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Rough Wasan Geometry: A Set-Theoretic Approach to Traditional Japanese Geometry

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Abstract.

A rough set approximates a target concept by constructing lower and upper approximations based on equivalence-class indiscernibility [1, 2]. Rough Geometry studies geometric objects under approximation, boundary regions, and indiscernibility, extending classical geometry with uncertainty representation. Wasan Geometry is a traditional Japanese branch of mathematics that focuses on elegant synthetic solutions to problems involving circles, polygons, and other geometric figures. In this paper, we extend Wasan Geometry within the framework of rough sets and investigate the resulting structure, which we call *Rough Wasan Geometry*.

Keywords. Rough Geometry, Geometry, Rough Wasan Geometry, Wasan Geometry

Mathematics Subject Classification (2010). 51M04

1 Preliminaries

This section collects the basic notions used throughout the paper. Unless explicitly stated otherwise, all sets and structures considered here are finite.

1.1 Rough Geometry

A rough set models a concept through a pair of approximations derived from an indiscernibility relation—that is, an equivalence relation—on a given universe [1, 2, 2]. Several related extensions of rough sets have been developed, including Fuzzy Rough Sets [3, 4], Neutrosophic Rough Sets [5–7], Soft Rough Sets [8, 9], and HyperRough Sets [10, 11]. Rough Geometry applies the principles of rough sets to geometric contexts, examining shapes and configurations under approximation (cf. [12–14]). It emphasizes the study of lower and upper geometric representations, boundary regions, and indistinguishability, thus extending classical geometric theory to accommodate uncertainty in a systematic and mathematically rigorous manner.

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Definition 1.1 (Rough Set Approximation)**.** [15–17] Let $X \neq \varnothing$ be a universe and let $R \subseteq X \times X$ be an equivalence (indiscernibility) relation. For $x \in X$, write

$$[x]_R \;:=\; \{\, y \in X : (x, y) \in R \,\}.$$

Given any $U \subseteq X$, the *lower* and *upper* R-approximations of U are

$$\underline{U} \;:=\; \{\, x \in X : [x]_R \subseteq U \,\}, \qquad \overline{U} \;:=\; \{\, x \in X : [x]_R \cap U \neq \varnothing \,\}.$$

Then

$$\underline{U} \;\subseteq\; U \;\subseteq\; \overline{U}.$$

Indeed, if $x \in \underline{U}$ then $[x]_R \subseteq U$, and since $x \in [x]_R$, we have $x \in U$. Conversely, if $x \in U$ then $[x]_R \cap U \supseteq \{x\} \neq \varnothing$, hence $x \in \overline{U}$.

Example 1.2 (Rough Set: a finite computation of lower/upper approximations)**.** Let the universe be $X = \{a, b, c, d, e, f\}$ and let R be an equivalence relation whose equivalence classes are

$$C_1 = \{a, b\}, \qquad C_2 = \{c, d\}, \qquad C_3 = \{e, f\}.$$

Fix $U = \{a, b, c\} \subseteq X$. Then, by definition,

$$\underline{U} = \{\, x \in X : [x]_R \subseteq U \,\}, \qquad \overline{U} = \{\, x \in X : [x]_R \cap U \neq \varnothing \,\}.$$

Since $C_1 \subseteq U$ while $C_2 \nsubseteq U$ and $C_3 \nsubseteq U$, we obtain

$$\underline{U} = C_1 = \{a, b\}.$$

Moreover, $C_1 \cap U \neq \varnothing$ and $C_2 \cap U \neq \varnothing$ (because $c \in U$), whereas $C_3 \cap U = \varnothing$. Hence

$$\overline{U} = C_1 \cup C_2 = \{a, b, c, d\}.$$

Consequently, the boundary region is $\overline{U} \setminus \underline{U} = \{c, d\}$, and the negative region is $X \setminus \overline{U} = \{e, f\}$.

