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Flanks, new flanks, generalized flanks and their properties

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Abstract. Flanks are a concept that Floor van Lamoen gave in 2001 on Forum Geometricorum. In this paper we will give new flanks, generalized flanks and their properties.

Keywords. Flanks, new flanks, generalized flanks. triangle geometry, Euclidean geometry.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Flanks

Floor van Lamoen gives the definition of flanks in [3]. We generalize this concept as follows. Given a triangle ABC with side lengths $BC = a, CA = b$, and $AB = c$. By erecting rectangulars AC_aC_bB, BA_bA_cC, and CB_cB_aA externally on the sides, we form new triangles AB_aC_a, BC_bA_b, and CA_cB_c, which we call the flanks of ABC. See figure 1.

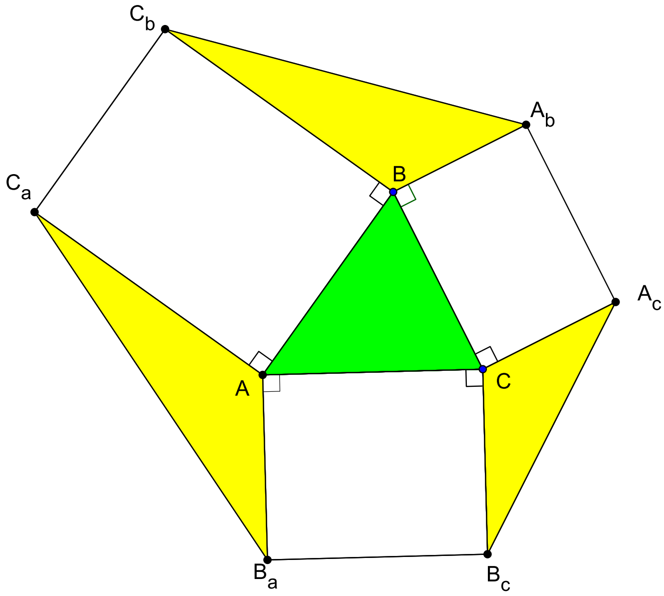

Figure 1. Flanks or new flanks for three outer rectangulars

We now give a generalization of flanks which is new flanks.

2. New flanks

Given a triangle ABC with side lengths $BC = a, CA = b$, and $AB = c$. By erecting rectangulars AC_aC_bB, BA_bA_cC, and CB_cB_aA on the sides, we form new triangles AB_aC_a, BC_bA_b, and CA_cB_c, which we call the new flanks of ABC.

New flanks is a concept contained flanks. New flanks do not need the external orientation. New flanks have arbitrary orientation. See new flanks through figure 1, figure 2, figure 3, figure 4.

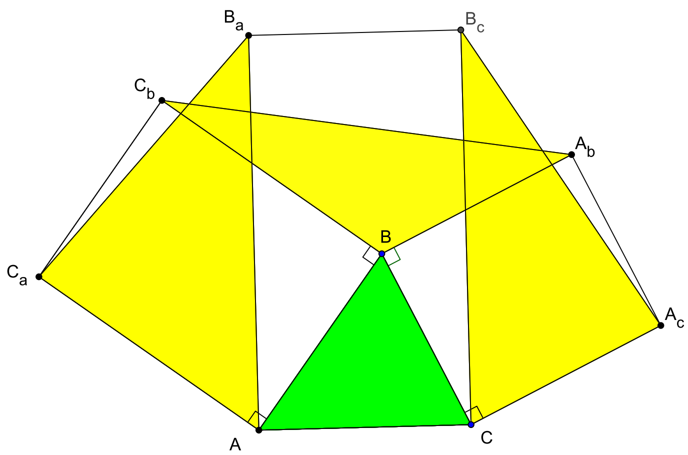

Figure 2. New flanks for one inner rectangular and two outer rectangulars

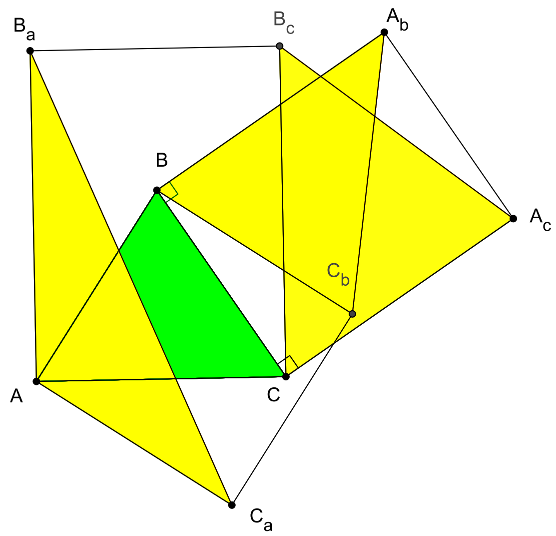

Figure 3. New flanks for one outer rectangular and two inner rectangulars

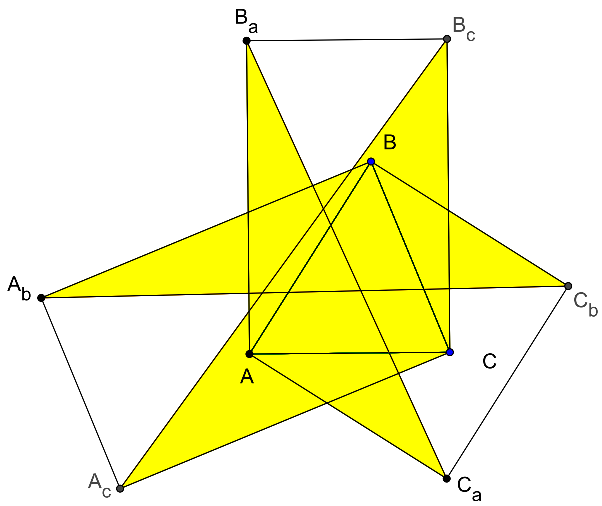

Figure 4. New flanks for three inner rectangulars

3. GENERALIZED FLANKS

Given a triangle ABC with side lengths $BC = a, CA = b$, and $AB = c$. By constructing similar isosceles trapezoids AC_aC_bB, BA_bA_cC, and CB_cB_aA on the sides, we form new triangles AB_aC_a, BC_bA_b, and CA_cB_c, which we call the generalized flanks of ABC. See the figures 5, 6, 7, 8.

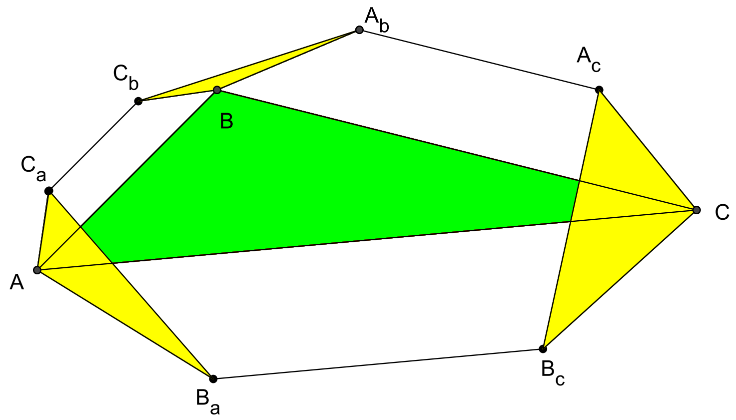

Figure 5. Generalized flanks for three outer isosceles trapezoids

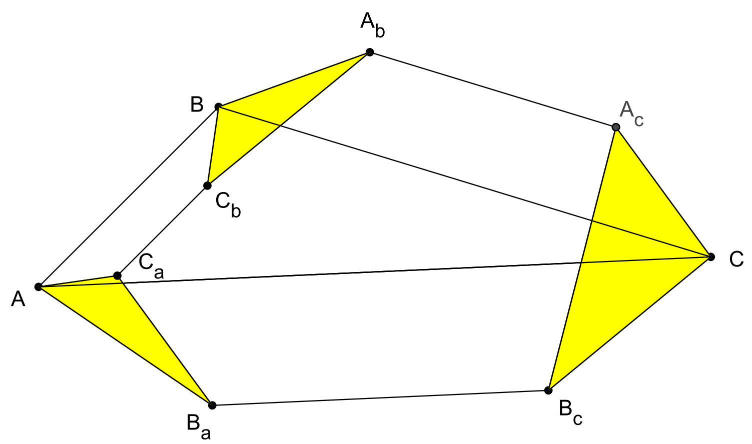

Figure 6. Generalized flanks for one inner isosceles trapezoid and two outer isosceles trapezoids

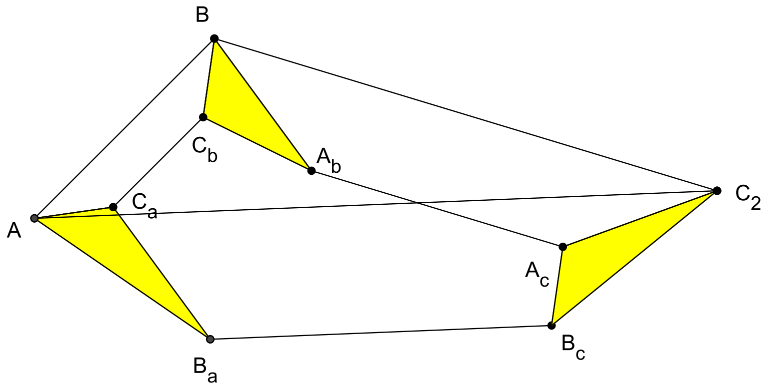

Figure 7. Generalized flanks for two inner isosceles trapezoids and one outer isosceles trapezoid

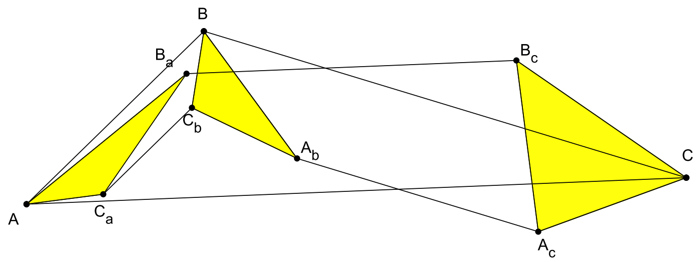

Figure 8. Generalized flanks for three inner isosceles trapezoids

4. The properties of flanks, new flanks and generalized flanks

We now give some properties around flanks, new flanks and generalized flanks.

Theorem 4.1. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of C_bA_b, A_cB_c, B_aC_a perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 9, 10, 11, 12.

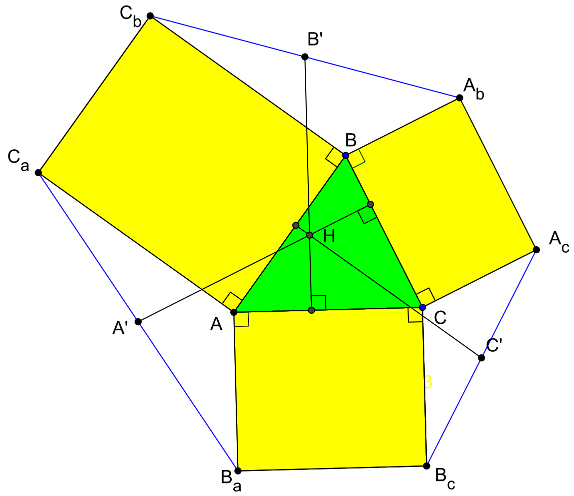

Figure 9. New flanks for three outer rectangulars

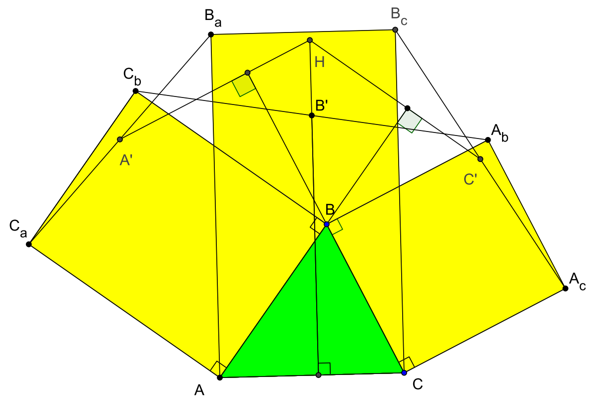

Figure 10. New flanks for one inner rectangular and two outer rectangulars

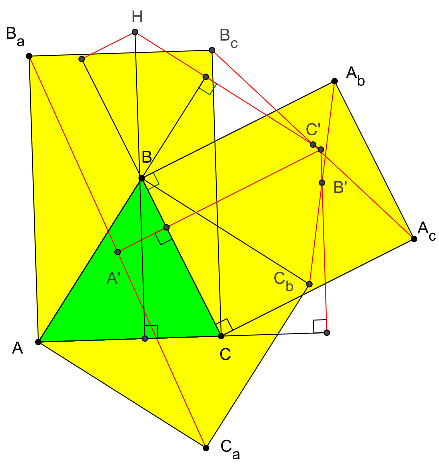

Figure 11. New flanks for one outer rectangular and two inner rectangulars

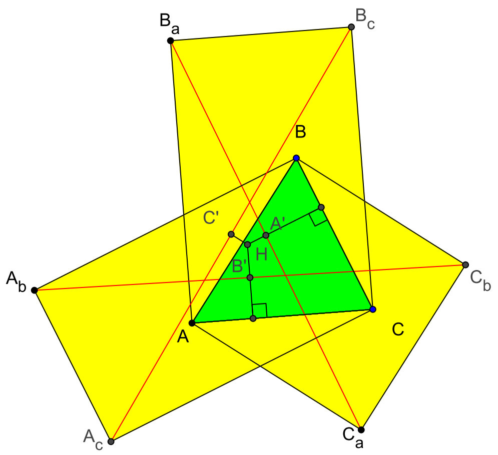

Figure 12. New flanks for three inner rectangulars

Theorem 4.2. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through B, C, A perpendicularly to C_bA_b, A_cB_c, B_aC_a respectively are concurrent.*

See figures 13, 14, 15, 16.

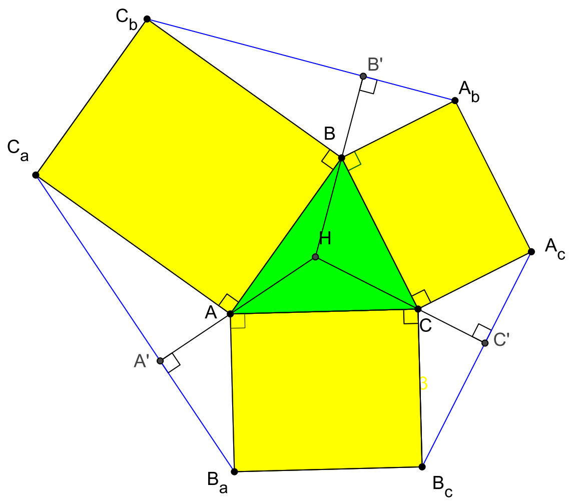

Figure 13. New flanks for three outer rectangulars

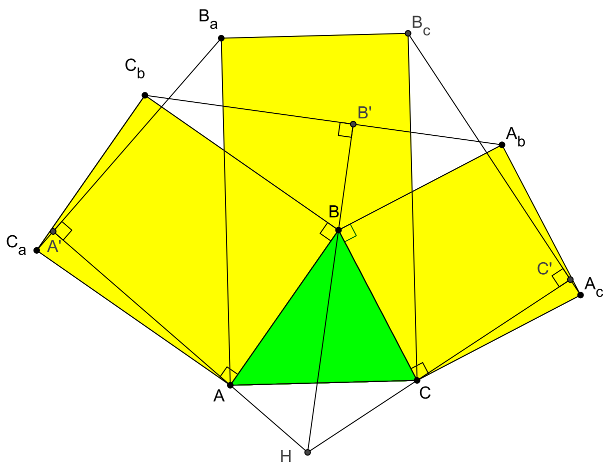

Figure 14. New flanks for one inner rectangular and two outer rectangulars

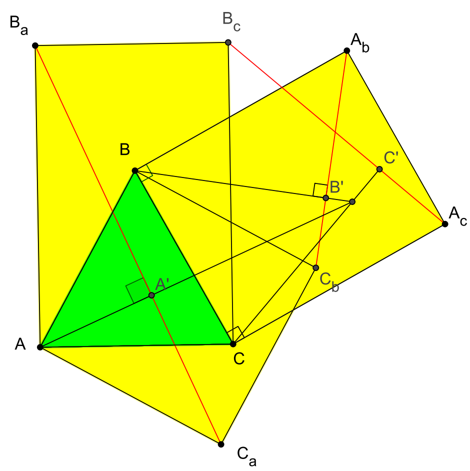

Figure 15. New flanks for one outer rectangular and two inner rectangulars

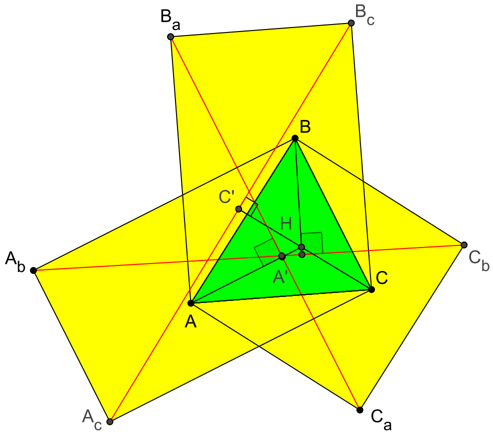

Figure 16. New flanks for three inner rectangulars

Theorem 4.3. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through B, C, A and the circumcenters of triangles $BA_bC_b, CA_cB_c, AB_aC_a$ respectively are concurrent.*

See figures 17, 18, 19, 20.

