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Creation of new theorems from flanks

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Abstract. We continue our study of flanks and we give new theorems from flanks.

Keywords. Flanks, new flanks, creation of flanks.

1. Introduction

Floor van Lamoen gave the definition of flanks in [1]. We gave the definition of new flanks and generalized flanks in [2]. Let us go to the basic concepts as follows.

Given a triangle ABC. By erecting rectangulars AC_aC_bB, BA_bA_cC and CB_cB_aA externally on the sides, we form new triangles AB_aC_a, BC_bA_b and CA_cB_c, which we call the *flanks* of ABC.

Given a triangle ABC. By erecting rectangulars AC_aC_bB, BA_bA_cC and CB_cB_aA on the sides, we form new triangles AB_aC_a, BC_bA_b and CA_cB_c, which we call the *new flanks* of ABC.

Given a triangle ABC. By erecting similar isosceles trapezoids AC_aC_bB, BA_bA_cC and CB_cB_aA on the sides, we form new triangles AB_aC_a, BC_bA_b and CA_cB_c, which we call the *generalized flanks* of ABC.

2. New theorems from flanks, new flanks and generalized flanks

2.1. **Creation to new theorems from theorem 1.** We give the following theorem in [2]:

Theorem 2.1. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of C_bA_b, A_cB_c, B_aC_a perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 1, 2, 3 4.

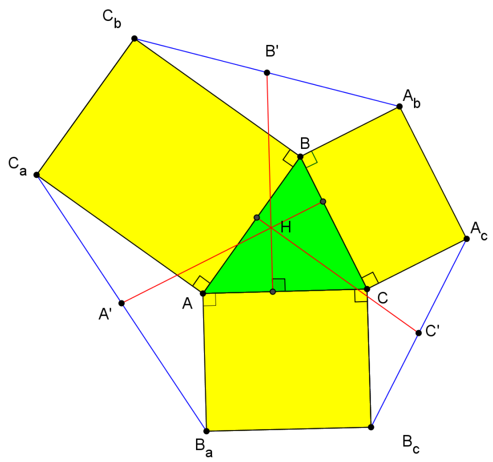

Figure 1. Three outer rectangulars

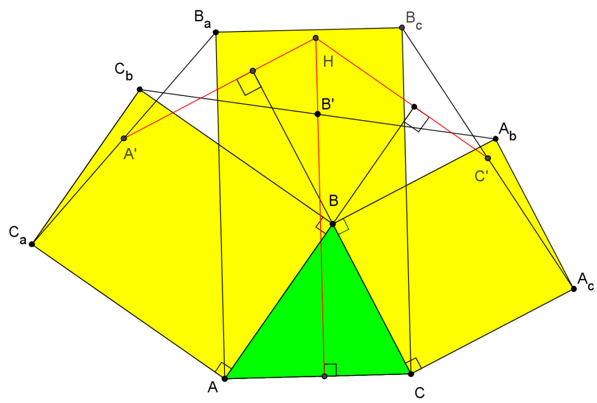

Figure 2. One inner rectangular and two outer rectangulars

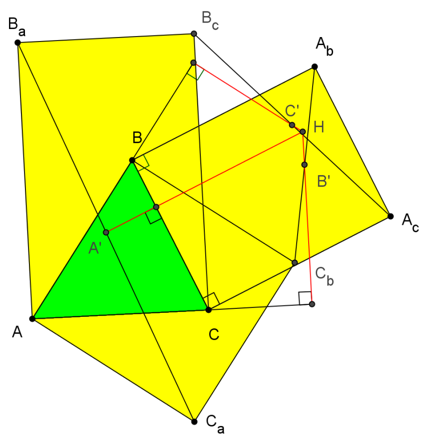

Figure 3. One outer rectangular and two inner rectangulars

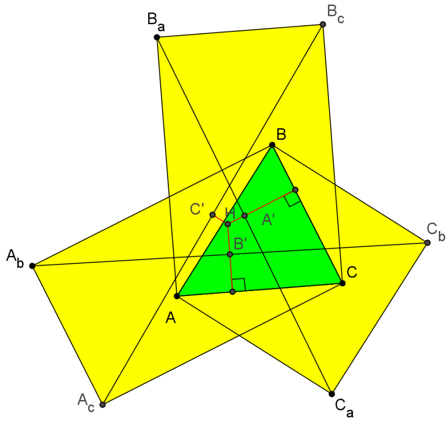

Figure 4. Three inner rectangulars

From the theorem 1, we recognize that the structure of theorem contains the following factors: *The triangle ABC and rectangulars errected on the sides of triangle; B' is the midpoint of A_bC_b and the line passing through B' perpendicularly to AC. Similarly to C' and A'.* Replacing one of these factors by a similar factor, we obtain the following theorems.

Theorem 2.2. *Given a triangle . Three arbitrary rectangulars are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of CA, AB, BC perpendicularly to C_bA_b, A_cB_c, B_aC_a respectively are concurrent.*

See figures 5, 6, 7, 8.