Definition 1.3 (Rough Geometry)**.** [12] Let $(U, \mathcal{R})$ be an approximation space, where $U \neq \varnothing$ and $\mathcal{R} \subseteq U \times U$ is an equivalence relation. The *rough geometry* on $(U, \mathcal{R})$ is a pair

$$\mathcal{G}_{\text{rough}} = (\mathcal{S}, \mathcal{A}),$$

with the following components:

(i) $\mathcal{S}$ is a family of *rough geometric objects*, each object being a pair (X^ℓ, X^u) obtained from some $X \subseteq U$ via

$$X^\ell = \{\, x \in U :\; [x]_{\mathcal{R}} \subseteq X \,\}, \qquad X^u = \{\, x \in U :\; [x]_{\mathcal{R}} \cap X \neq \varnothing \,\},$$

and satisfying $X^\ell \subseteq X^u$.

(ii) $\mathcal{A}$ is a collection of *rough geometric operations* acting componentwise on such pairs. For instance, for $A = (A^\ell, A^u)$ and $B = (B^\ell, B^u)$ in $\mathcal{S}$,

$$A \sqcup B := (A^\ell \cup B^\ell,\; A^u \cup B^u), \quad A \sqcap B := (A^\ell \cap B^\ell,\; A^u \cap B^u), \quad A^{\complement} := (\, U \setminus A^u,\; U \setminus A^\ell \,),$$

which are the standard rough union, intersection, and complement.

Example 1.4 (Rough Geometry: a pixelated shape under 2×2 indiscernibility)**.** Let $U = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ be a 4×4 grid of lattice points. Define an equivalence relation $\mathcal{R}$ whose classes are the four 2×2 blocks:

$$B_{11} = \{(1,1), (1,2), (2,1), (2,2)\}, \quad B_{12} = \{(1,3), (1,4), (2,3), (2,4)\},$$
$$B_{21} = \{(3,1), (3,2), (4,1), (4,2)\}, \quad B_{22} = \{(3,3), (3,4), (4,3), (4,4)\}.$$

Consider the "diamond" (Manhattan ball) of radius 1 centered at $(2, 2)$,

$$D = \{(2,2), (1,2), (2,1), (2,3), (3,2)\}.$$

Augment it by $(1, 1)$ to obtain $X := D \cup \{(1, 1)\}$. Then the rough geometric object associated with X is (X^ℓ, X^u) where

$$X^\ell = \{\, p \in U : [p]_\mathcal{R} \subseteq X \,\}, \qquad X^u = \{\, p \in U : [p]_\mathcal{R} \cap X \neq \varnothing \,\}.$$

Since $B_{11} \subseteq X$ while $B_{12} \nsubseteq X$, $B_{21} \nsubseteq X$, $B_{22} \cap X = \varnothing$, we have

$$X^\ell = B_{11}, \qquad X^u = B_{11} \cup B_{12} \cup B_{21}.$$

Therefore the rough boundary is $X^u \setminus X^\ell = B_{12} \cup B_{21}$, which captures the uncertain "fringe" of the diamond under the 2×2 indiscernibility.

1.2 Wasan Geometry

Wasan Geometry concerns traditional Japanese geometric problem solving—especially configurations built from circles and polygons—and emphasizes synthetic, elegant arguments rather than analytic methods (cf. [18–22]).

Definition 1.5 (Wasan Geometry)**.** *Wasan Geometry* denotes the corpus of geometric techniques developed in Japan during the Edo era (1603–1868). Problems and solutions were frequently recorded on wooden votive tablets (*sangaku*) dedicated at shrines and temples, and typically featured circle tangencies, polygonal arrangements, and related symmetric figures [23–25].

Formally, a *Wasan Geometric Framework* is a pair

$$\mathcal{W} = (P, \mathcal{R}),$$

where

- P is a (finite or infinite) set of geometric primitives and figures (points, lines, circles, polygons, etc.),
- $\mathcal{R}$ is a family of relations and constructive rules reflecting classical Wasan methods (e.g., circle tangency conditions, area/length ratios, and nested circular or polygonal configurations).

Typical outcomes are closed-form expressions for lengths, areas, or radii in highly symmetric settings, obtained through synthetic and arithmetic reasoning rather than calculus.