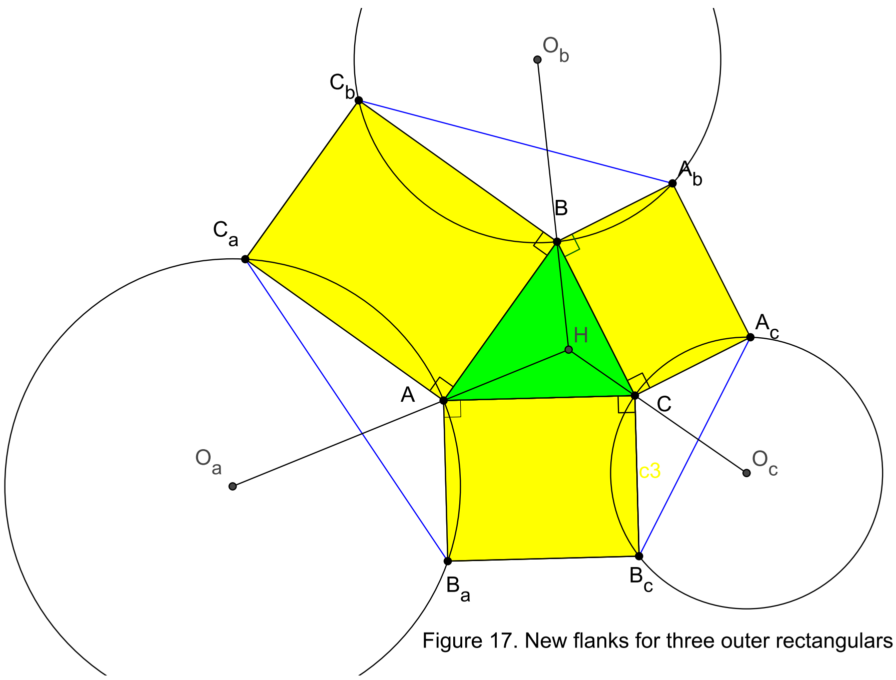

Figure 17. New flanks for three outer rectangulars

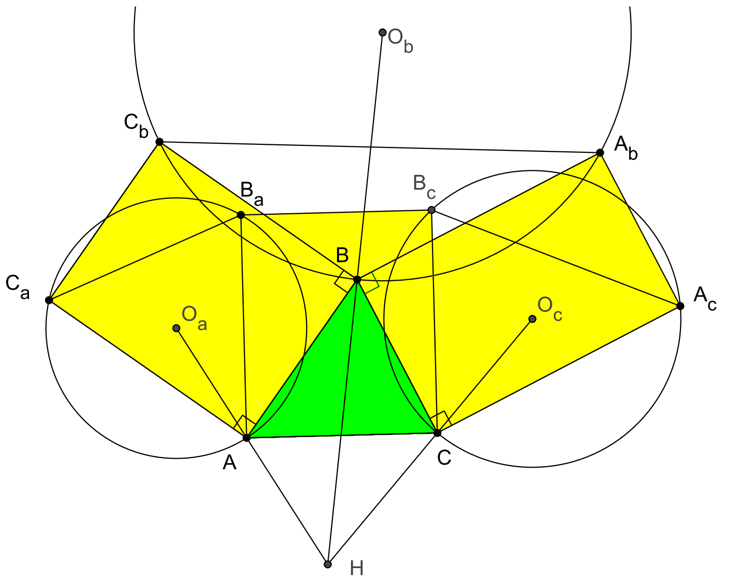

Figure 18. New flanks for one inner rectangular and two outer rectangulars

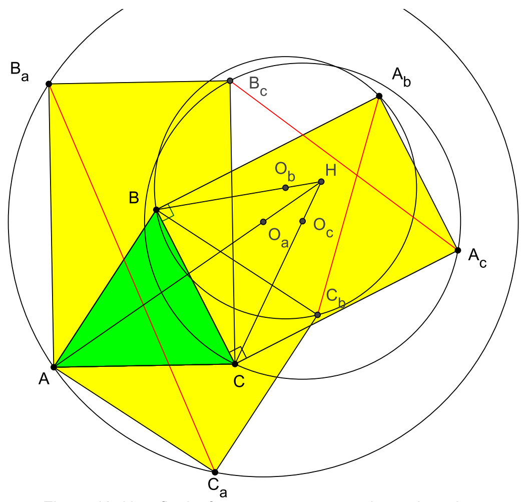

Figure 19. New flanks for one outer rectangular and two inner rectangulars

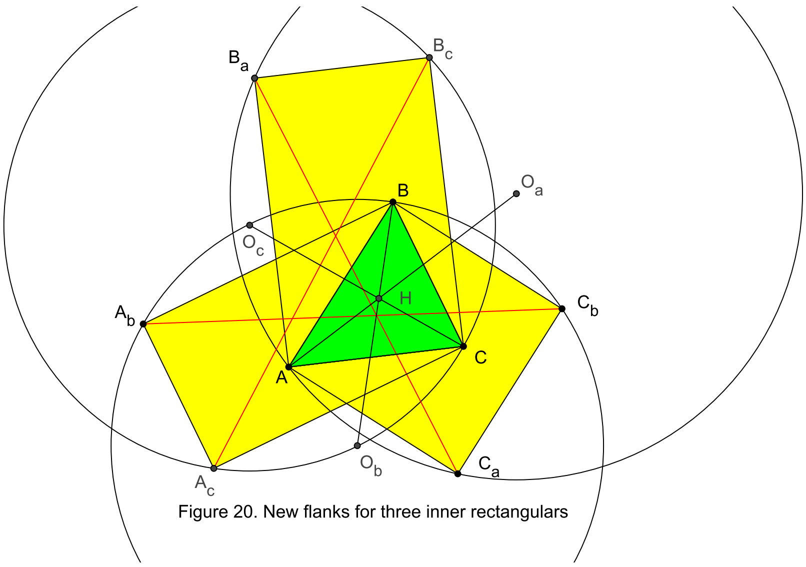

Figure 20. New flanks for three inner rectangulars

Theorem 4.4. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then the three midperpendiculars of segments A_bC_b, A_cB_c, B_aC_a are concurrent.*

See figures 21, 22, 23, 24.

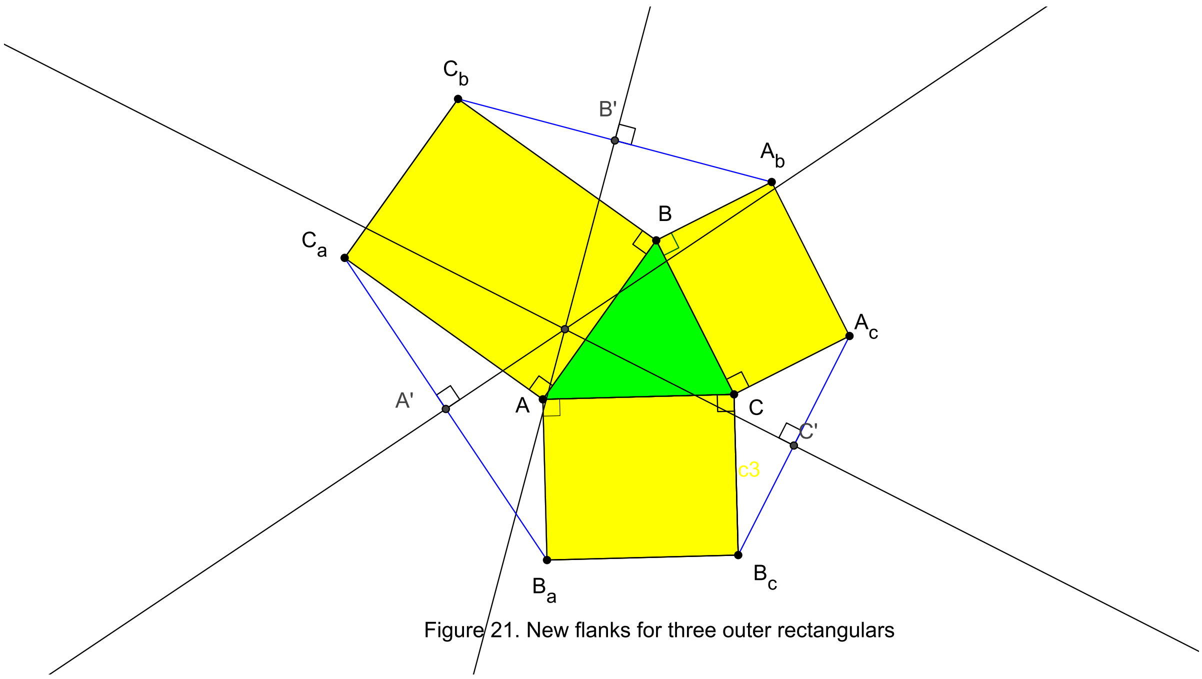

Figure 21. New flanks for three outer rectangulars

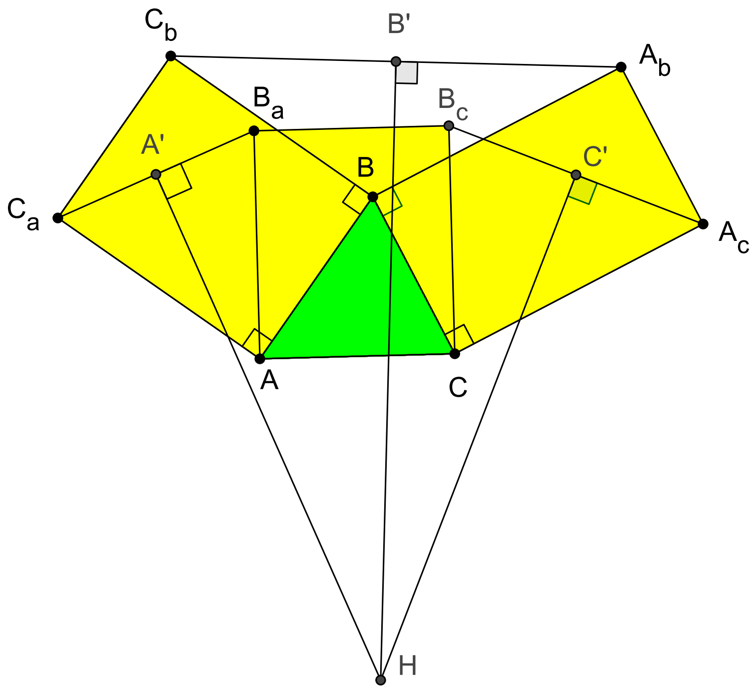

Figure 22. New flanks for one inner rectangular and two outer rectangulars

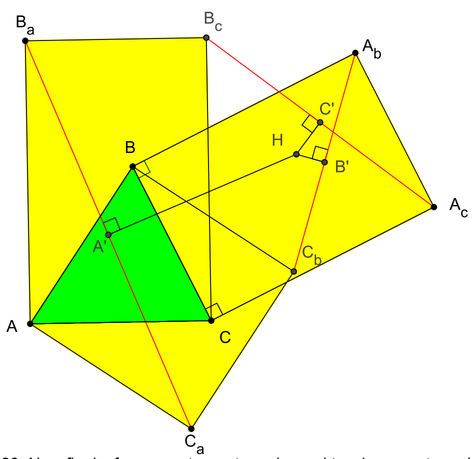

Figure 23. New flanks for one outer rectangular and two inner rectangulars

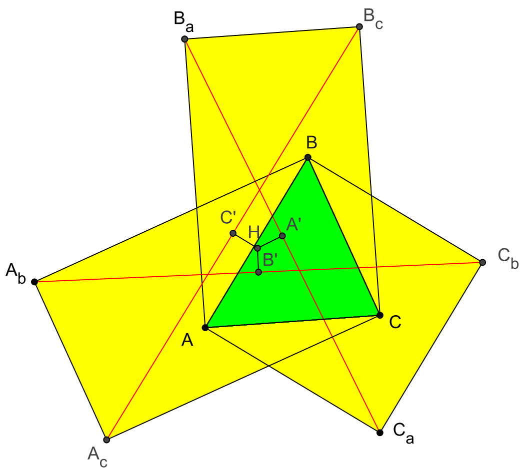

Figure 24. New flanks for three inner rectangulars

Theorem 4.5. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through three nine-point centers of triangles $BA_bC_b, CA_cB_c, AB_aC_a$ perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 25, 26, 27, 28.