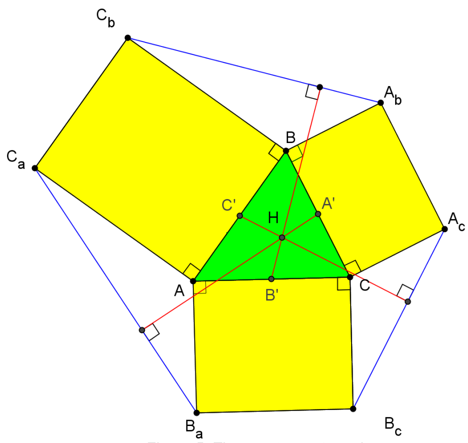

Figure 5. Three outer rectangulars

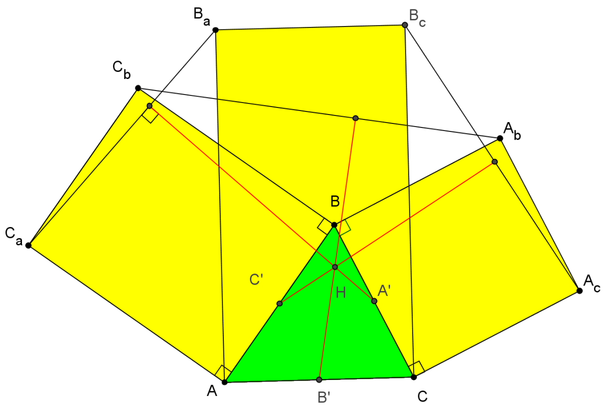

Figure 6. One inner rectangular and two outer rectangulars

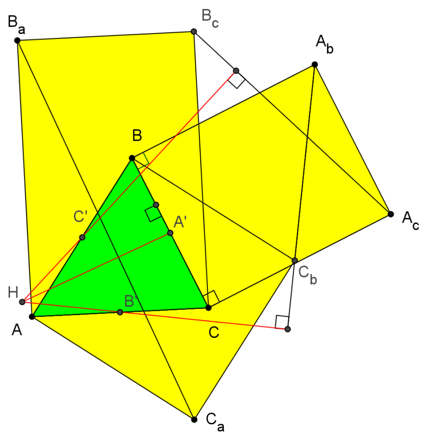

Figure 7. One outer rectangular and two inner rectangulars

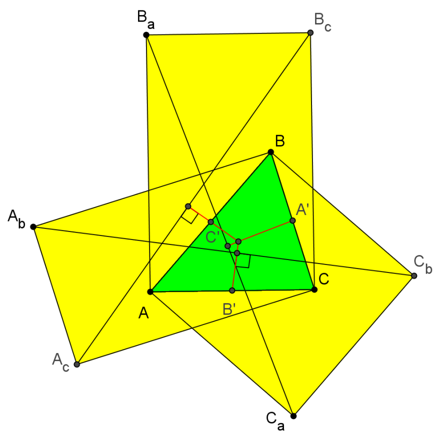

Figure 8. Three inner rectangulars

Theorem 2.3. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of A_cC_a, A_bB_a, B_cC_b perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 9, 10, 11, 12.

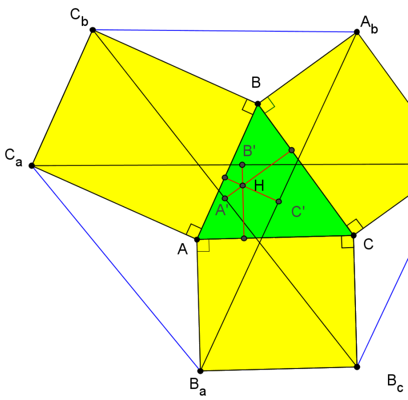

Figure 9. Three outer rectangulars

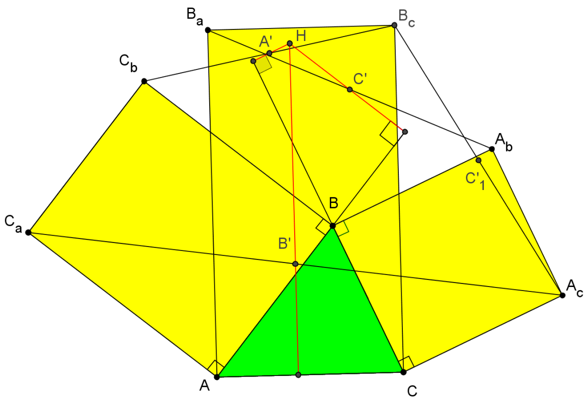

Figure 10. One inner rectangular and two outer rectangulars

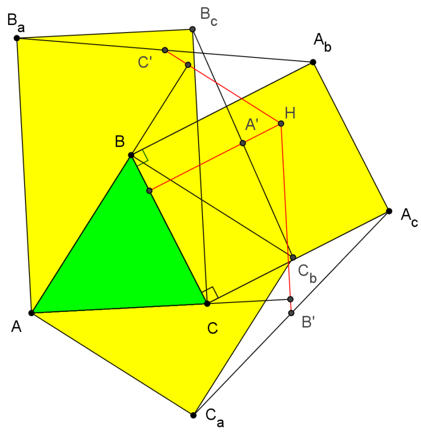

Figure 11. One outer rectangular and two inner rectangulars

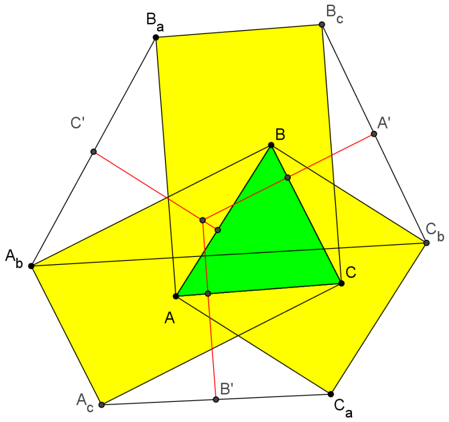

Figure 12. Three inner rectangulars

Theorem 2.4. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then the mid-perpendiculars of segments A_cC_a, A_bB_a, B_cC_b are concurrent.*

See figures 13, 14, 15, 16.