Example 1.6 (Wasan Geometry: two circles tangent to both axes and to each other)**.** Place two circles in the first quadrant, each tangent to both coordinate axes. Let the larger circle have radius R and center (R, R); the smaller has radius r and center (r, r) with $0 < r \leq R$. External tangency yields that the distance between centers equals $R + r$:

$$\sqrt{(R - r)^2 + (R - r)^2} = R + r \iff \sqrt{2}\,(R - r) = R + r.$$

Solving,

$$\sqrt{2}\,R - \sqrt{2}\,r = R + r \iff (\sqrt{2} + 1)r = (\sqrt{2} - 1)R$$

$$\iff r = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}\,R = \frac{(\sqrt{2} - 1)^2}{2 - 1}\,R = (3 - 2\sqrt{2})\,R.$$

Thus the smaller radius is $r = (3 - 2\sqrt{2})R$, a classic sangaku-style relation.

2 Result: Rough Wasan Geometry

Rough Wasan Geometry extends classical Wasan geometry by integrating rough set approximations, modeling traditional Japanese geometric objects under uncertainty.

Definition 2.1 (Rough Wasan Geometry)**.** Let $\mathcal{W} = (P, \mathcal{R}_{\mathrm{W}})$ be a Wasan framework and let $\approx$ be an equivalence (indiscernibility) relation on P. For any $X \subseteq P$ set

$$X^{\ell} := \{\, p \in P : [p]_{\approx} \subseteq X \,\}, \qquad X^{u} := \{\, p \in P : [p]_{\approx} \cap X \neq \varnothing \,\}.$$

Define the universe of rough objects by

$$\mathcal{S}_{\mathrm{RW}} := \{\, (X^{\ell}, X^{u}) \mid X \subseteq P \,\}.$$

Define the basic rough operations componentwise, for $A = (A^{\ell}, A^{u})$ and $B = (B^{\ell}, B^{u})$ in $\mathcal{S}_{\mathrm{RW}}$,

$$A \sqcup B := (A^{\ell} \cup B^{\ell},\ A^{u} \cup B^{u}), \qquad A \sqcap B := (A^{\ell} \cap B^{\ell},\ A^{u} \cap B^{u}), \qquad A^{\complement} := (\,P \setminus A^{u},\ P \setminus A^{\ell}\,).$$

For each m-ary $R \in \mathcal{R}_{\mathrm{W}}$, its *rough lifting* is the relation $\widehat{R} \subseteq \mathcal{S}_{\mathrm{RW}}^{m}$ defined by

$$\big((A_1^{\ell}, A_1^{u}), \ldots, (A_m^{\ell}, A_m^{u})\big) \in \widehat{R} \iff A_1^{\ell} \times \cdots \times A_m^{\ell} \subseteq R \subseteq A_1^{u} \times \cdots \times A_m^{u}.$$

The *Rough Wasan Geometry* associated with $(\mathcal{W}, \approx)$ is

$$\mathcal{RW}(\mathcal{W}, \approx) := \big(\mathcal{S}_{\mathrm{RW}},\ \sqcup,\ \sqcap,\ {}^{\complement},\ \{\widehat{R}\}_{R \in \mathcal{R}_{\mathrm{W}}}\big).$$

Example 2.2 (Right-triangle incircle under indiscernibility (Rough Wasan))**.** Fix a Wasan framework whose relations include $\mathsf{Tangent}(\cdot,\cdot)$ between a circle and a line. Let ABC be a right triangle with side lengths $(a, b, c) = (3, 4, 5)$ and incircle radius

$$r = \frac{a + b - c}{2} = \frac{3 + 4 - 5}{2} = 1.$$

Let $P = \{\triangle ABC,\ s_a,\ s_b,\ s_c,\ I,\ I'\}$, where s_a, s_b, s_c are the three sides, I is the incircle, and I' is an indistinguishable "duplicate" of the incircle produced by measurement noise. Define an indiscernibility relation $\approx$ on P by the equivalence classes

$$C_{\triangle} = \{\triangle ABC\}, \qquad C_{\mathrm{side}} = \{s_a, s_b, s_c\}, \qquad C_{\mathrm{inc}} = \{I, I'\}.$$

Let the intended configuration be

$$X = \{\triangle ABC,\ s_a,\ s_b,\ s_c,\ I\} \subseteq P.$$

Then the rough object (X^{ℓ}, X^{u}) is given by

$$X^{\ell} = \{\triangle ABC\} \cup \{s_a, s_b, s_c\} \quad (\text{since } C_{\triangle}, C_{\mathrm{side}} \subseteq X),$$

$$X^{u} = X^{\ell} \cup \{I, I'\} \quad (\text{since } C_{\mathrm{inc}} \cap X = \{I\} \neq \varnothing).$$

Thus the boundary region is $X^{u} \setminus X^{\ell} = \{I, I'\} \setminus \{I\} = \{I'\}$, encoding the uncertainty about the incircle.