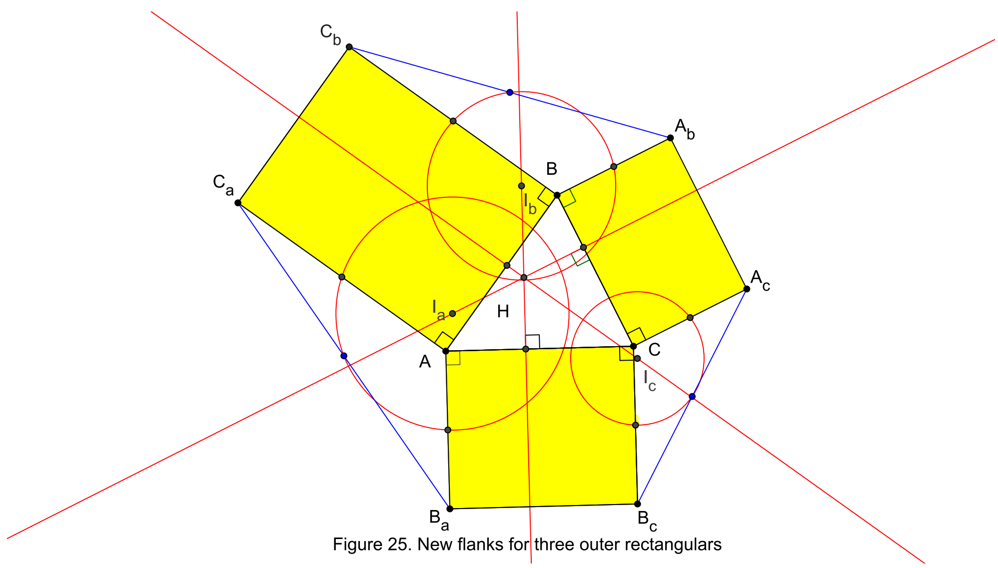

Figure 25. New flanks for three outer rectangulars

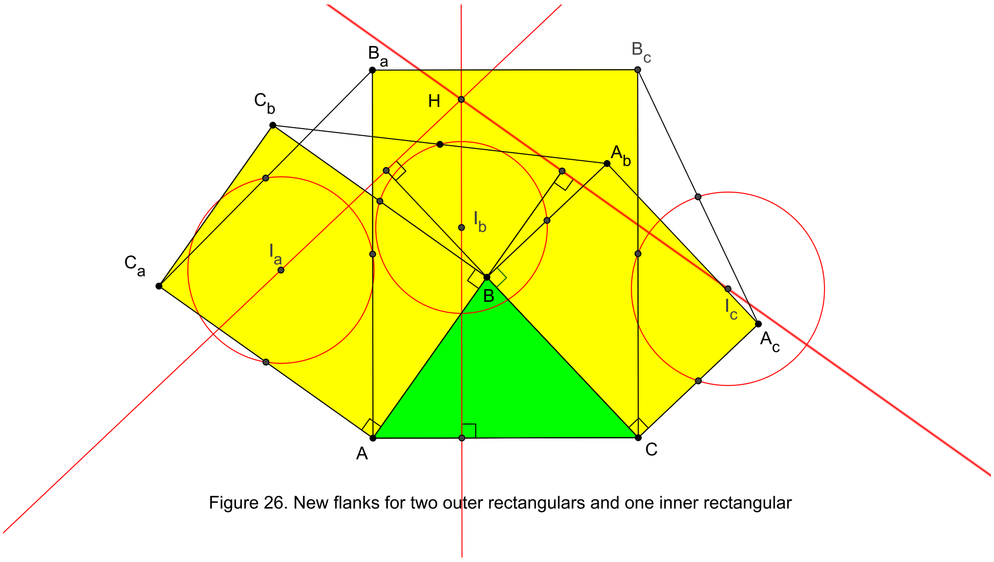

Figure 26. New flanks for two outer rectangulars and one inner rectangular

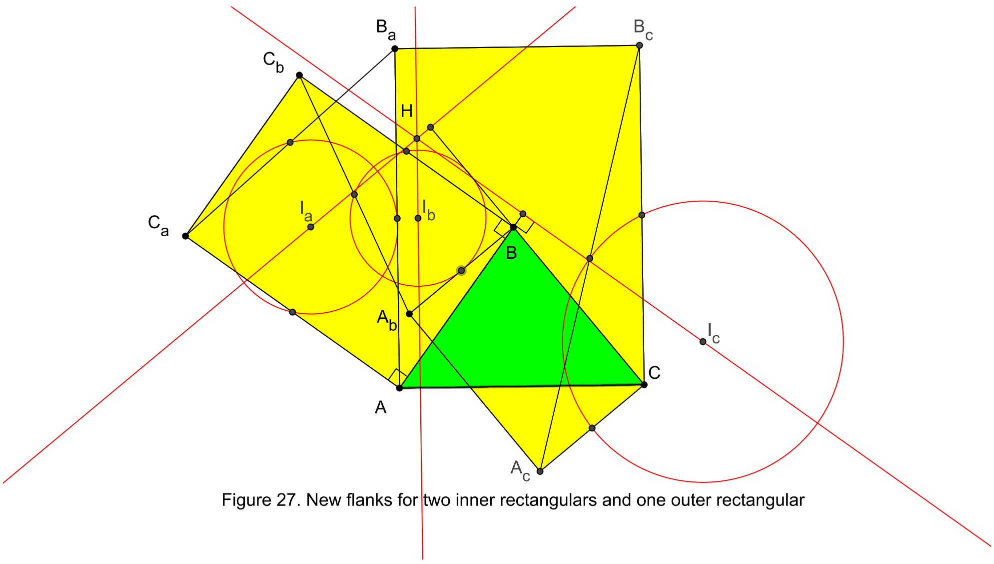

Figure 27. New flanks for two inner rectangulars and one outer rectangular

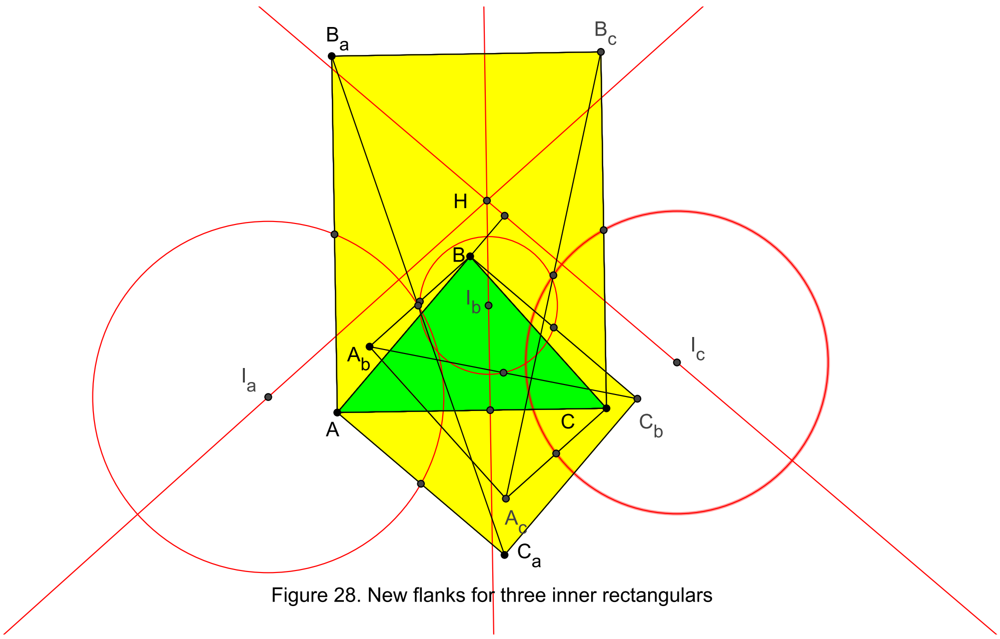

Figure 28. New flanks for three inner rectangulars

Theorem 4.6. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of C_bA_b, A_cB_c, B_aC_a perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 29, 30, 31, 32.

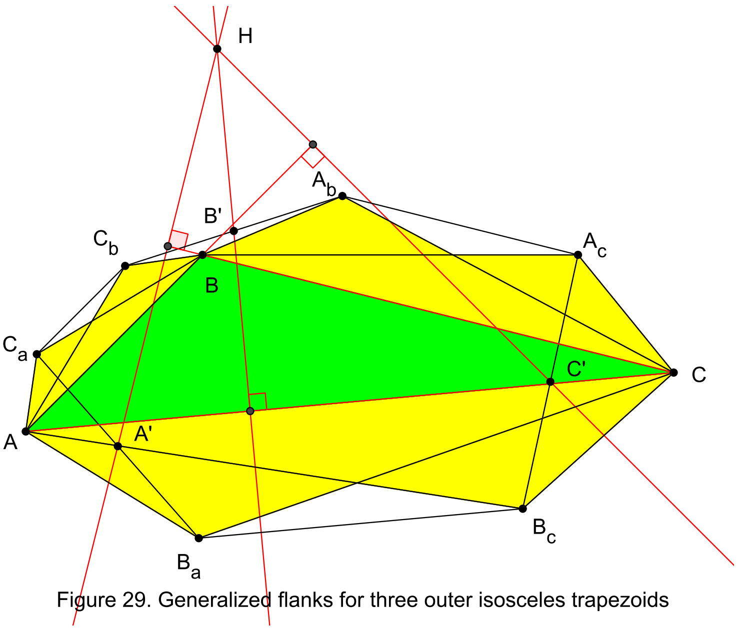

Figure 29. Generalized flanks for three outer isosceles trapezoids

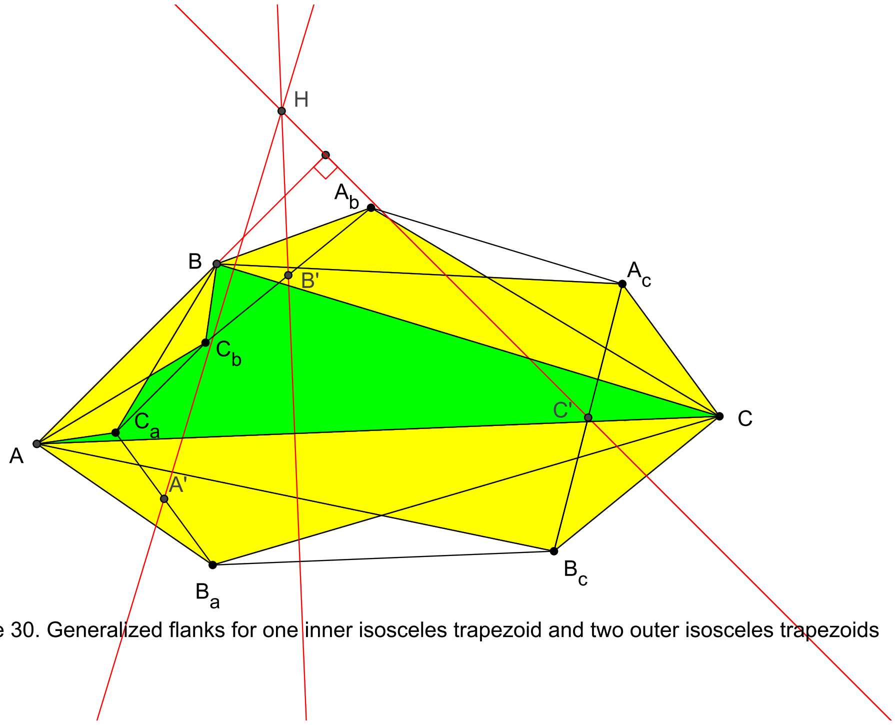

Figure 30. Generalized flanks for one inner isosceles trapezoid and two outer isosceles trapezoids

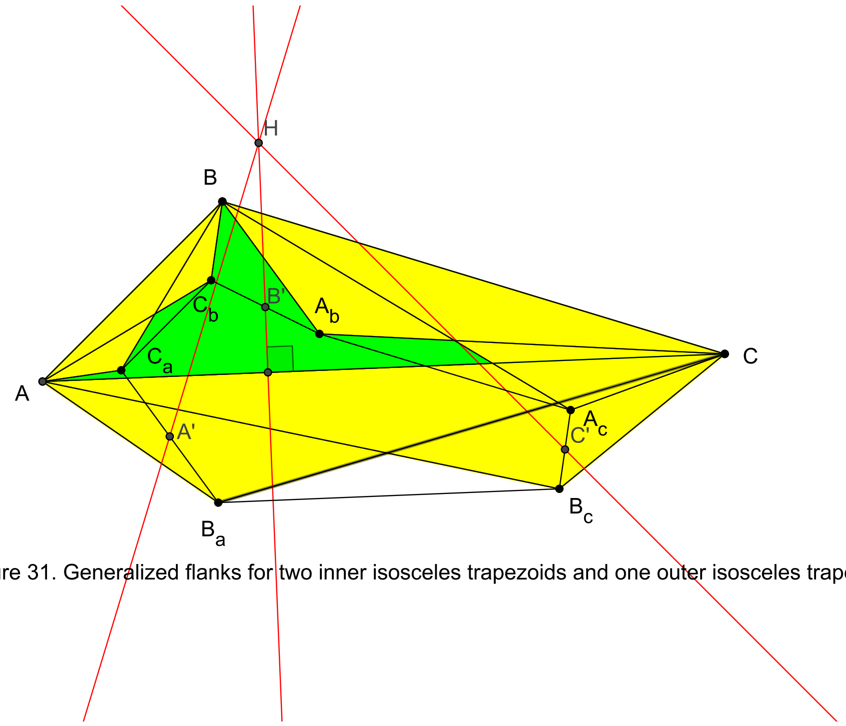

Figure 31. Generalized flanks for two inner isosceles trapezoids and one outer isosceles trapezoid

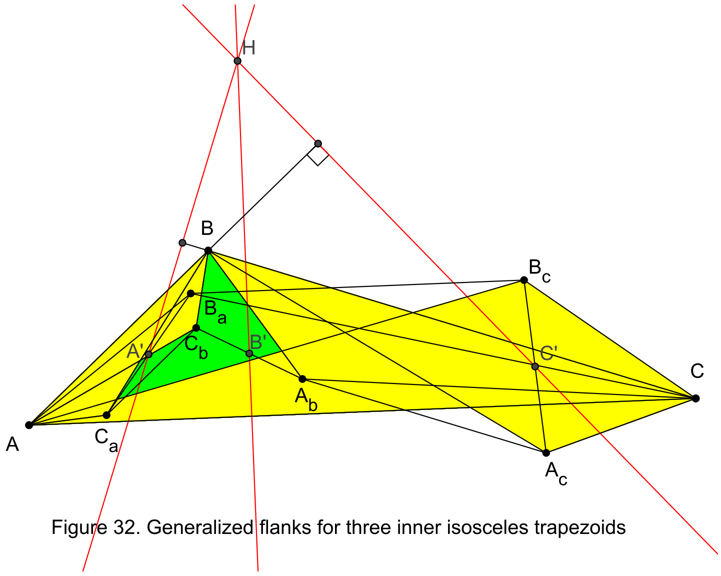

Figure 32. Generalized flanks for three inner isosceles trapezoids

Theorem 4.7. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through B, C, A perpendicularly to C_bA_b, A_cB_c, B_aC_a respectively are concurrent.*

See figures 33, 34, 35, 36.

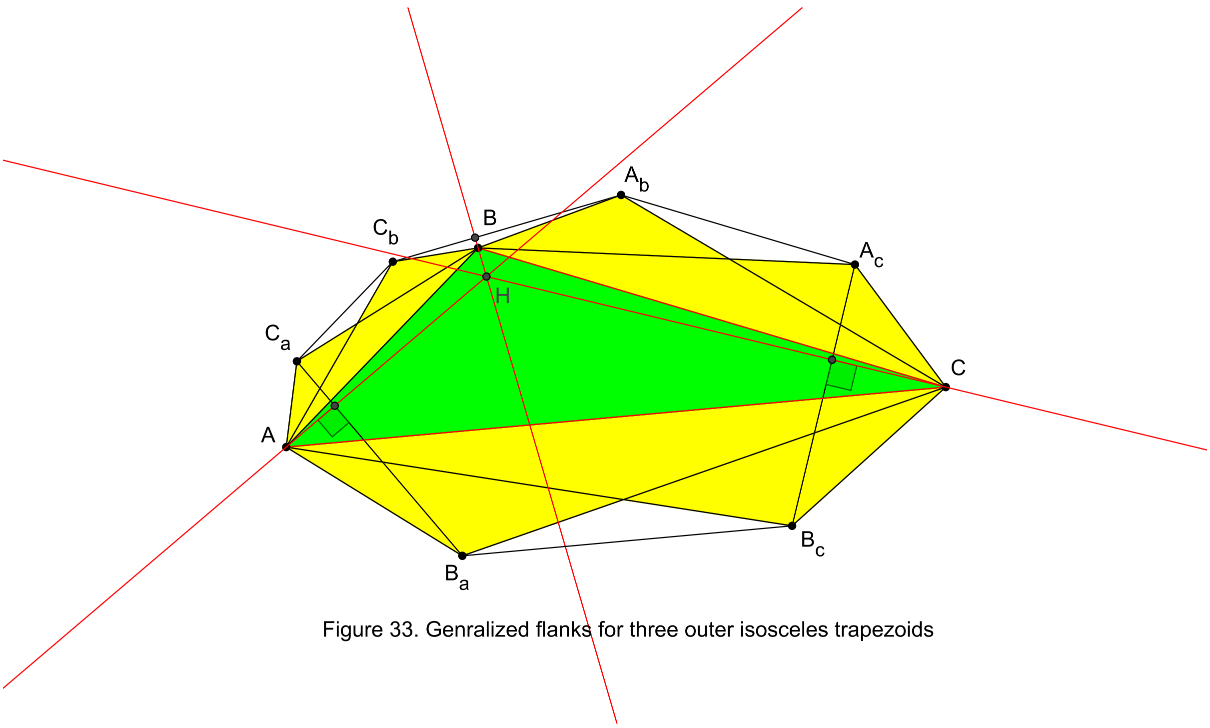

Figure 33. Genralized flanks for three outer isosceles trapezoids

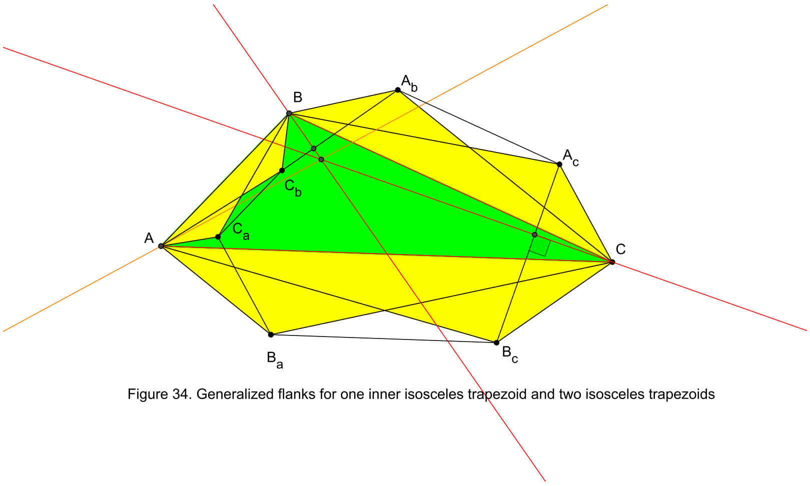

Figure 34. Generalized flanks for one inner isosceles trapezoid and two isosceles trapezoids