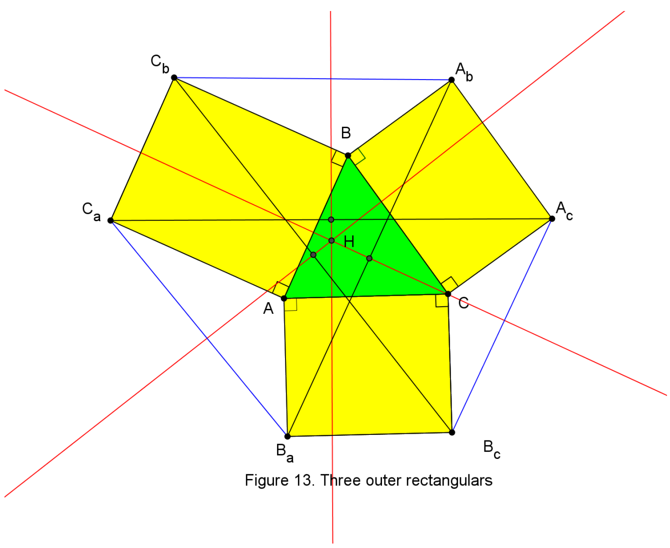

Figure 13. Three outer rectangulars

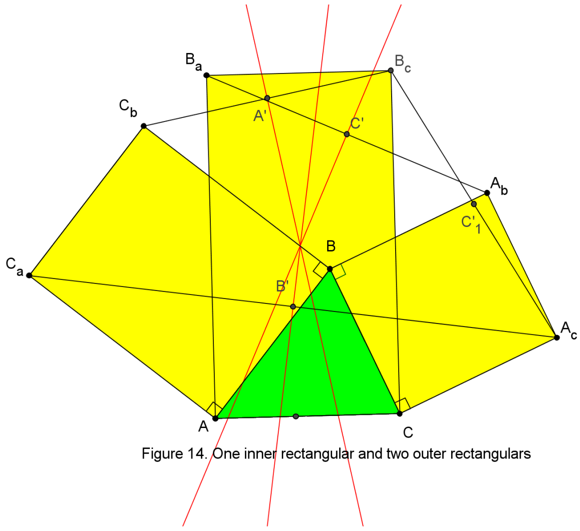

Figure 14. One inner rectangular and two outer rectangulars

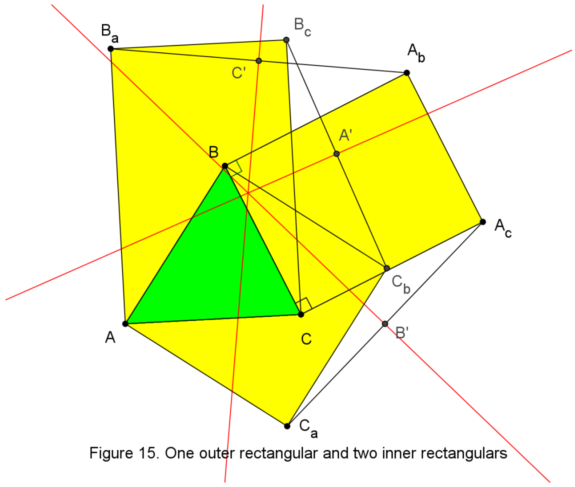

Figure 15. One outer rectangular and two inner rectangulars

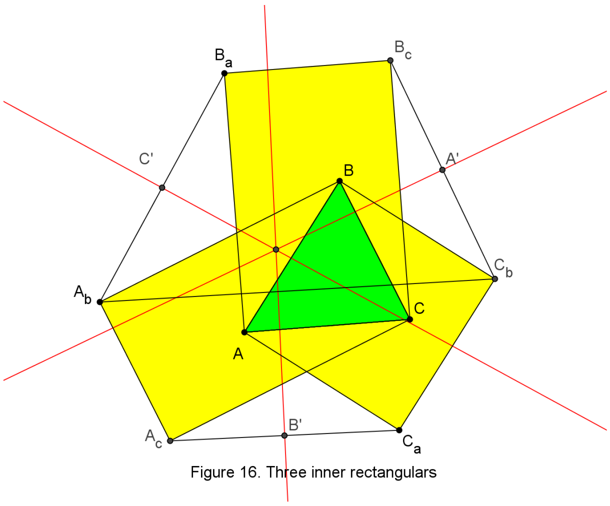

Figure 16. Three inner rectangulars

Theorem 2.5. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the circumcenters of triangles BA_bC_b, CB_cA_c, AC_aB_a perpendicularly to A_cC_a, A_bB_a, B_cC_b respectively are concurrent.*

See figures 17, 18 19, 20.