Let $\widehat{\mathsf{Tangent}}$ be the rough lift of $\mathsf{Tangent}$. If every $J \in C_{\mathrm{inc}}$ is tangent to each side $s \in C_{\mathrm{side}}$ (true for I, and by construction for I'), then

$$X_{\mathrm{inc}} := (\varnothing,\ C_{\mathrm{inc}}), \qquad X_{\mathrm{side}} := (C_{\mathrm{side}},\ C_{\mathrm{side}})$$

satisfy

$$X_{\mathrm{inc}}^{\ell} \times X_{\mathrm{side}}^{\ell} = \varnothing \subseteq \mathsf{Tangent} \subseteq C_{\mathrm{inc}} \times C_{\mathrm{side}} = X_{\mathrm{inc}}^{u} \times X_{\mathrm{side}}^{u},$$

hence $\big(X_{\mathrm{inc}}, X_{\mathrm{side}}\big) \in \widehat{\mathsf{Tangent}}$.

Example 2.3 (Two circles tangent to both axes (sangaku-style) with rough small circle)**.** Work in a Wasan framework with relations $\mathsf{TangentAxis}(\cdot,\cdot)$ and $\mathsf{ExtTangent}(\cdot,\cdot)$. Place the coordinate axes as lines L_x, L_y. Let C_R be the circle of radius $R > 0$ centered at (R, R), tangent to both axes. The smaller circle tangent to both axes and externally tangent to C_R has radius

$$r = (3 - 2\sqrt{2})\, R.$$

Let S_1, S_2 be two indistinguishable candidates for this small circle (e.g., two reconstructions from noisy data). Set

$$P = \{L_x, L_y, C_R, S_1, S_2\}.$$

Define $\approx$ with the classes

$$C_{\text{axes}} = \{L_x, L_y\}, \quad C_{\text{big}} = \{C_R\}, \quad C_{\text{small}} = \{S_1, S_2\}.$$

Let the intended configuration be $X = \{L_x, L_y, C_R, S_1\}$. Then

$$X^\ell = C_{\text{axes}} \cup C_{\text{big}} = \{L_x, L_y, C_R\}, \qquad X^u = X^\ell \cup C_{\text{small}} = \{L_x, L_y, C_R, S_1, S_2\}.$$

Thus the boundary is $X^u \setminus X^\ell = \{S_2\}$, capturing uncertainty in the small circle.

For the rough lifts:

$$X_{\text{ax}} := (C_{\text{axes}}, C_{\text{axes}}), \quad X_{\text{big}} := (C_{\text{big}}, C_{\text{big}}), \quad X_{\text{sm}} := (\varnothing, C_{\text{small}}).$$

Since each S_i is tangent to L_x, L_y, we have

$$X^\ell_{\text{sm}} \times X^\ell_{\text{ax}} = \varnothing \subseteq \mathsf{TangentAxis} \subseteq C_{\text{small}} \times C_{\text{axes}} = X^u_{\text{sm}} \times X^u_{\text{ax}}.$$

Likewise, each S_i is externally tangent to C_R, so

$$X^\ell_{\text{sm}} \times X^\ell_{\text{big}} = \varnothing \subseteq \mathsf{ExtTangent} \subseteq C_{\text{small}} \times C_{\text{big}} = X^u_{\text{sm}} \times X^u_{\text{big}}.$$

Hence $\big(X_{\text{sm}}, X_{\text{ax}}\big) \in \widehat{\mathsf{TangentAxis}}$ and $\big(X_{\text{sm}}, X_{\text{big}}\big) \in \widehat{\mathsf{ExtTangent}}$.