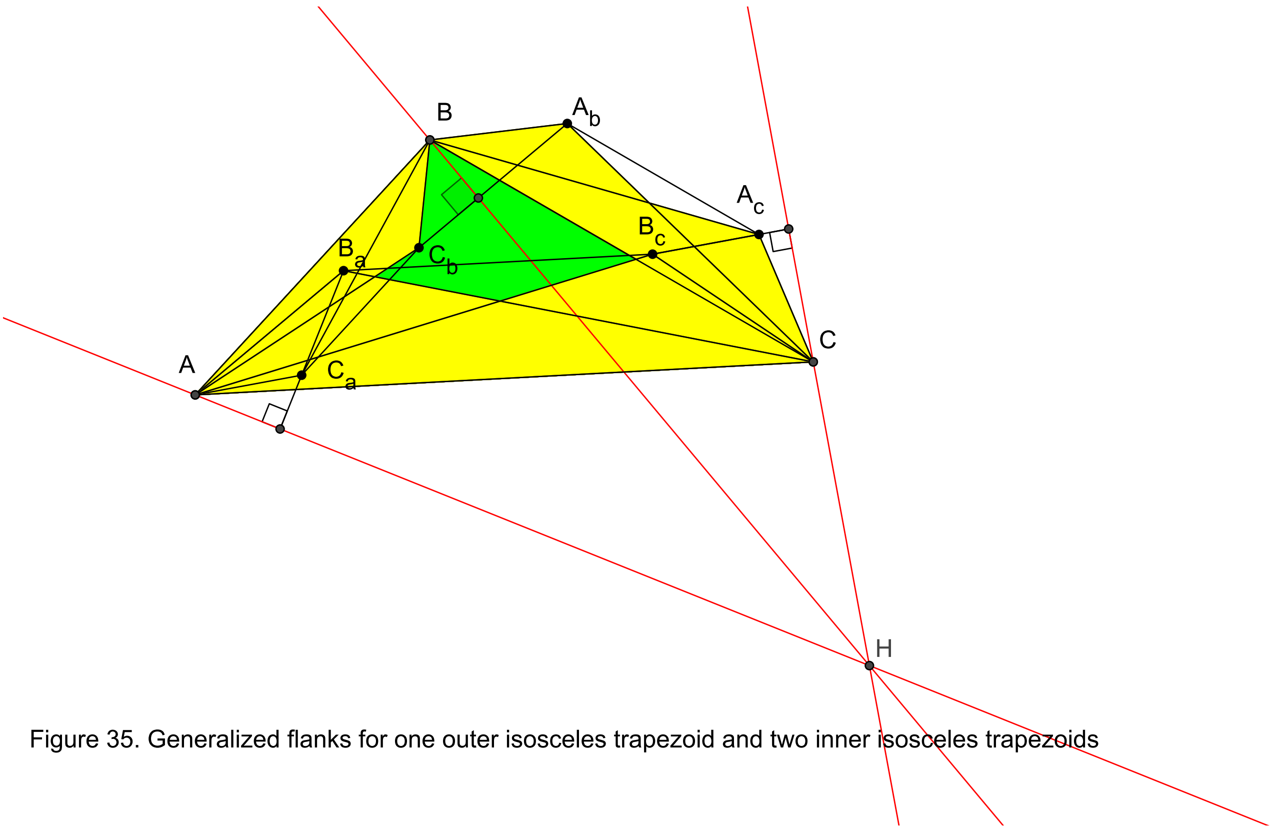

Figure 35. Generalized flanks for one outer isosceles trapezoid and two inner isosceles trapezoids

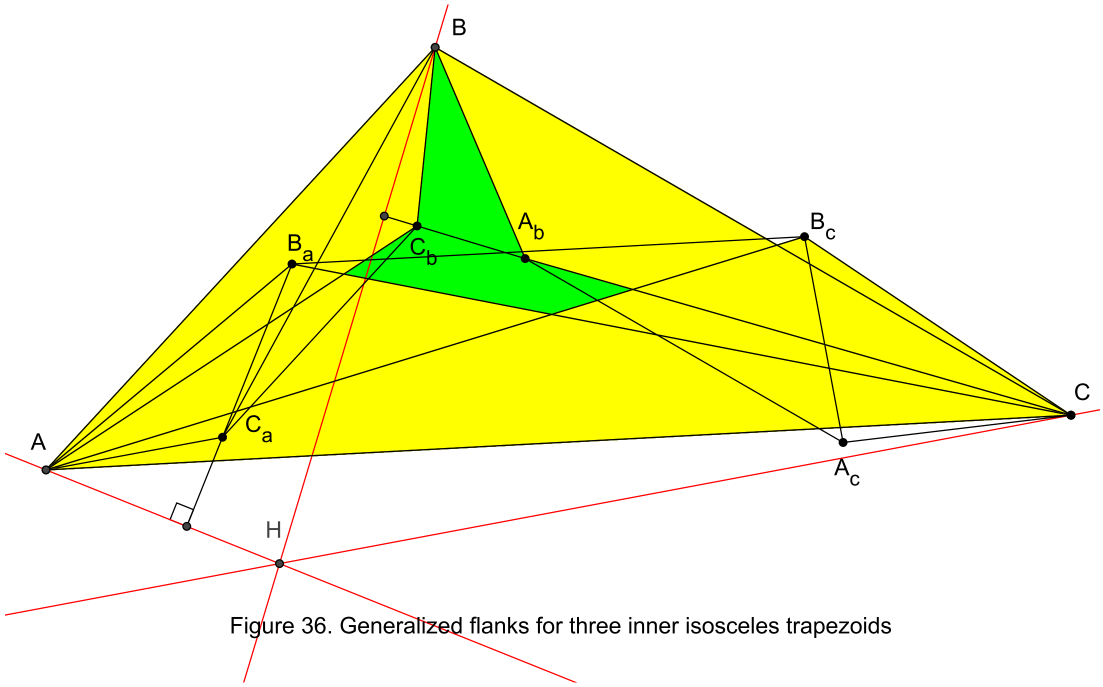

Figure 36. Generalized flanks for three inner isosceles trapezoids

Theorem 4.8. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then the three midperpendiculars of segments A_bC_b, A_cB_c, B_aC_a are concurrent.*

See figures 37, 38, 39, 40.

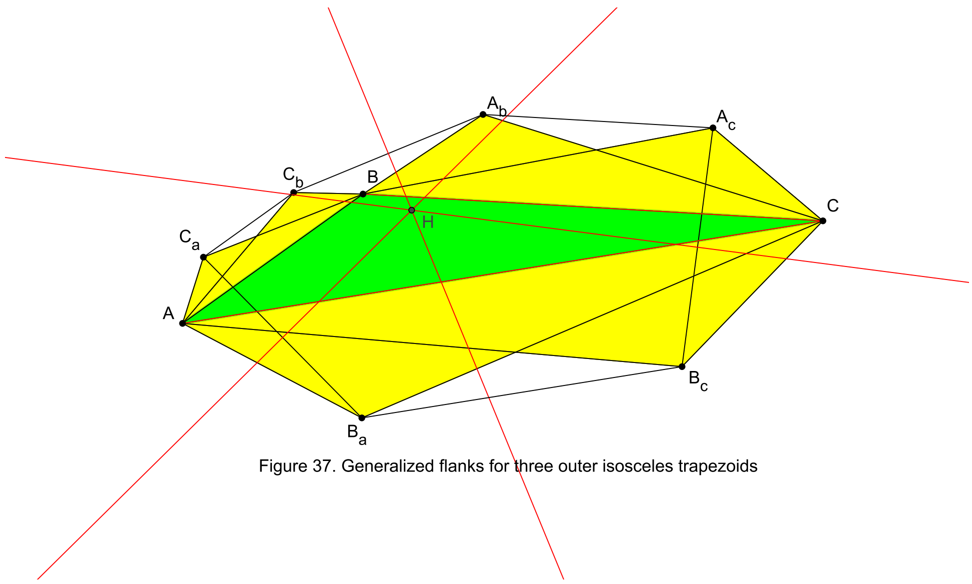

Figure 37. Generalized flanks for three outer isosceles trapezoids

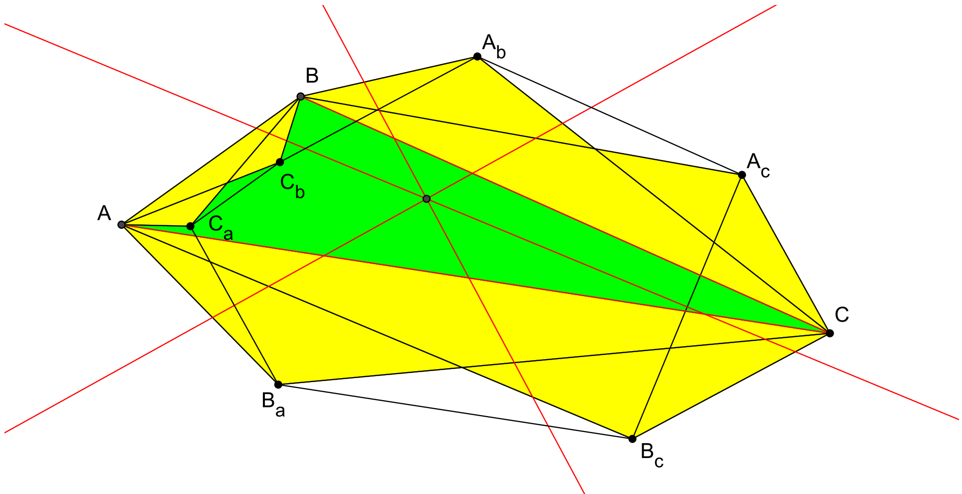

Figure 38. Generalized flanks for one inner isosceles trapezoid and two outer isosceles trapezoids

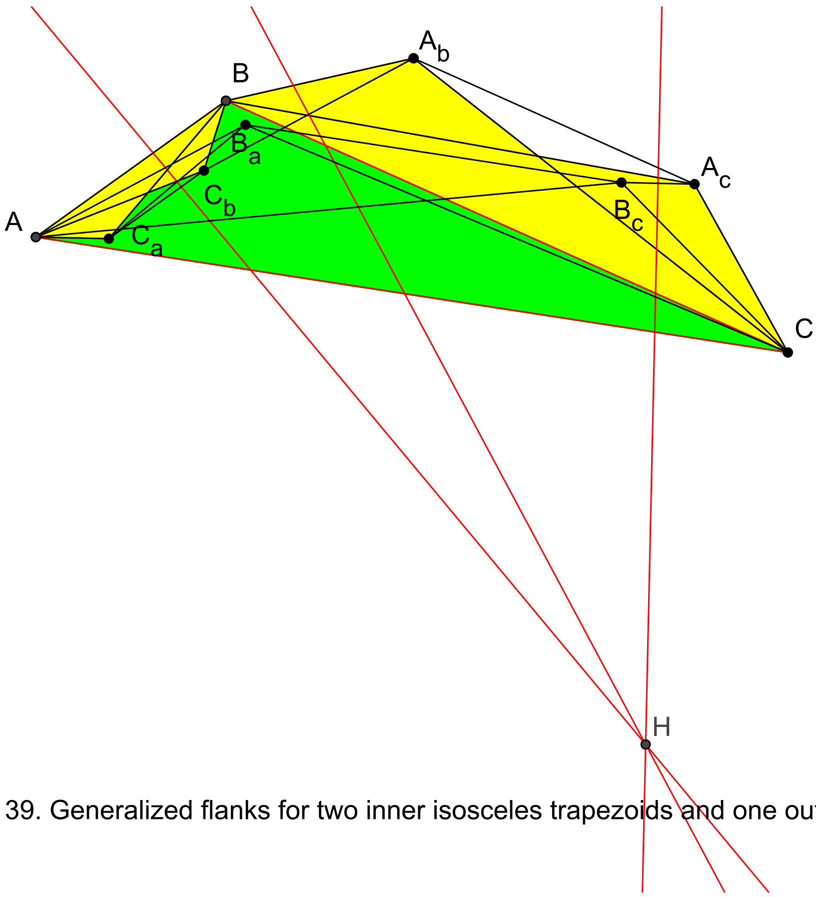

Figure 39. Generalized flanks for two inner isosceles trapezoids and one outer isosceles trapezoid

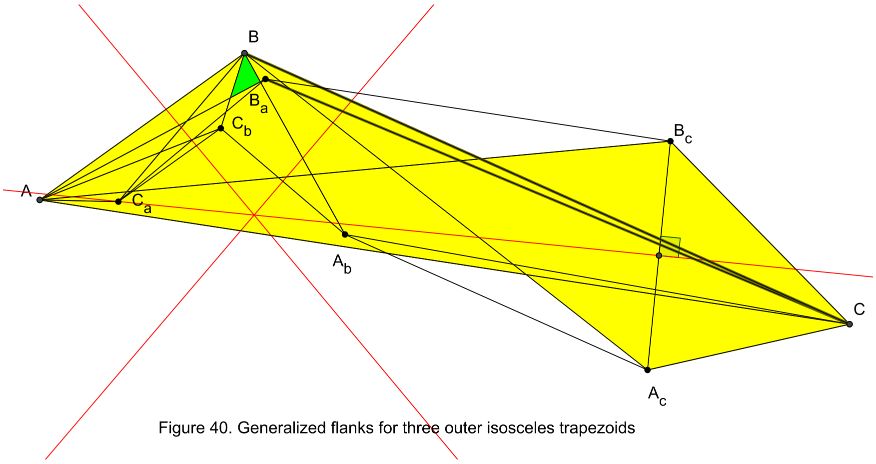

Figure 40. Generalized flanks for three outer isosceles trapezoids

Theorem 4.9. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through three nine-point centers of triangles BA_bC_b, CA_cB_c, AB_aC_a perpendicularly to A_bC_b, A_cB_c, B_aC_c respectively are concurrent.*

See figures 41, 42, 43, 44.

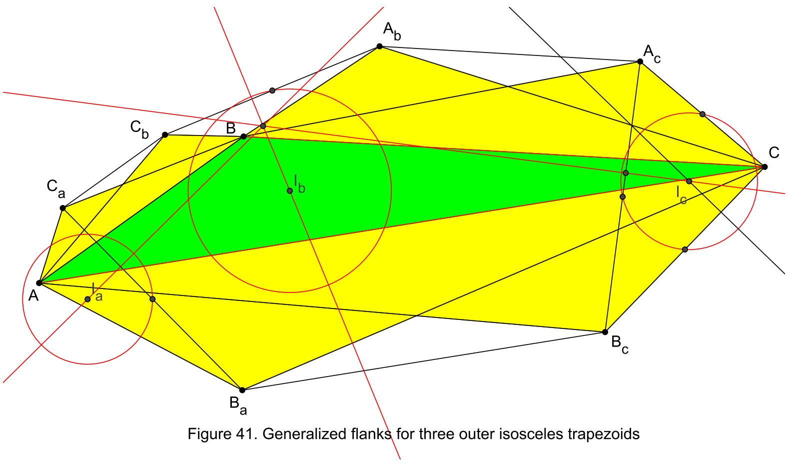

Figure 41. Generalized flanks for three outer isosceles trapezoids

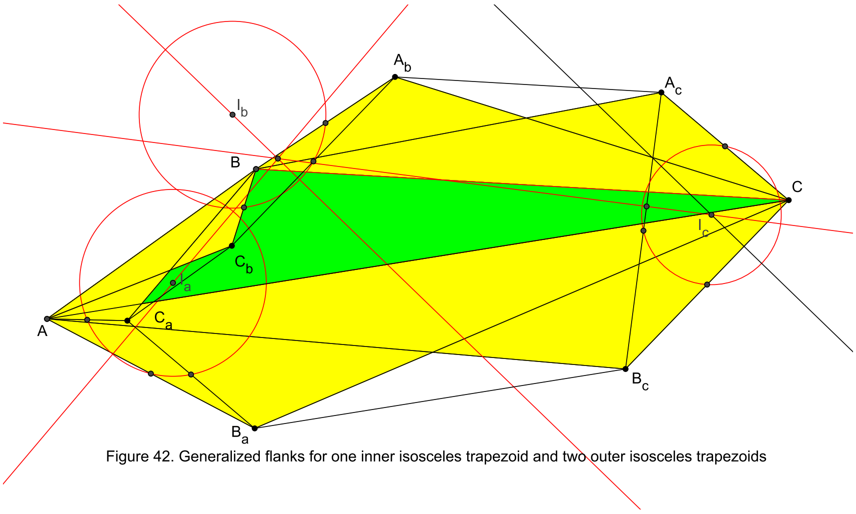

Figure 42. Generalized flanks for one inner isosceles trapezoid and two outer isosceles trapezoids