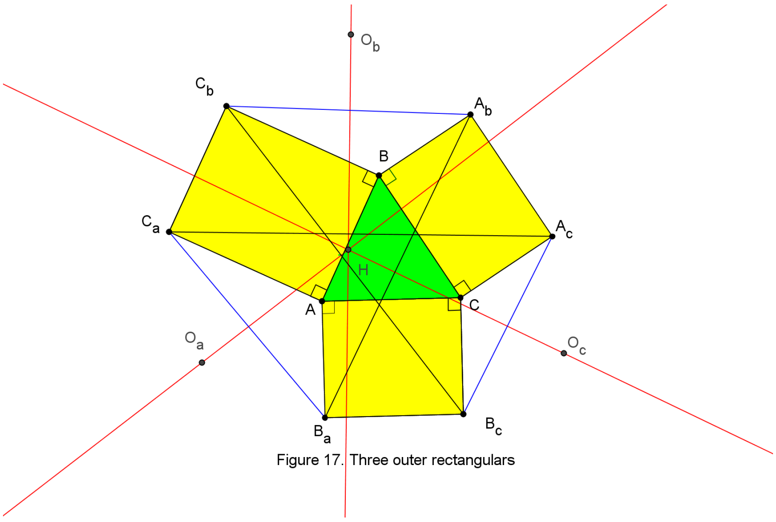

Figure 17. Three outer rectangulars

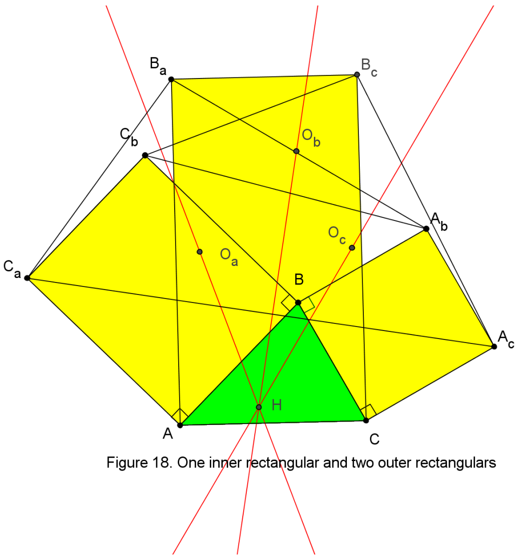

Figure 18. One inner rectangular and two outer rectangulars

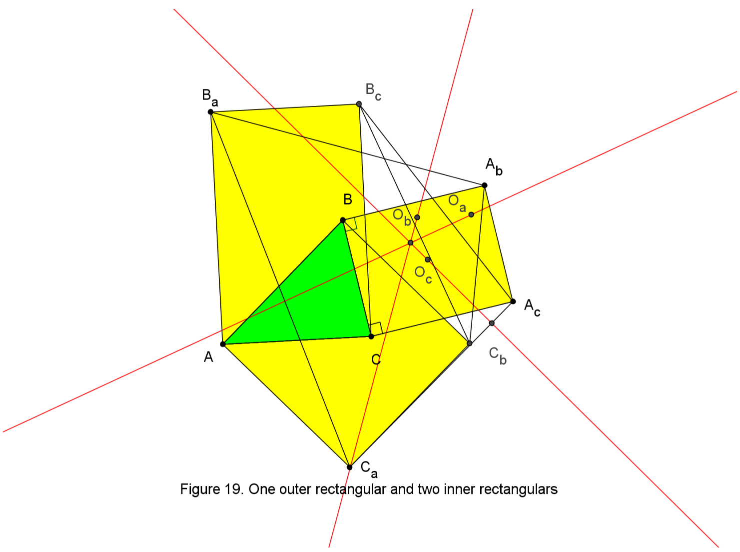

Figure 19. One outer rectangular and two inner rectangulars

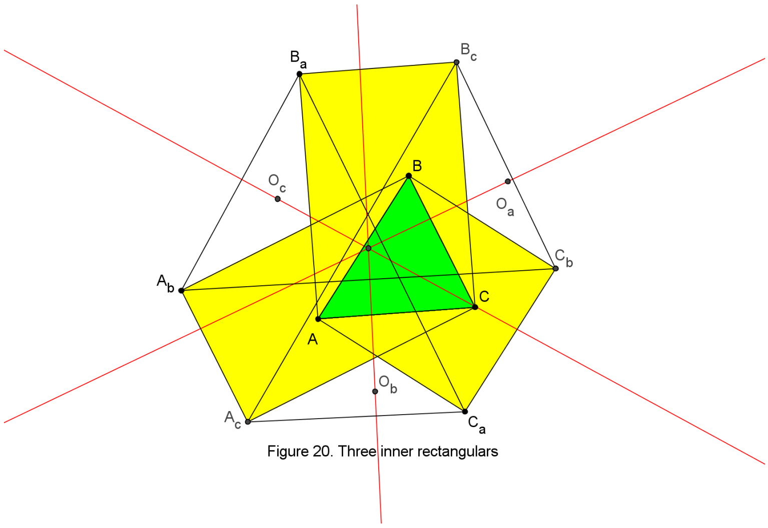

Figure 20. Three inner rectangulars

Theorem 2.6. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through three nine-point centers of triangles BA_bC_b, CB_cA_c, AC_aB_a perpendicularly to A_cC_a, A_bB_a, B_cC_b respectively are concurrent.*

See figures 21, 22, 23, 24.

