*(x_P-1)*x_P*y_C)/((y_C^2+x_C^2)*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+((2*x_C*y_C*(x_C*y_P-x_P*y_C)*(y_P^2+(x_P-1)^2))/((y_C^2+x_C^2)*(y_P*(y_C*(y_P-y_C)-(x_C-1)*x_C)+(x_P-1)*x_P*y_C))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0,$$

and

$$(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2+(y_P+(x_C-x_P)*(y_P-(-((((x_C-1)*y_P-(x_P-1)*y_C)*(x_C*y_P-x_P*y_C))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2)))))/(x_P-(2*x_P-(((x_C*y_P-x_P*y_C)*(y_C*(y_P-y_C)+(x_C-1)*(x_P-x_C)))/(y_C*((y_P-y_C)^2+(x_P-x_C)^2)))))-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2-(x_C-(((2*x_P-2*x_C+1)*y_C*y_P^2-(y_C^2+(x_C-1)*x_C*(4*x_P-1))*y_P+(x_P-1)*x_P*(2*(x_P+x_C)-1)*y_C)/(2*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+$$

$(x_P-1)*x_P*y_C))))^2-(x_C*(1-x_C)/y_C-((2*y_C^2*y_P^3-y_C*(3*y_C^2+(x_C-1)*x_C)*y_P^2+(y_C^4+2*(x_P-x_C)^2*y_C^2+x_C^4-2*x_C^3+x_C^2)*y_P+(x_P-1)*x_P*y_C^3-(x_C-1)*x_C*(x_P-1)*x_P*y_C)/(2*y_C*(y_C*y_P^2-(y_C^2+(x_C-1)*x_C)*y_P+(x_P-1)*x_P*y_C))))^2=0$
as desired. □

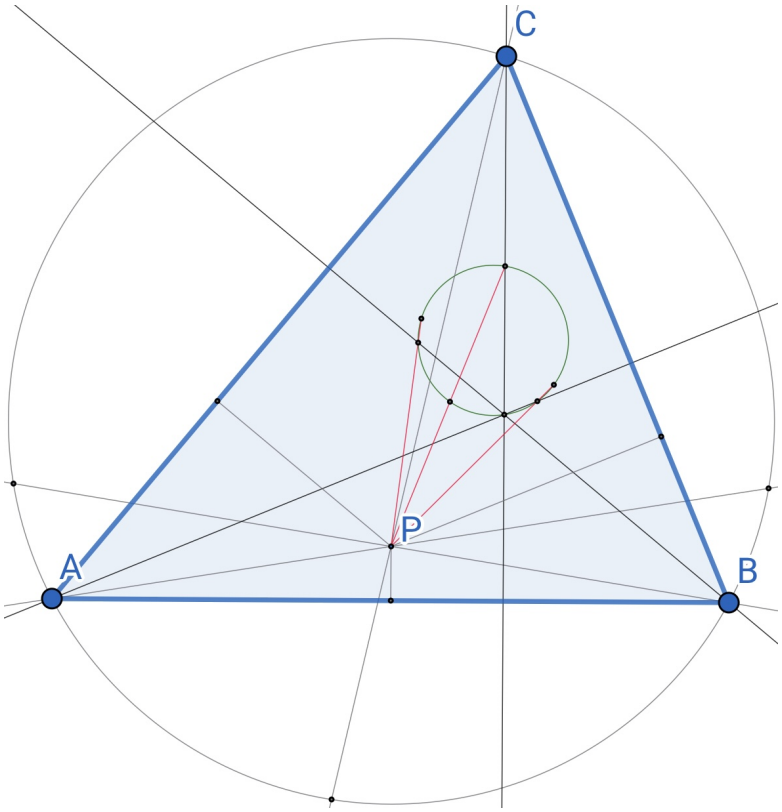

Figure 1. Illustration.

References

[1] Weisstein, Eric W. "Orthocenter." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/Orthocenter.html