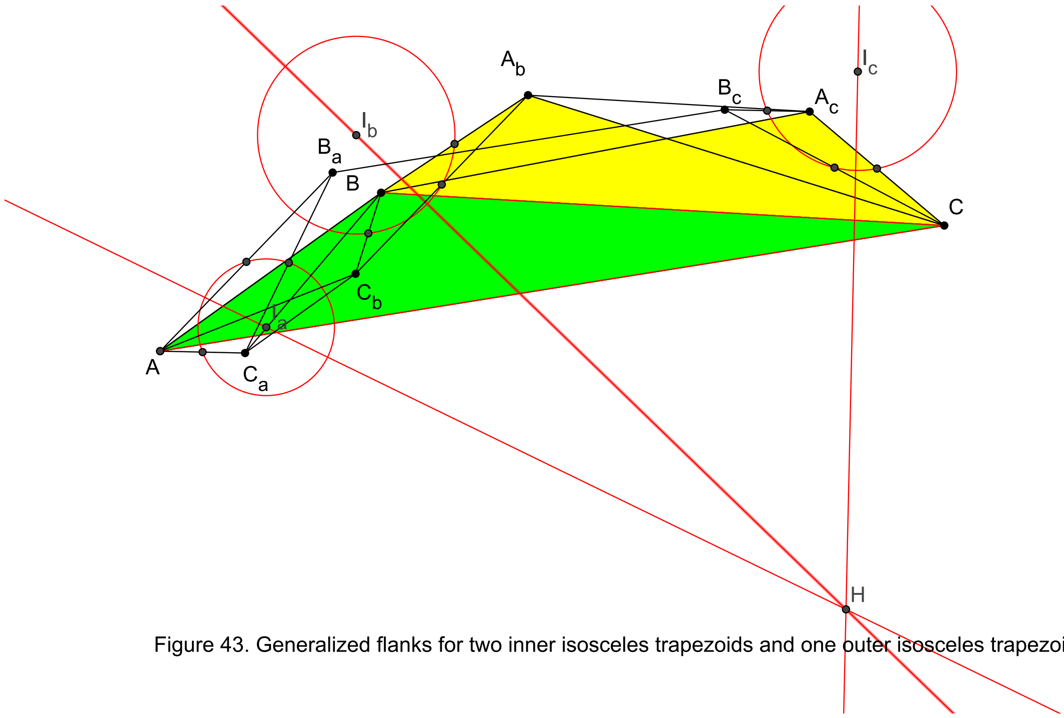

Figure 43. Generalized flanks for two inner isosceles trapezoids and one outer isosceles trapezoid

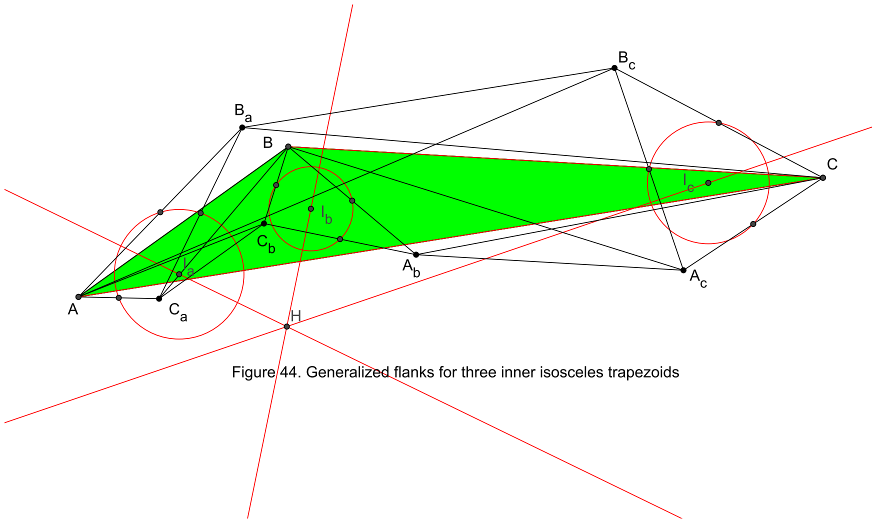

Figure 44. Generalized flanks for three inner isosceles trapezoids

Theorem 4.10. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let A', B', C' be the intersection points of of A_bC_b, A_cB_c; B_cA_c, B_aC_a; C_aB_a, C_bA_b. Then three lines passing through the circumcenters of triangles $A'A_bA_c$, $B'B_aB_c, C'C_bC_a$ perpendicularly to B_aC_a, C_bA_b, A_cB_c respectively are concurrent.*

See figures 45, 46, 47, 48.

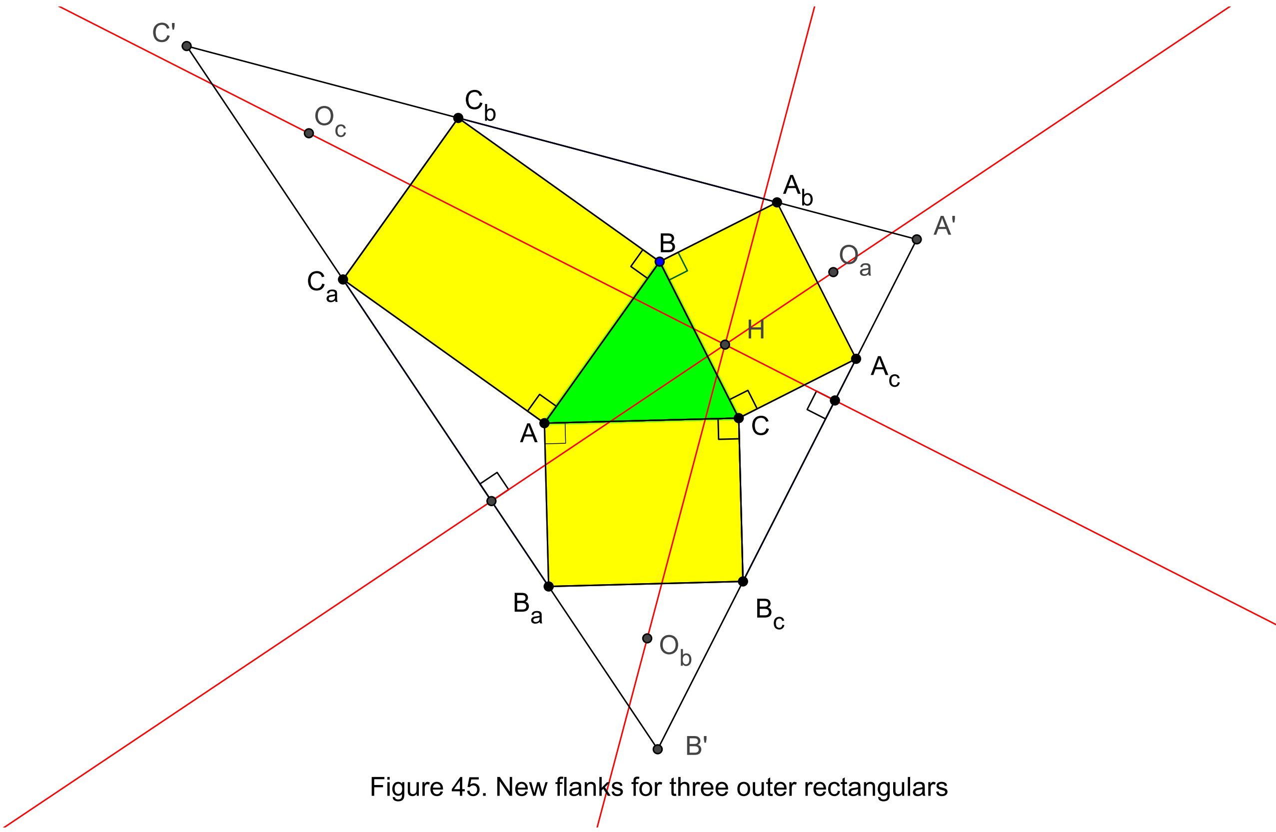

Figure 45. New flanks for three outer rectangulars

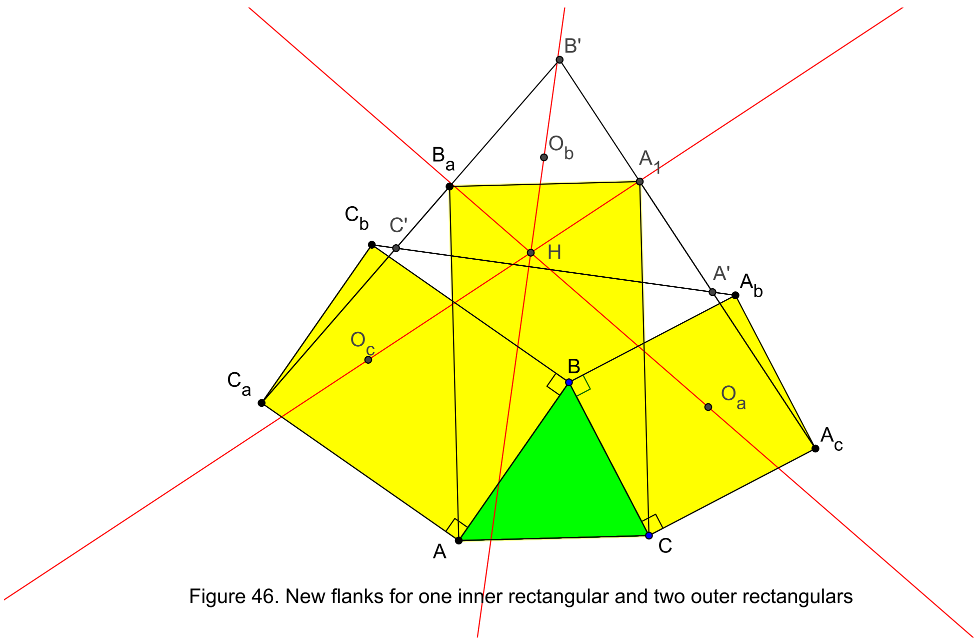

Figure 46. New flanks for one inner rectangular and two outer rectangulars

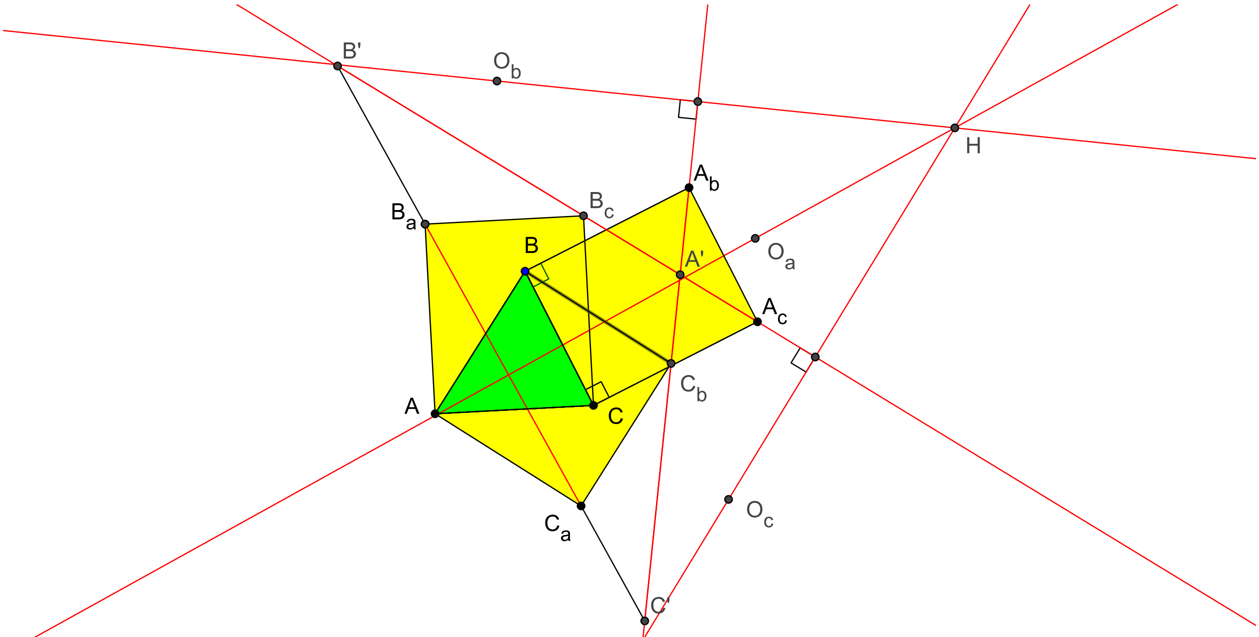

Figure 47. New flanks for two inner rectangulars and one outer rectangular

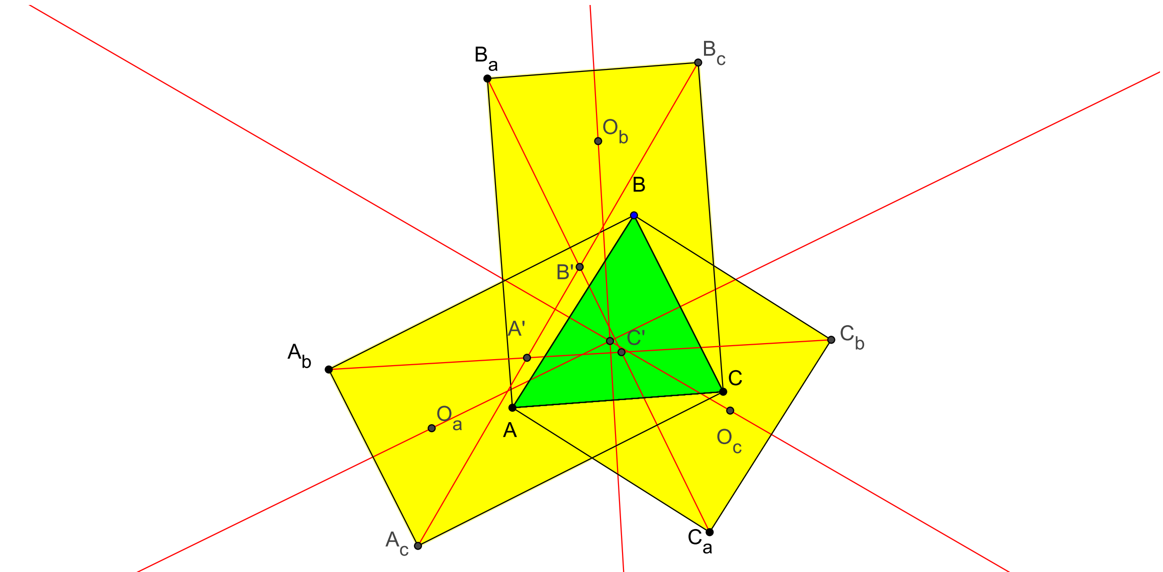

Figure 48. New flanks for three inner rectangulars

Theorem 4.11. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let A', B', C' be the intersection points of A_bC_b, A_cB_c; B_cA_c, B_aC_a; C_aB_a, C_bA_b. Then three lines passing through the circumcenters of triangles $A'A_bA_c, B'B_aB_c, C'C_bC_a$ perpendicularly to B_aC_a, C_bA_b, A_cB_c respectively are concurrent.*

See figures 49, 50, 51, 52.