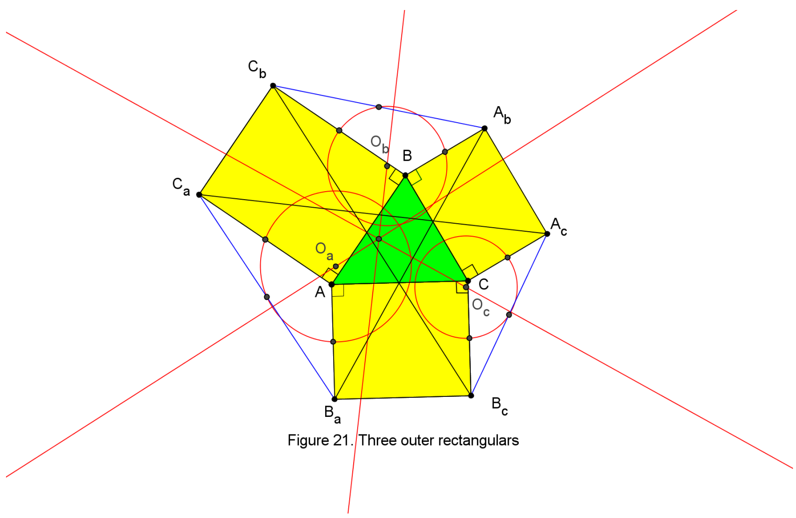

Figure 21. Three outer rectangulars

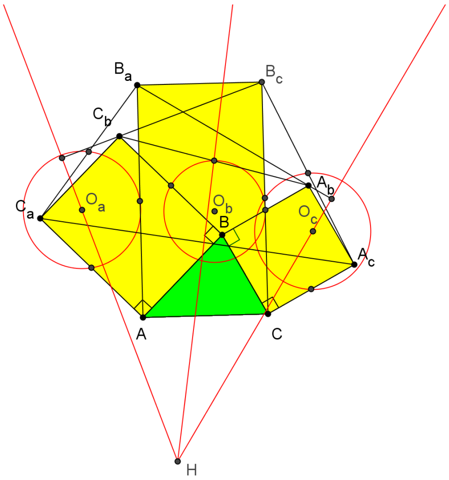

Figure 22. One inner rectangular and two outer rectangulars

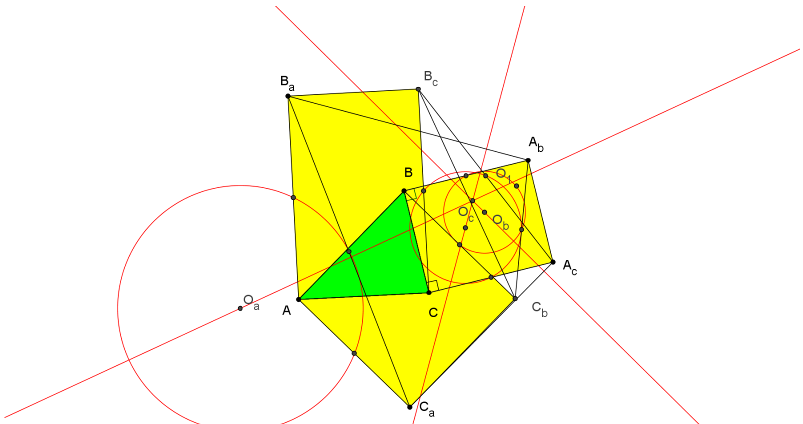

Figure 23. One outer rectangular and two inner rectangulars

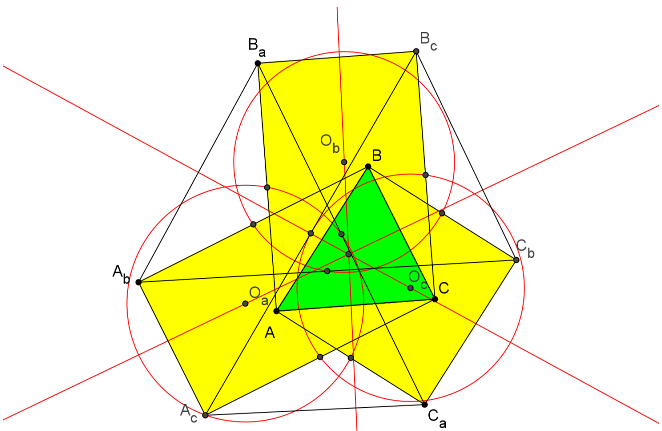

Figure 24. Three inner rectangulars

Theorem 2.7. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines AA', BB', CC' are concurrent.*

See figures 25, 26, 27, 28.