Example 2.4 (Square inscribed in a semicircle with uncertain square candidate)**.** Consider a semicircle of radius $R > 0$ centered at the origin with diameter on the x-axis; let S denote this semicircle and L its diameter line. A square of side s centered on the diameter and inscribed in the semicircle satisfies

$$\left(\frac{s}{2}\right)^2 + s^2 = R^2 \quad \Longrightarrow \quad s = \frac{2}{\sqrt{5}}\, R.$$

Let Q_1 be the intended inscribed square and Q_2 a nearly indistinguishable candidate produced by coarse imaging. Set

$$P = \{S,\ L,\ Q_1,\ Q_2\},$$

and define $\approx$ with classes

$$C_{\text{semi}} = \{S\}, \qquad C_{\text{diam}} = \{L\}, \qquad C_{\square} = \{Q_1, Q_2\}.$$

Choose $X = \{S, L, Q_1\}$. Then

$$X^\ell = C_{\text{semi}} \cup C_{\text{diam}} = \{S, L\}, \qquad X^u = X^\ell \cup C_{\square} = \{S, L, Q_1, Q_2\}.$$

Thus the rough boundary is $X^u \setminus X^\ell = \{Q_1, Q_2\} \setminus \{Q_1\} = \{Q_2\}$.

Let $\mathsf{InscribedSquare}(\cdot,\cdot)$ be the relation "square is inscribed in semicircle." Define

$$X_{\square} := (\varnothing,\ C_{\square}), \qquad X_{\text{semi}} := (C_{\text{semi}},\ C_{\text{semi}}).$$

Since Q_1 is inscribed in S (and Q_2 may or may not be), we have

$$X^\ell_{\square} \times X^\ell_{\text{semi}} = \varnothing \ \subseteq\ \mathsf{InscribedSquare} \ \subseteq\ C_{\square} \times C_{\text{semi}} = X^u_{\square} \times X^u_{\text{semi}},$$

so $\big(X_{\square}, X_{\text{semi}}\big) \in \widehat{\mathsf{InscribedSquare}}$.

Theorem 2.5 (Roughness and Generalization)**.** *For every Wasan framework $\mathcal{W} = (P, \mathcal{R}_{\mathrm{W}})$ and every equivalence relation $\approx$ on P, the structure $\mathcal{RW}(\mathcal{W}, \approx)$ satisfies:*

(1) $\mathcal{RW}(\mathcal{W}, \approx)$ is a rough geometry *on the approximation space $(P, \approx)$; namely, its objects are pairs (X^{ℓ}, X^{u}) induced by lower/upper approximations and its operations are the standard rough union, intersection, and complement acting componentwise.*

(2) $\mathcal{RW}(\mathcal{W}, \approx)$ generalizes *the classical Wasan framework: the map*

$$\iota : P \longrightarrow \mathcal{S}_{\mathrm{RW}}, \qquad \iota(p) := (\{p\}, \{p\}),$$

is an injective homomorphism of relational structures $(P, \mathcal{R}_{\mathrm{W}}) \hookrightarrow (\mathcal{S}_{\mathrm{RW}}, \{\widehat{R}\}_{R \in \mathcal{R}_{\mathrm{W}}})$. Moreover, when $\approx = \Delta_P$ (the identity relation), every rough object is crisp, i.e. $(X^{\ell}, X^{u}) = (X, X)$, and the lifted relations $\widehat{R}$ reduce to R. Hence the classical Wasan geometry is recovered as a special case.

Proof. (1) *Rough-geometry axioms.* Fix $\approx$ and write $C := P/\!\approx$ for the set of equivalence classes. Recall the standard equalities for any $X, Y \subseteq P$:

$$(X \cup Y)^{\ell} = \{\, p : [p] \subseteq X \cup Y \,\} = \{\, p : [p] \subseteq X \text{ or } [p] \subseteq Y \,\} = X^{\ell} \cup Y^{\ell},$$

$$(X \cup Y)^{u} = \{\, p : [p] \cap (X \cup Y) \neq \varnothing \,\} = \{\, p : [p] \cap X \neq \varnothing \text{ or } [p] \cap Y \neq \varnothing \,\} = X^{u} \cup Y^{u},$$

$$(X \cap Y)^{\ell} = \{\, p : [p] \subseteq X \cap Y \,\} = \{\, p : [p] \subseteq X \text{ and } [p] \subseteq Y \,\} = X^{\ell} \cap Y^{\ell},$$

$$(P \setminus X)^{\ell} = \{\, p : [p] \subseteq P \setminus X \,\} = \{\, p : [p] \cap X = \varnothing \,\} = P \setminus X^{u},$$

$$(P \setminus X)^{u} = \{\, p : [p] \cap (P \setminus X) \neq \varnothing \,\} = \{\, p : [p] \not\subseteq X \,\} = P \setminus X^{\ell}.$$