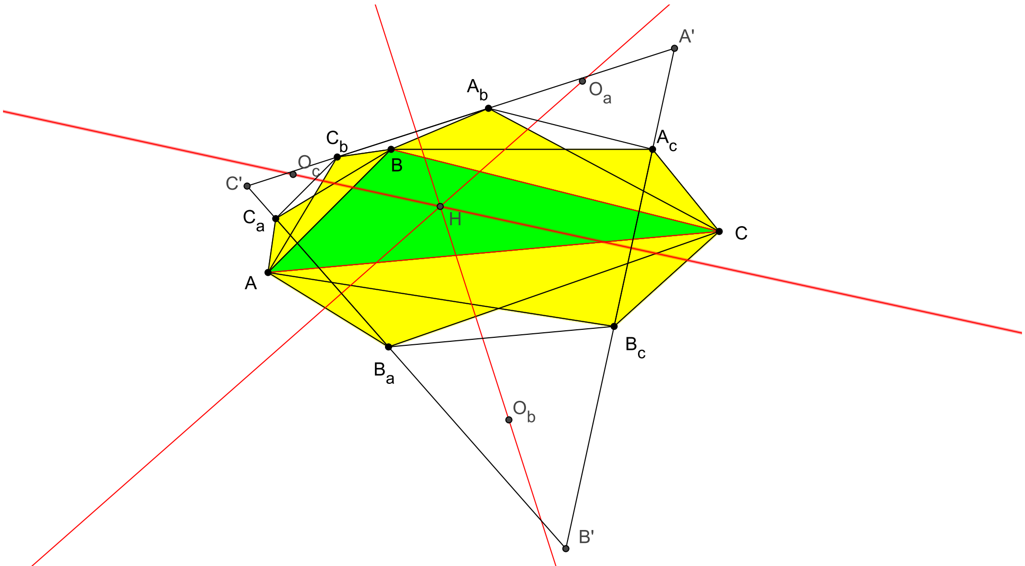

Figure 49. Generalized flanks for three outer isosceles trapezoids

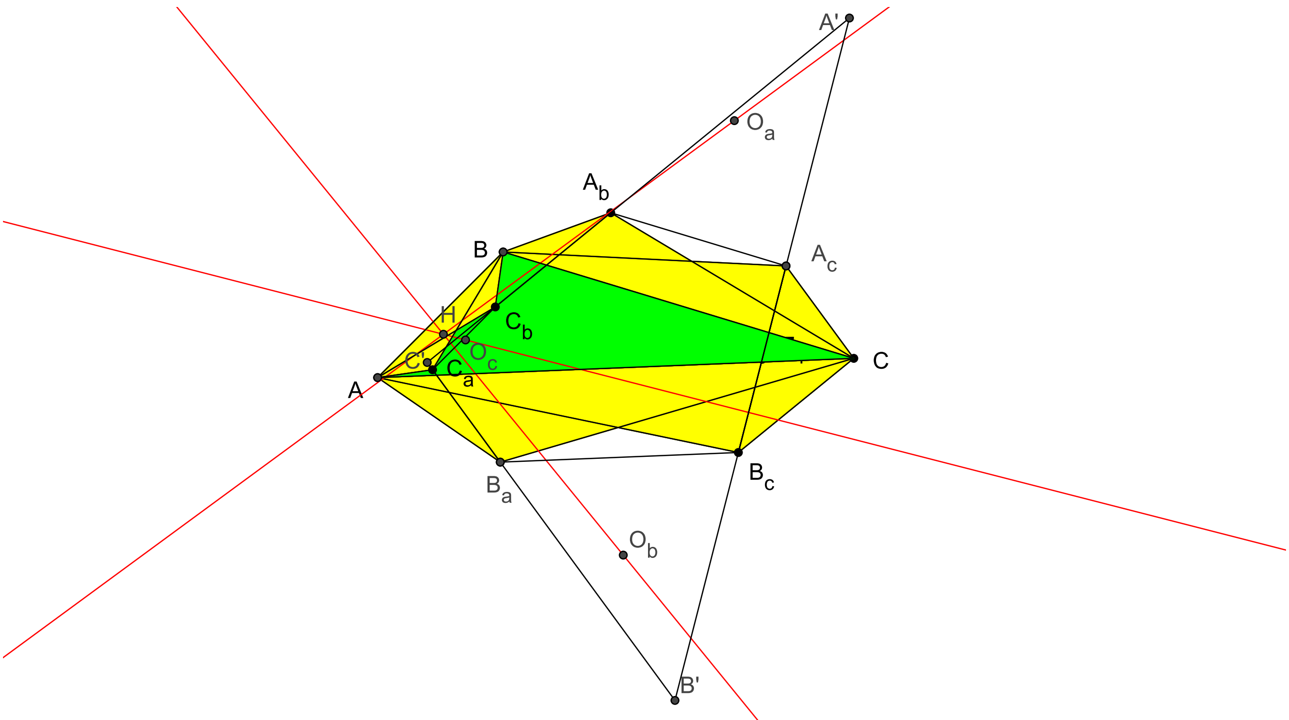

Figure 50. Generalized flanks for one inner isosceles trapezoid and two outer isosceles trapezoids

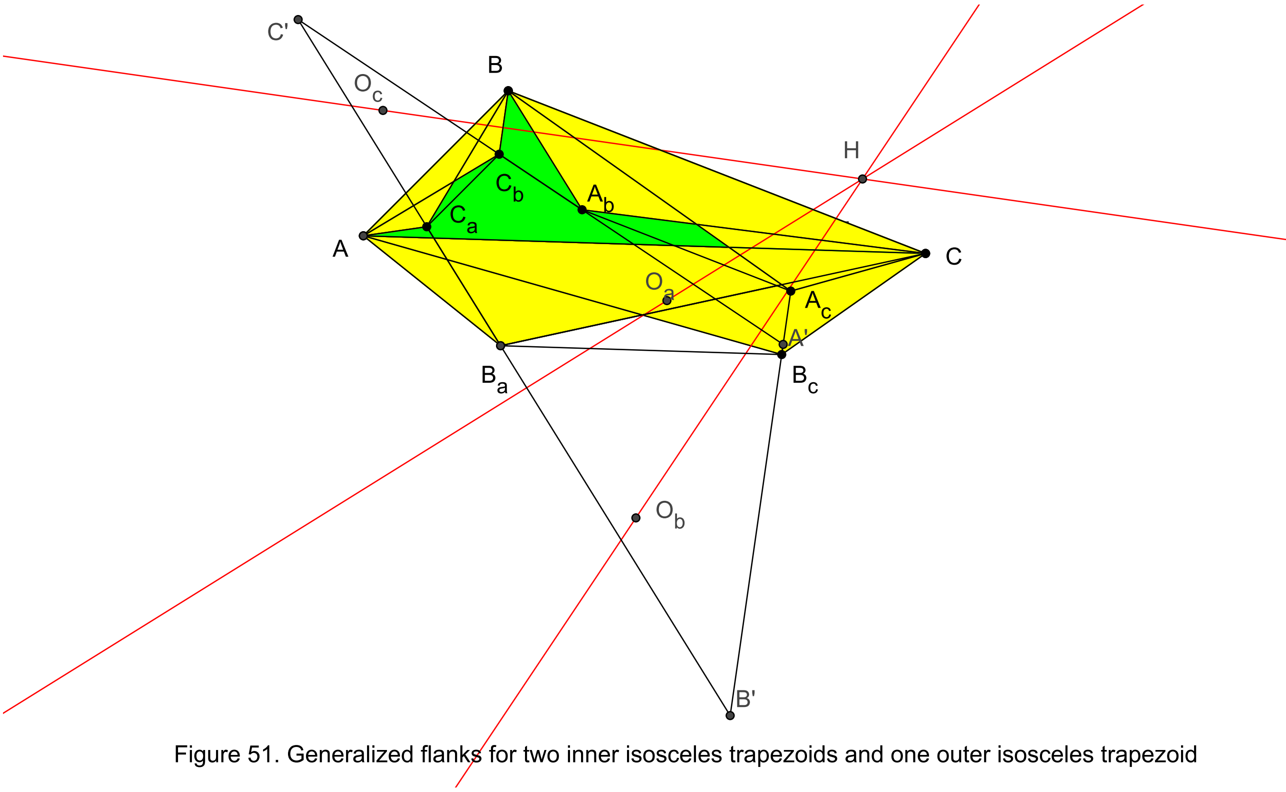

Figure 51. Generalized flanks for two inner isosceles trapezoids and one outer isosceles trapezoid

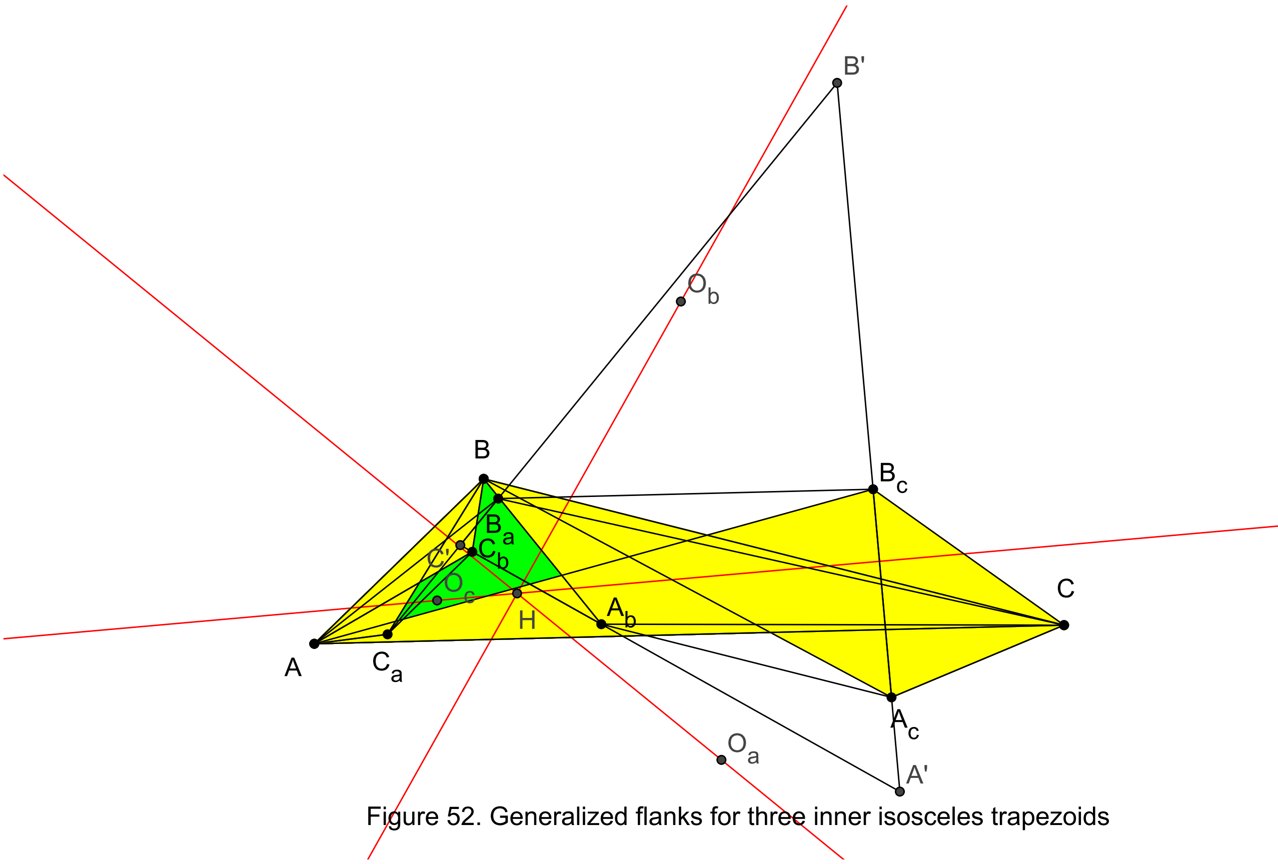

Figure 52. Generalized flanks for three inner isosceles trapezoids

Theorem 4.12. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let C' be the intersection point of the perpendicular line of C_bA_b at C_b and the perpendicular line of C_aB_a at C_a. Similarly to A', B'. Then three lines AA', BB', CC' are concurrent.*

See figures 53, 54, 54, 56.