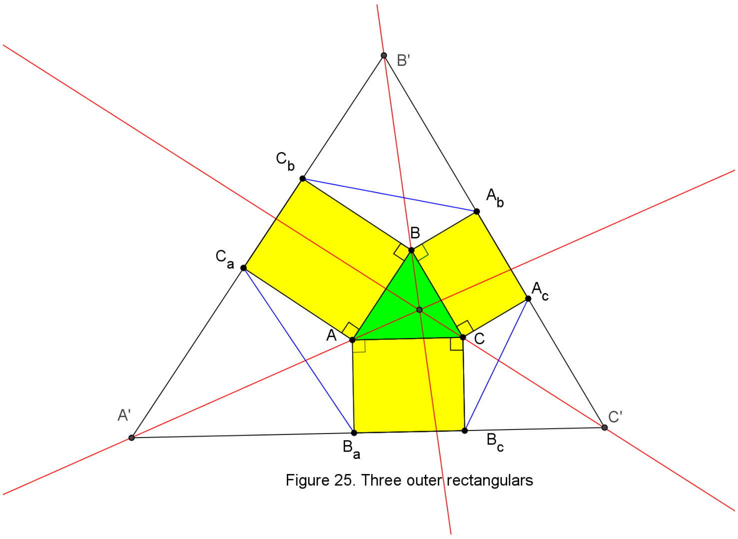

Figure 25. Three outer rectangulars

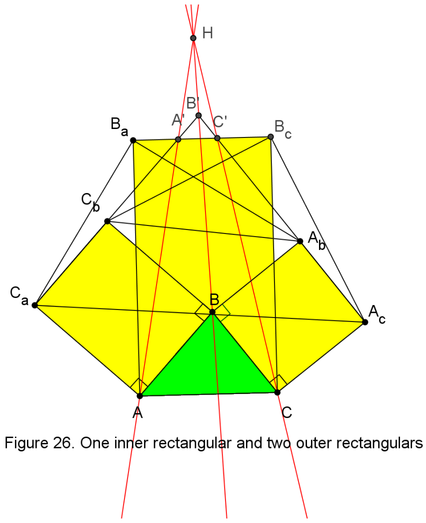

Figure 26. One inner rectangular and two outer rectangulars

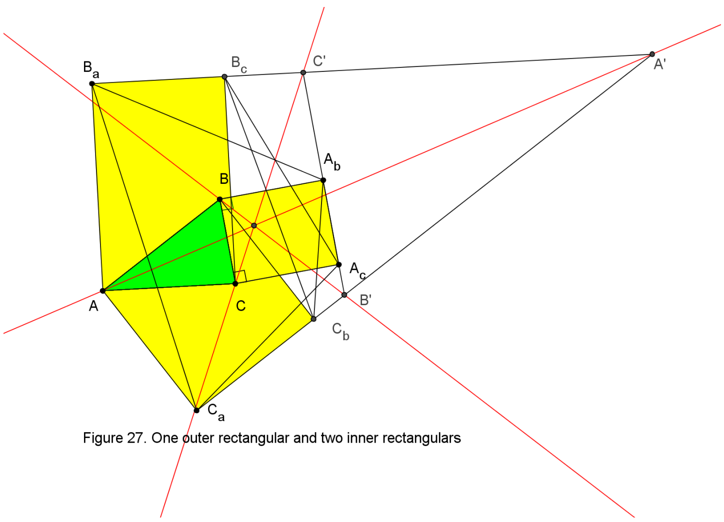

Figure 27. One outer rectangular and two inner rectangulars

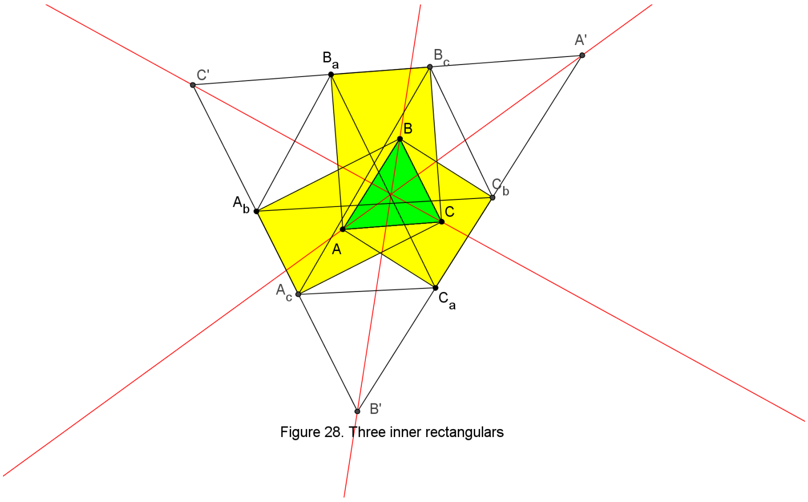

Figure 28. Three inner rectangulars

Theorem 2.8. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines passing through perpendicularly to A_bC_b, B_cA_c, C_aB_a respectively are concurrent.*

See figures 29, 30, 31, 32.