From these identities we obtain immediately the componentwise formulas for $\sqcup$ and $^{\complement}$ in the definition above. For $\sqcap$, observe first that $X^{\ell} \cap Y^{\ell}$ and $X^{u} \cap Y^{u}$ are unions of $\approx$-classes (hence "definable"), and $X^{\ell} \cap Y^{\ell} \subseteq X^{u} \cap Y^{u}$. To see that $(X^{\ell} \cap Y^{\ell},\ X^{u} \cap Y^{u})$ is again realized by some subset $Z \subseteq P$, pick one representative $z_C \in C$ for each class $C \in C$ with $C \subseteq (X^{u} \cap Y^{u}) \setminus (X^{\ell} \cap Y^{\ell})$, and set

$$Z \;:=\; (X^{\ell} \cap Y^{\ell}) \;\cup\; \{\, z_C : C \subseteq (X^{u} \cap Y^{u}) \setminus (X^{\ell} \cap Y^{\ell}) \,\}.$$

Then

$$Z^{\ell} = X^{\ell} \cap Y^{\ell}, \qquad Z^{u} = X^{u} \cap Y^{u},$$

because every class $C \subseteq X^{\ell} \cap Y^{\ell}$ lies wholly in Z, and every class $C \subseteq X^{u} \cap Y^{u}$ meets Z by construction. Therefore $\sqcap$ is closed in $\mathcal{S}_{\mathrm{RW}}$, and all three operations coincide with the standard rough union/intersection/complement. Consequently,

$$\mathcal{RW}(\mathcal{W}, \approx) = \big(\mathcal{S}_{\mathrm{RW}},\ \sqcup,\ \sqcap,\ ^{\complement}\big)$$

is a rough geometry on $(P, \approx)$.

(2) *Generalization of Wasan geometry.* Define $\iota : P \to \mathcal{S}_{\mathrm{RW}}$ by $\iota(p) = (\{p\}, \{p\})$. It is injective. For any m-ary $R \in \mathcal{R}_{\mathrm{W}}$ and any $(p_1, \ldots, p_m) \in P^m$, we have

$$\big(\iota(p_1), \ldots, \iota(p_m)\big) \in \widehat{R} \iff \{(p_1, \ldots, p_m)\} \;\subseteq\; R \;\subseteq\; \{(p_1, \ldots, p_m)\},$$

which is equivalent to $(p_1, \ldots, p_m) \in R$. Thus ι is a homomorphism from $(P, \mathcal{R}_{\mathrm{W}})$ into $(\mathcal{S}_{\mathrm{RW}}, \{\widehat{R}\})$.

If, in addition, $\approx = \Delta_P$ (identity), then for every $X \subseteq P$ we have $X^{\ell} = X^{u} = X$. Hence

$$\mathcal{S}_{\mathrm{RW}} = \{(X, X) : X \subseteq P\}, \quad \widehat{R} = \{((A_1, A_1), \ldots, (A_m, A_m)) :\ A_1 \times \cdots \times A_m \subseteq R\}.$$

In particular, on singletons this reduces exactly to R. Therefore the crisp Wasan geometry (on objects P and relations $\mathcal{R}_{\mathrm{W}}$) is recovered inside $\mathcal{RW}(\mathcal{W}, \Delta_P)$ via ι, showing that Rough Wasan Geometry strictly generalizes the classical one. □

3 Conclusion

In this paper, we extended Wasan Geometry by incorporating the framework of rough sets and investigated the resulting structure, termed *Rough Wasan Geometry*. As future work, we aim to explore further generalizations using Neutrosophic Sets [26–28], Vague Sets [29, 30], Uncertain Sets [31, 32], Plithogenic Sets [33, 34], and other uncertainty-based mathematical structures.

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Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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