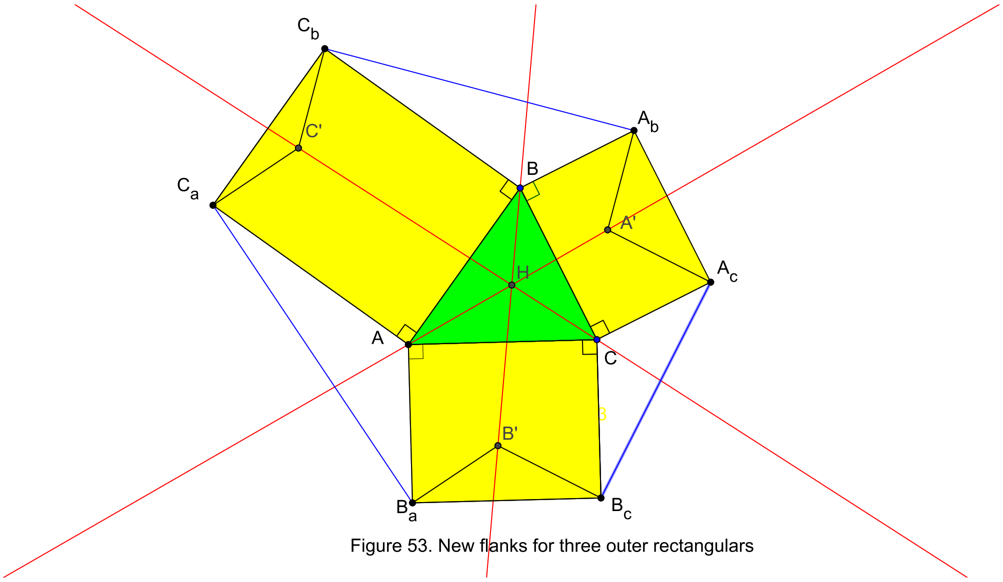

Figure 53. New flanks for three outer rectangulars

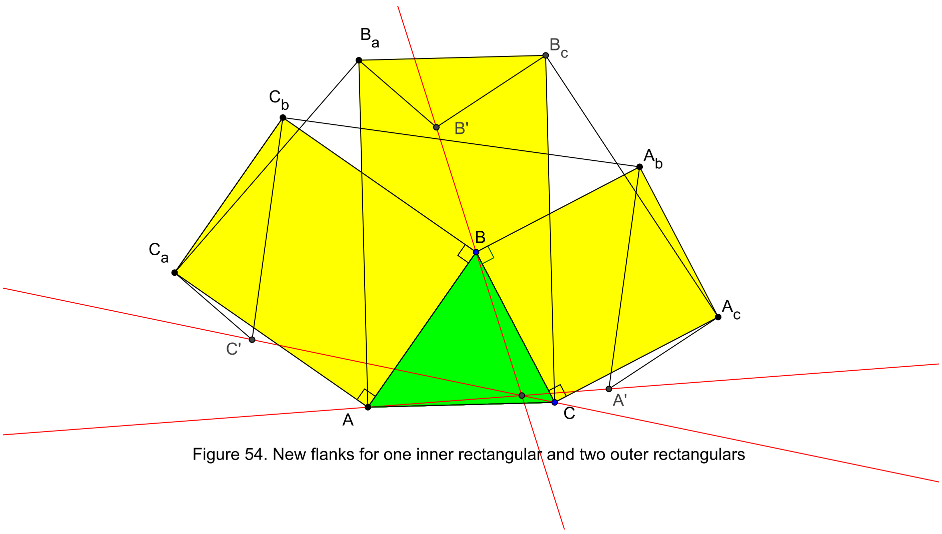

Figure 54. New flanks for one inner rectangular and two outer rectangulars

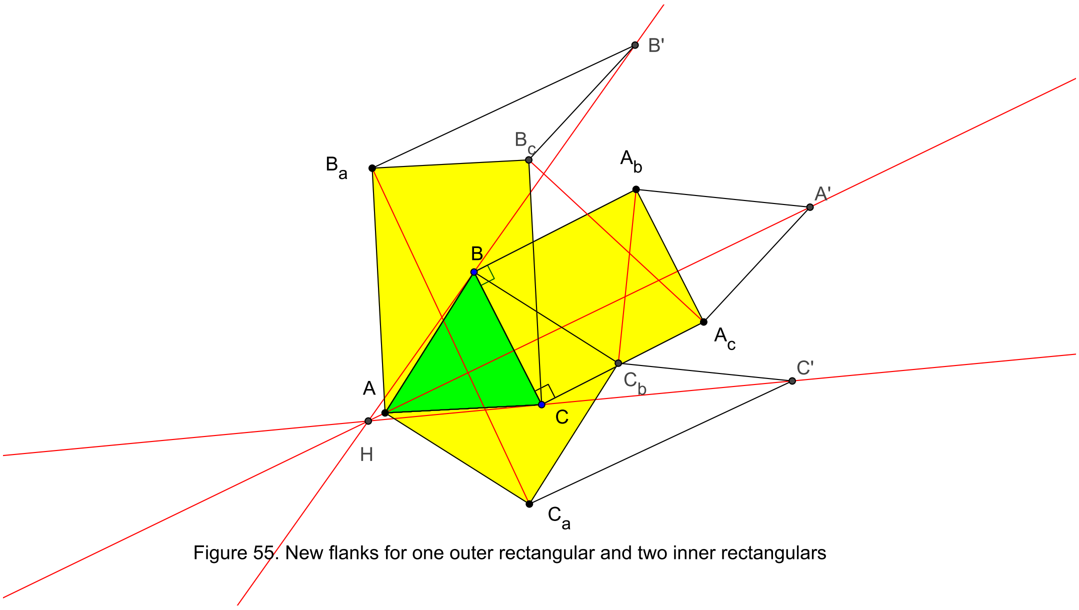

Figure 55. New flanks for one outer rectangular and two inner rectangulars

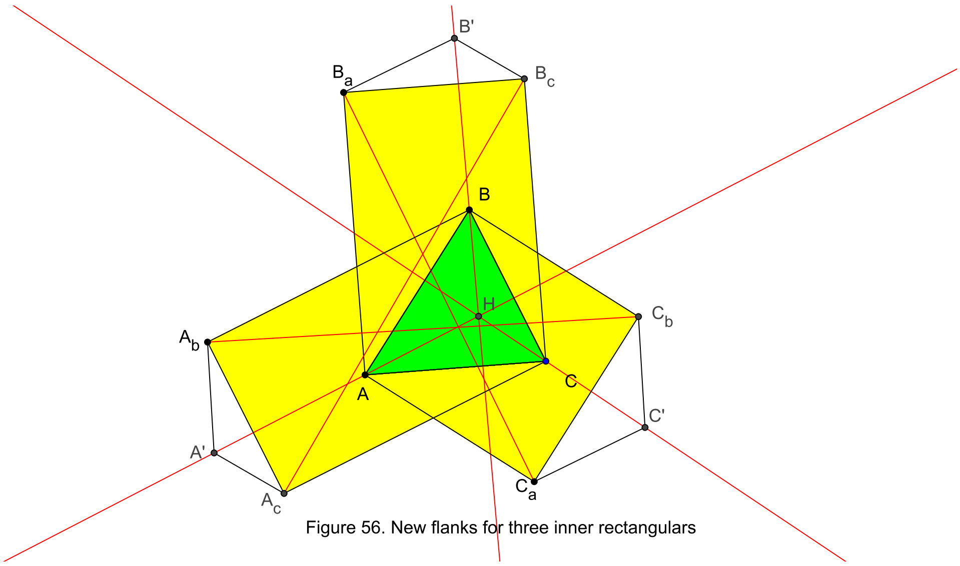

Figure 56. New flanks for three inner rectangulars

Theorem 4.13. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let C' be the intersection point of the perpendicular line of C_bA_b at C_b and the perpendicular line of C_aB_a at C_a. Similarly to A', B'. Then the lines passing through A', B', C' perpendicularly to B_aC_a, A_bC_b, A_cB_c respectively are concurrent.*

See figures 57, 58, 59, 60

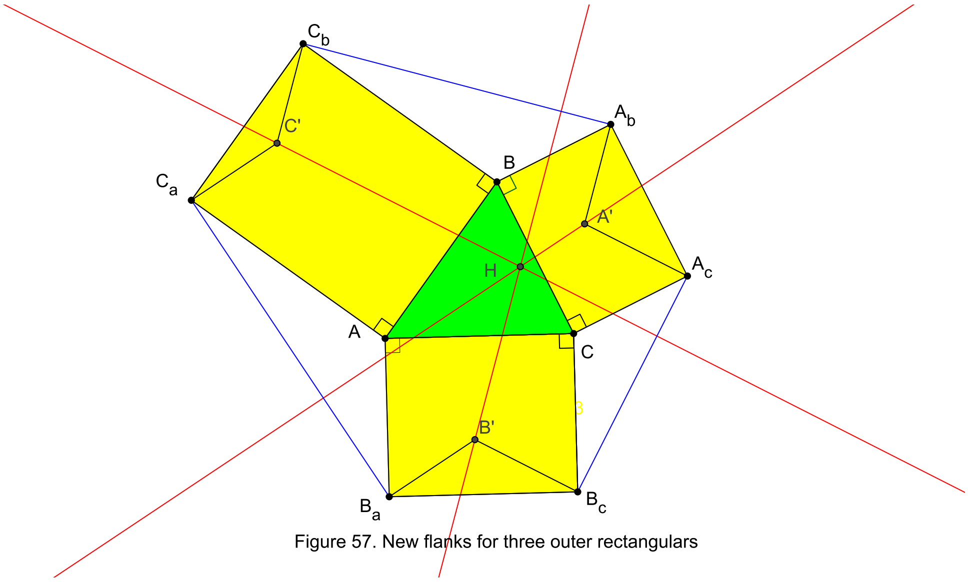

Figure 57. New flanks for three outer rectangulars

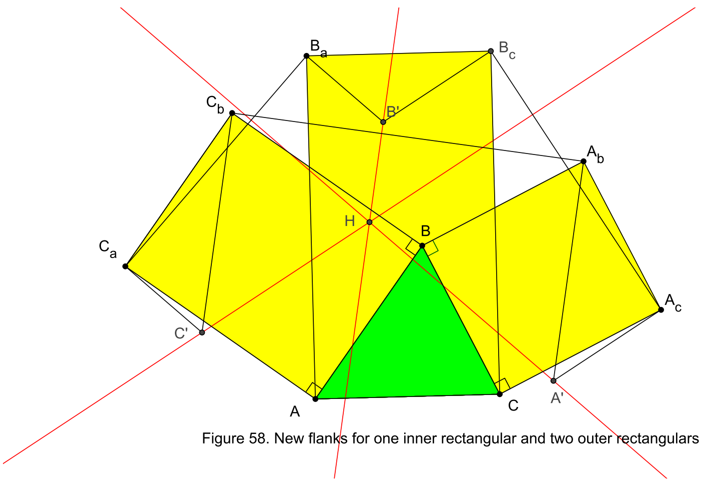

Figure 58. New flanks for one inner rectangular and two outer rectangulars

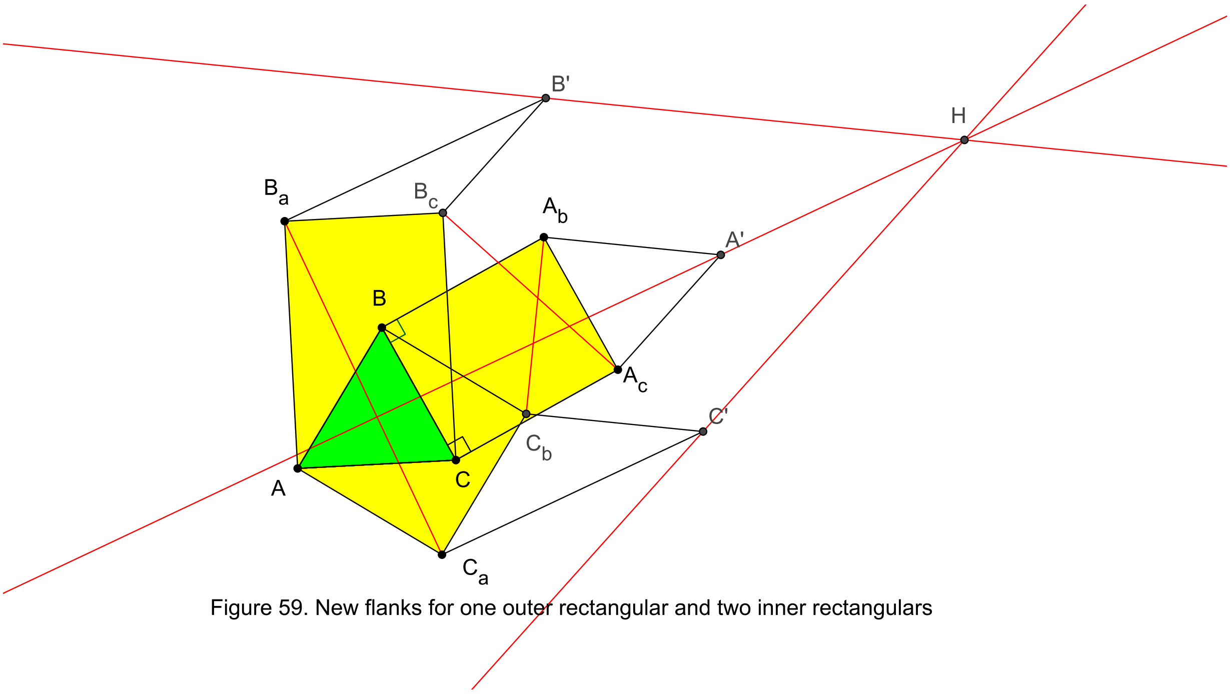

Figure 59. New flanks for one outer rectangular and two inner rectangulars

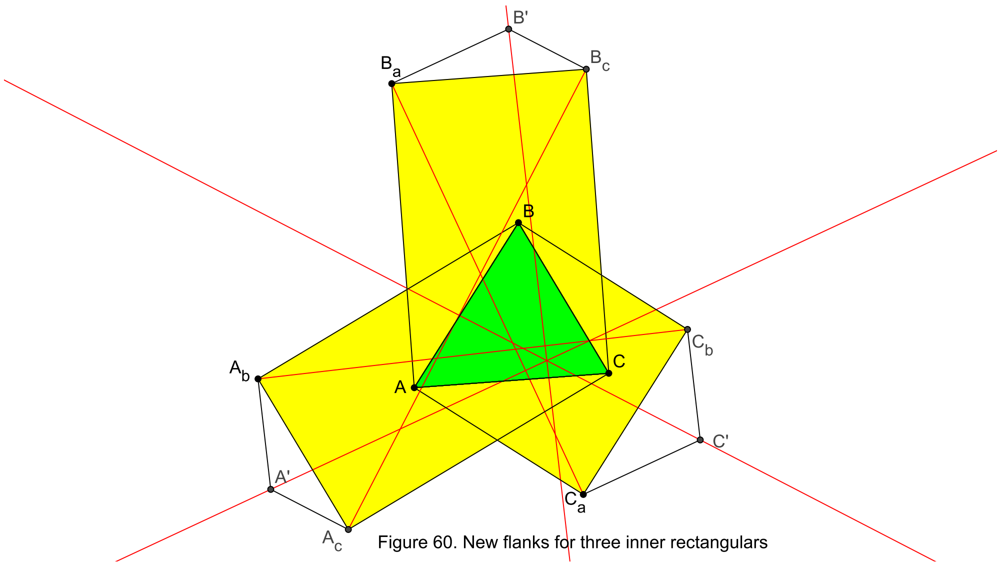

Figure 60. New flanks for three inner rectangulars

Theorem 4.14. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let C' be the intersection point of the perpendicular line of C_bA_b at C_b and the perpendicular line of C_aB_a at C_a. Similarly to A', B'. Then the lines passing through A', B', C' perpendicularly to BC, CA, AB respectively are concurrent.*

See figures 61, 62, 63, 64.

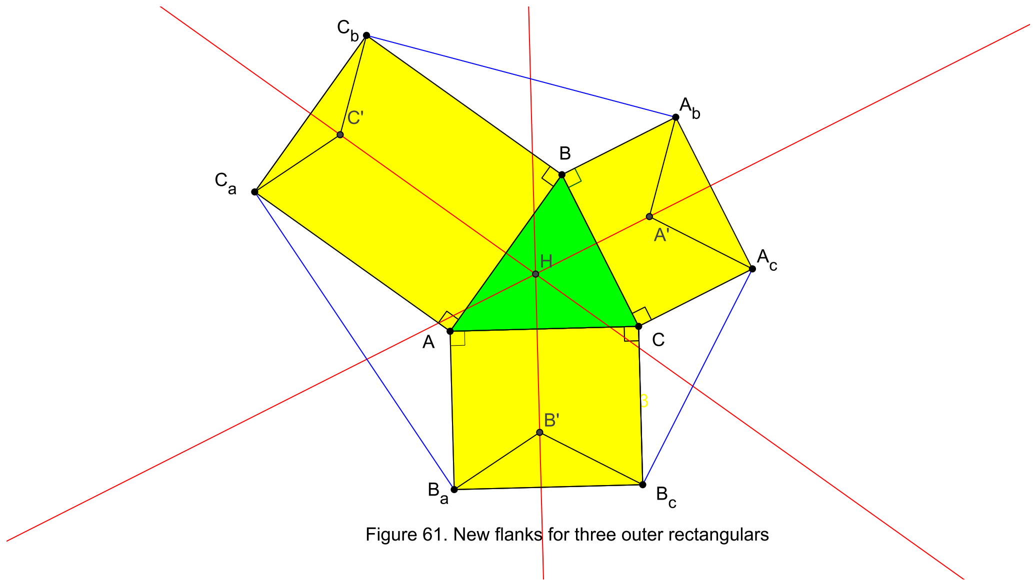

Figure 61. New flanks for three outer rectangulars

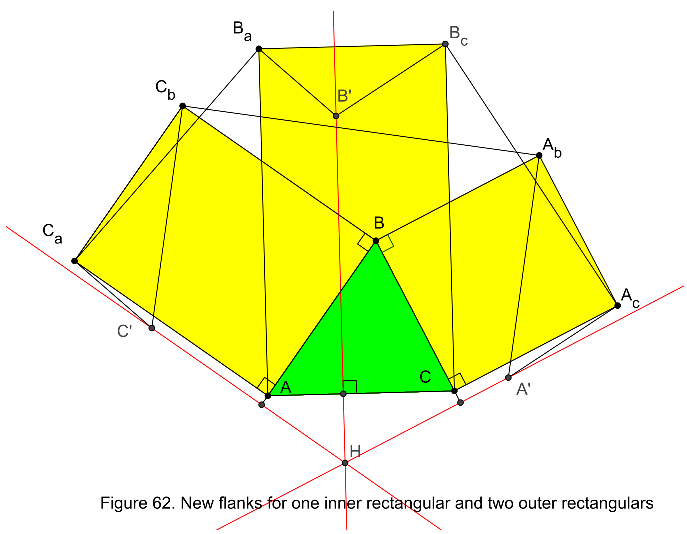

Figure 62. New flanks for one inner rectangular and two outer rectangulars

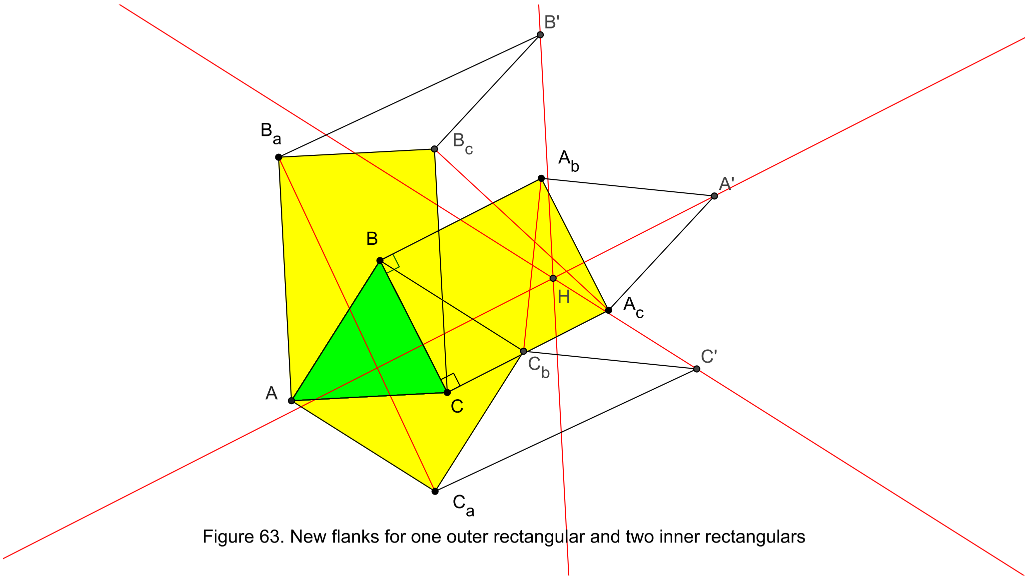

Figure 63. New flanks for one outer rectangular and two inner rectangulars

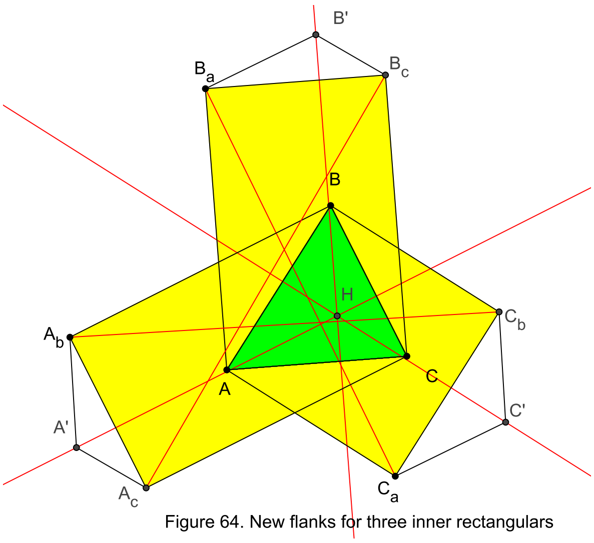

Figure 64. New flanks for three inner rectangulars

Theorem 4.15. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let C' be the intersection point of the perpendicular line of C_bA_b at C_b and the perpendicular line of C_aB_a at C_a. Similarly to A', B'. Then the lines passing through the circumcenters of triangles $A'A_cA_b$, $B'B_aB_c, C'C_aC_b$ perpendicularly to B_aC_a, C_bA_b, A_cB_c respectively are concurrent.*

See figures 65, 66, 67, 68.