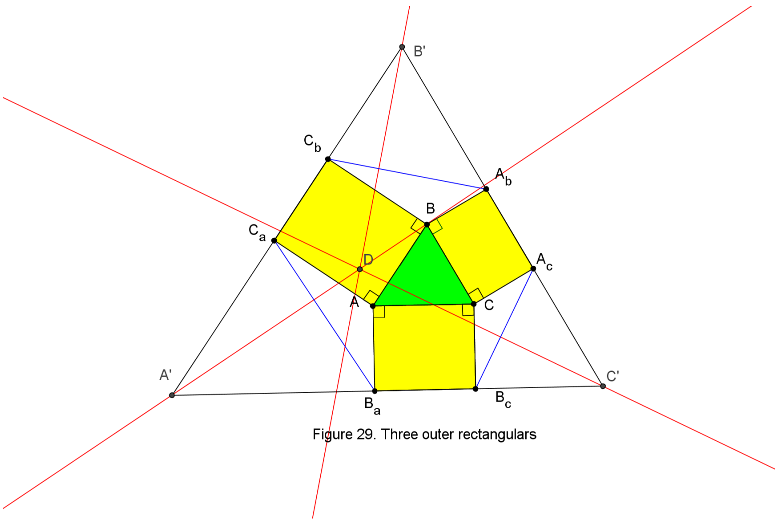

Figure 29. Three outer rectangulars

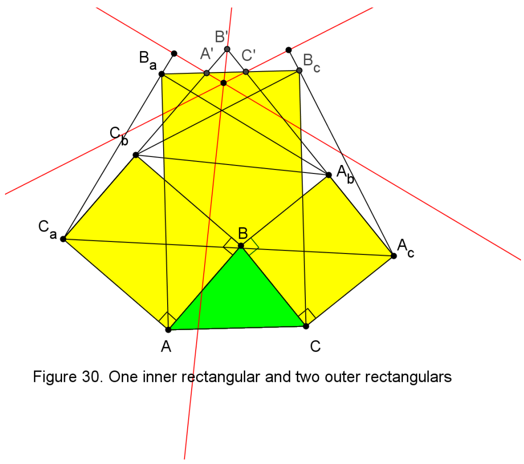

Figure 30. One inner rectangular and two outer rectangulars

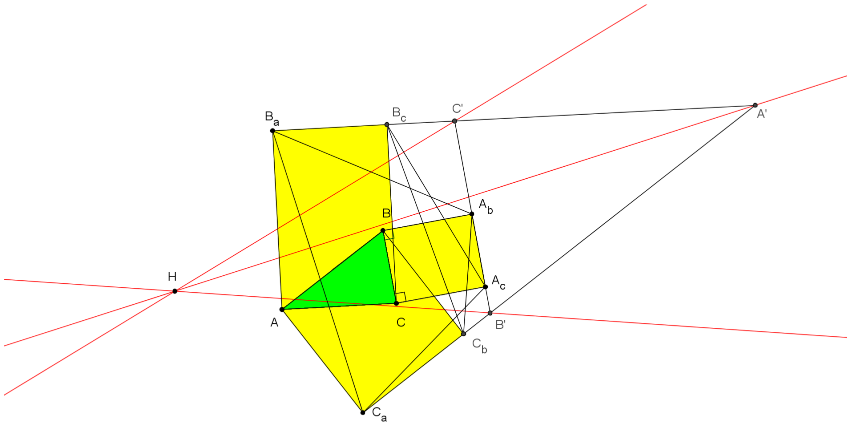

Figure 31. One outer rectangular and two inner rectangulars

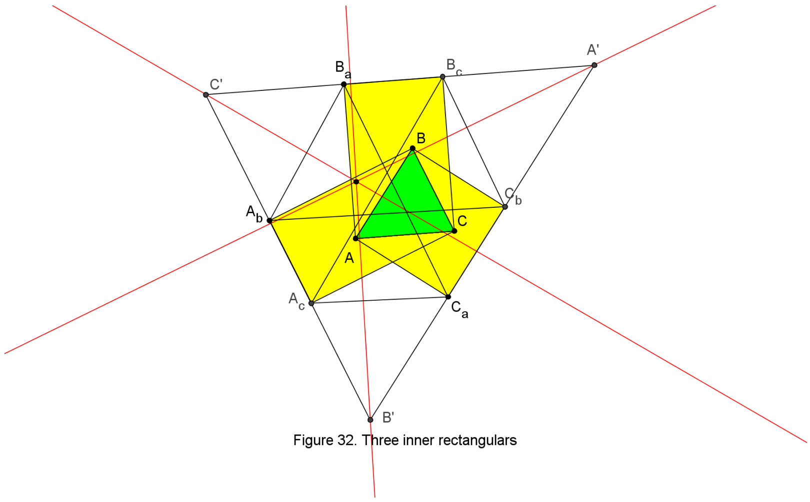

Figure 32. Three inner rectangulars

Theorem 2.9. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines passing through B', C', A' perpendicularly to A_cC_a, B_aA_b, C_bB_c respectively are concurrent.*

See figures 33, 34, 35, 36.