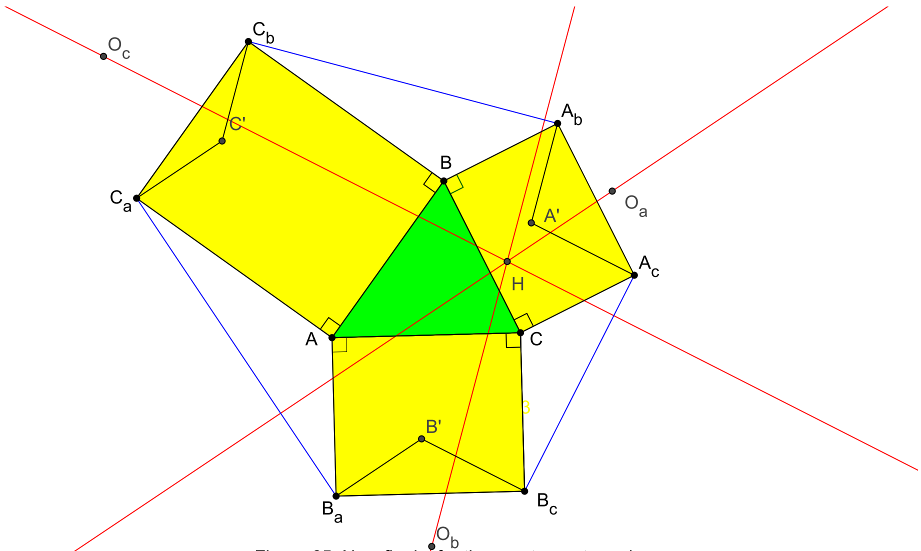

Figure 65. New flanks for three outer rectangulars

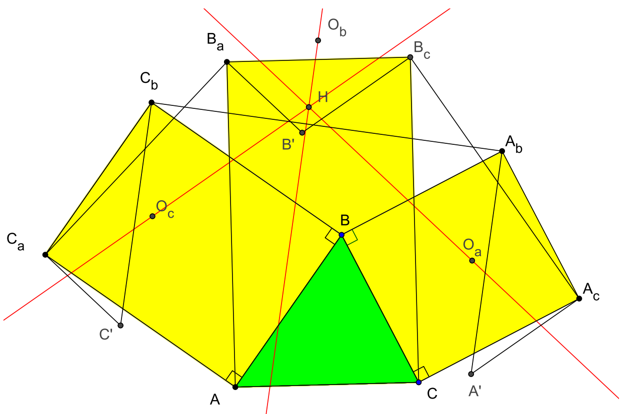

Figure 66. New flanks for one inner rectangular and two outer rectangulars

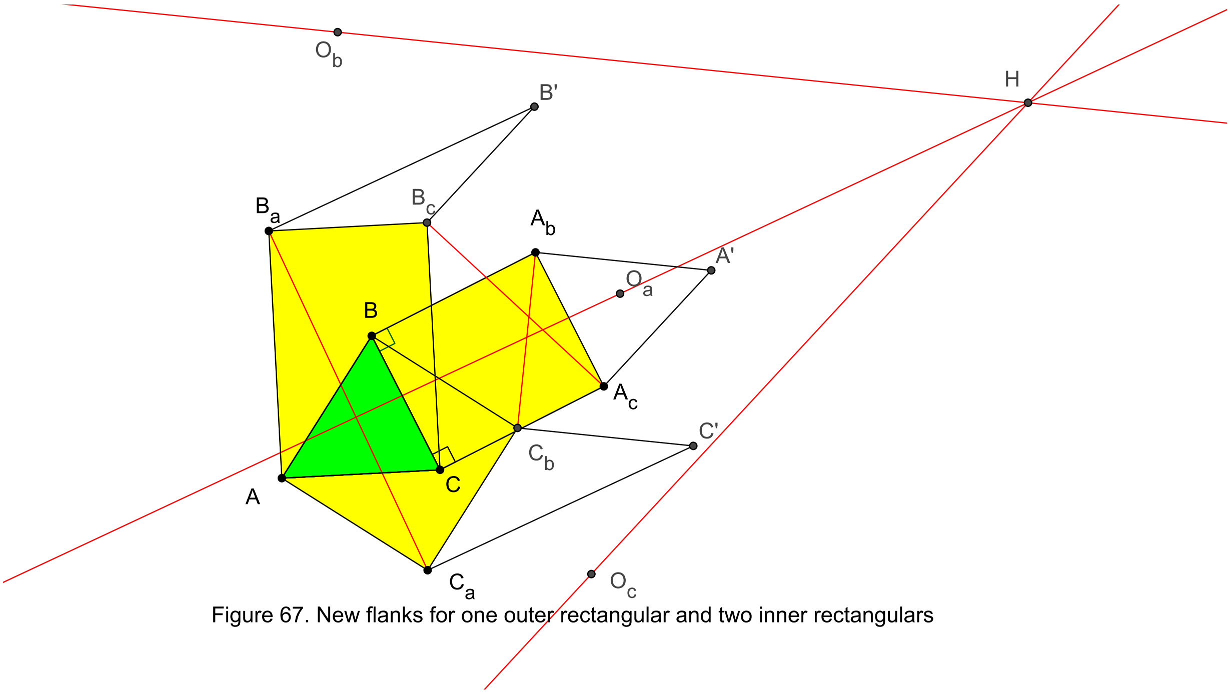

Figure 67. New flanks for one outer rectangular and two inner rectangulars

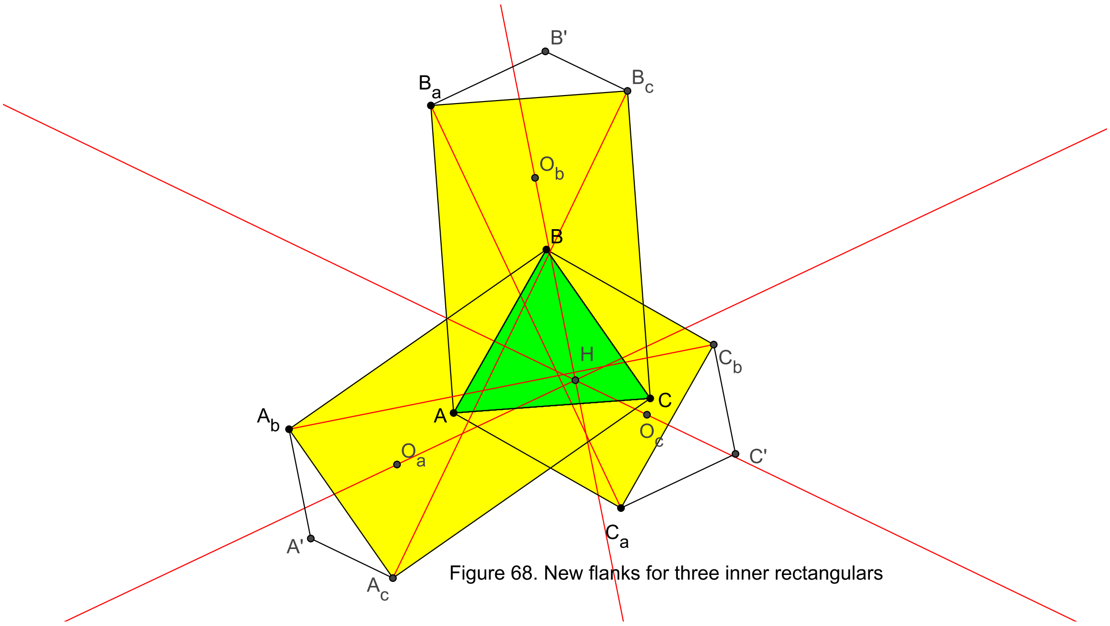

Figure 68. New flanks for three inner rectangulars

Theorem 4.16. *Given a triangle ABC. Four similar rectangulars ABC_bC_a, BCA_cA_b, $A_cA_bA'_bA'_c$, CAB_aB_c are constructed on three sides having the same orientation (inner or outer orientation). Let O be the intersection of point of $A_bA'_c$, $A_cA'_b$. Then three lines C_aA_c, B_aA_b, AO are concurrent.*

See figures 69, 70.

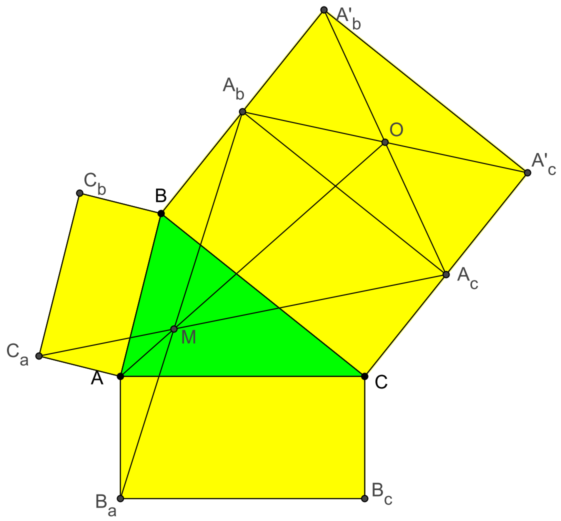

Figure 69. Three outer rectangulars

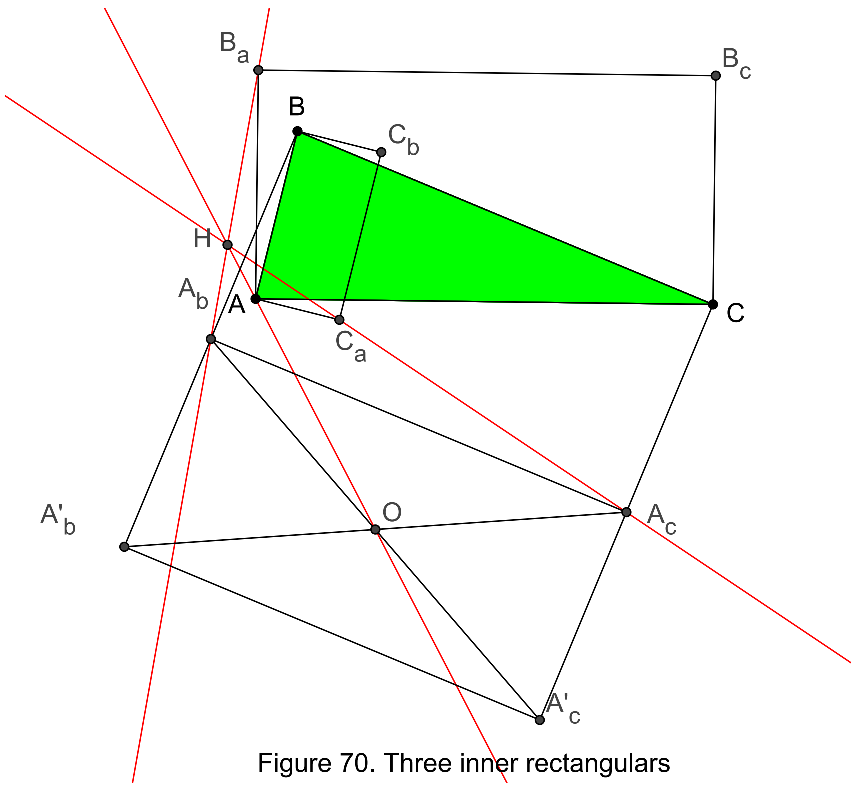

Figure 70. Three inner rectangulars

Theorem 4.17. *Given a triangle ABC. Three similar rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the same orientation (inner or outer orientation). Let O_a, O_b, O_c be the centers of rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c. Then three lines AO_a, BO_b, CO_c are concurrent.*

See figures 71, 72.

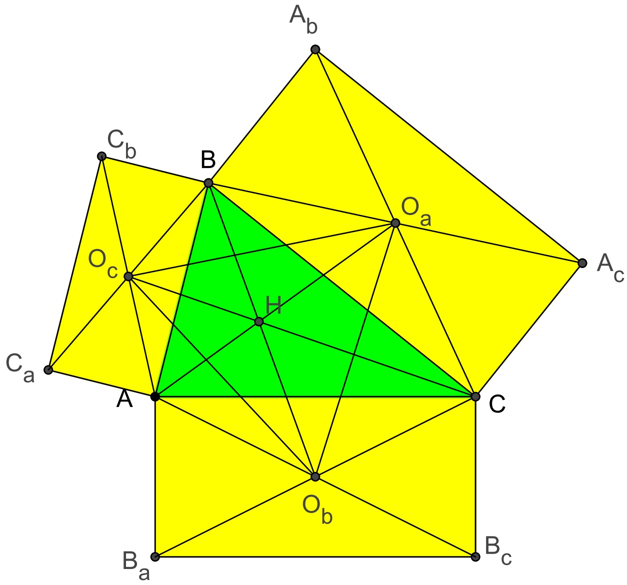

Figure 71. Three outer rectangulars

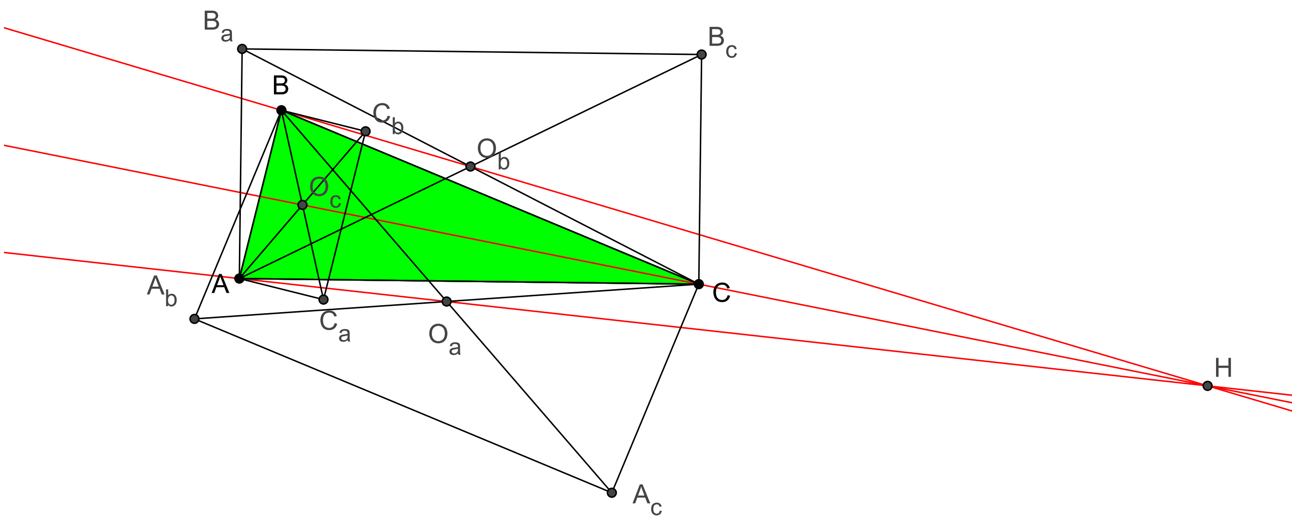

Figure 72. Three inner rectangulars

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[2] Nikolaos Dergiades and Floor van Lamoen, *Rectangles Attached to Sides of a Triangle*, Forum Geom., 2 (2003) 145–149.
[3] F. M. van Lamoen(2001), *Friendship among triangle centers*, Forum Geom., 1 1–6.
[4] Vietnamese Forum: *Nice problem, nice solutions, the passion of mathematics*