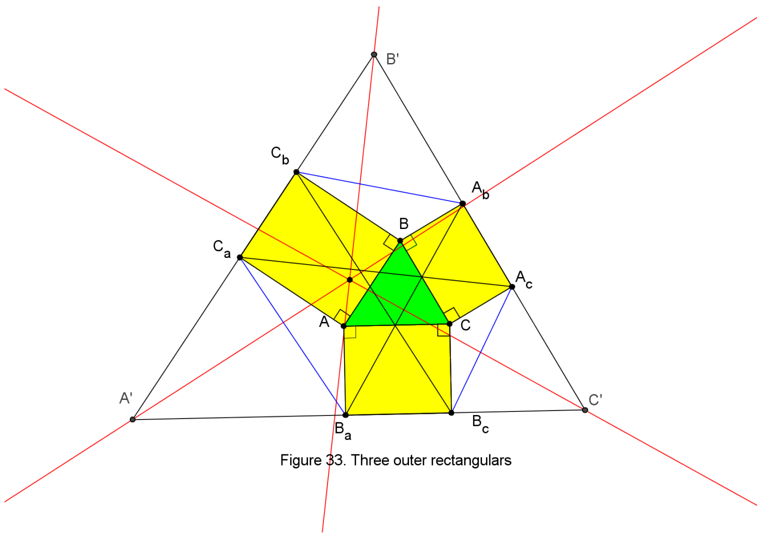

Figure 33. Three outer rectangulars

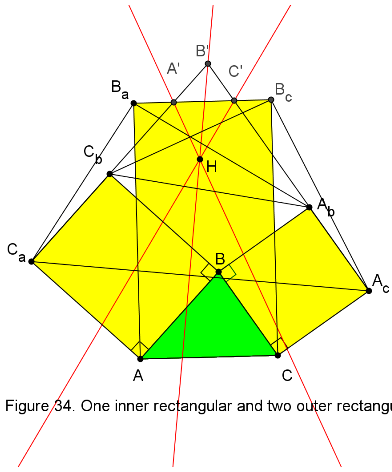

Figure 34. One inner rectangular and two outer rectangulars

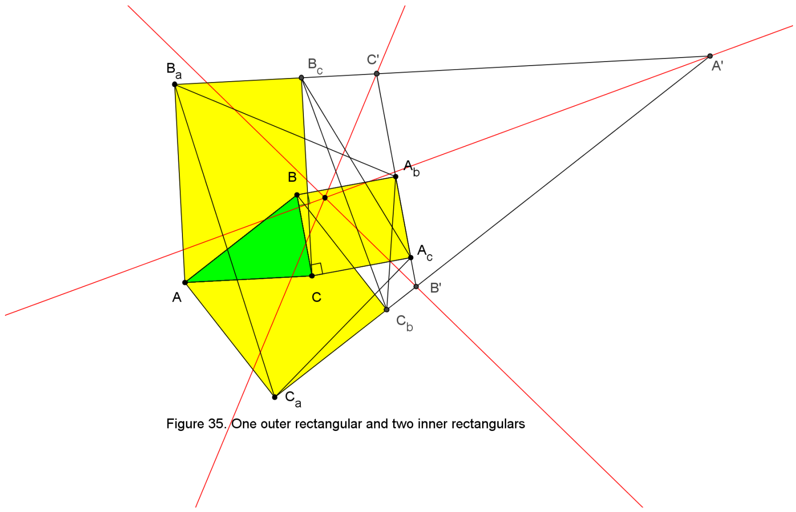

Figure 35. One outer rectangular and two inner rectangulars

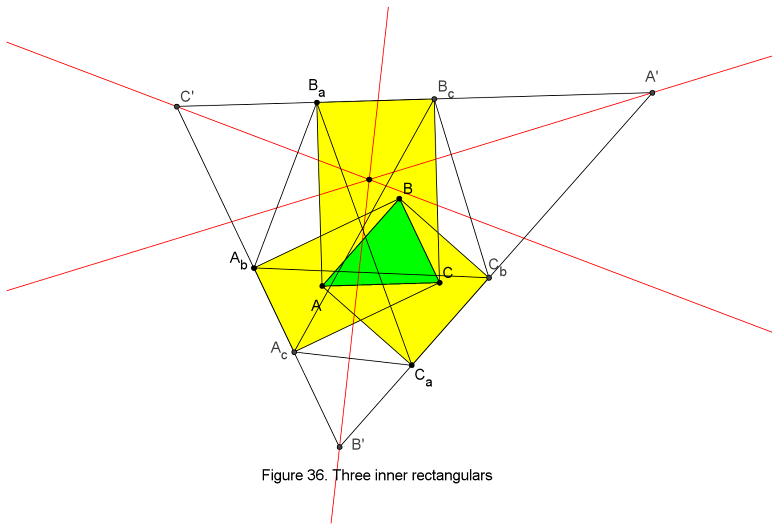

Figure 36. Three inner rectangulars

Theorem 2.10. *Given a triangle ABC. Three arbitrary rectangulars ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Construct three arbitrary rectangulars $A_bC_bC'_bA'_b$, $C_aB_aB'_aC'_a$, $B_cA_cA'_cB'_c$. Then three lines passing through the midpoints of $C'_bC'_a$, $B'_aB'_c$, $A'_cA'_b$ perpendicularly to A_cB_c, C_bA_b, B_aC_a respectively are concurrent*

See figures 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 ,51, 52.

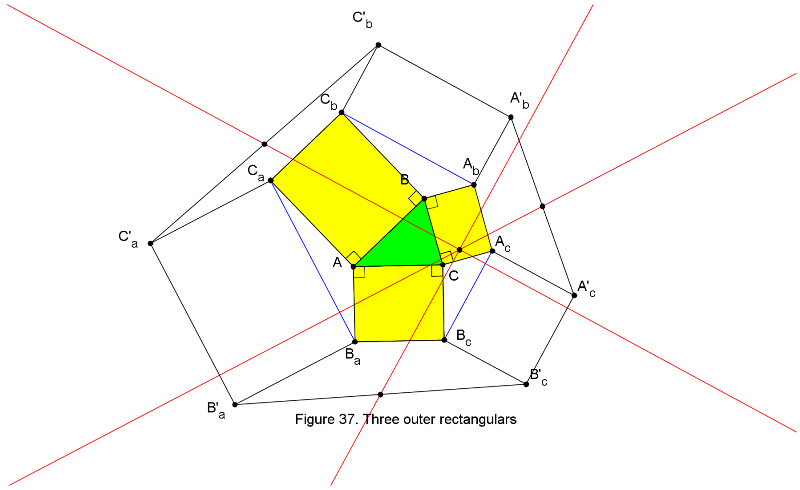

Figure 37. Three outer rectangulars

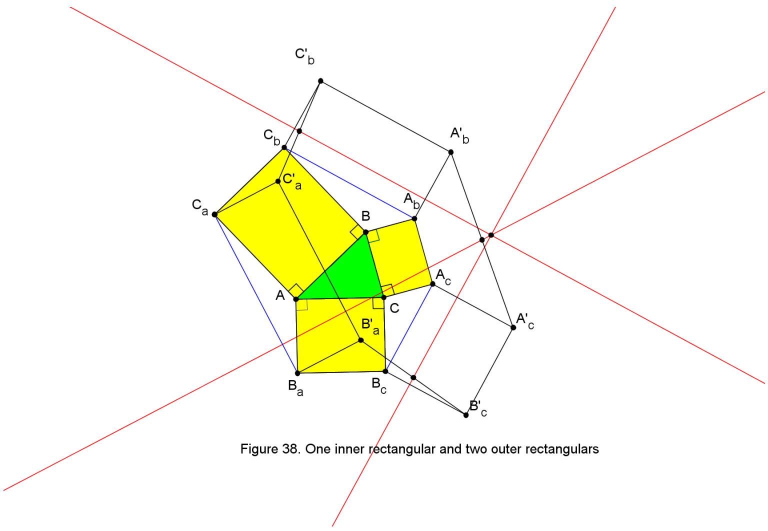

Figure 38. One inner rectangular and two outer rectangulars

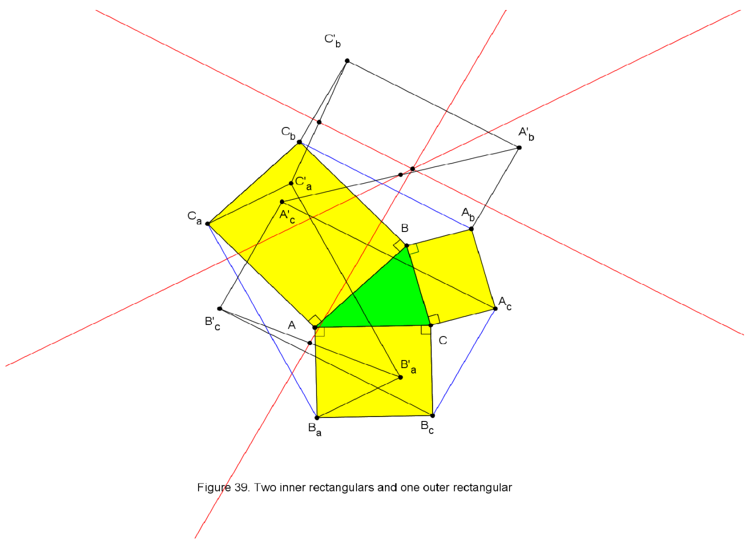

Figure 39. Two inner rectangulars and one outer rectangular

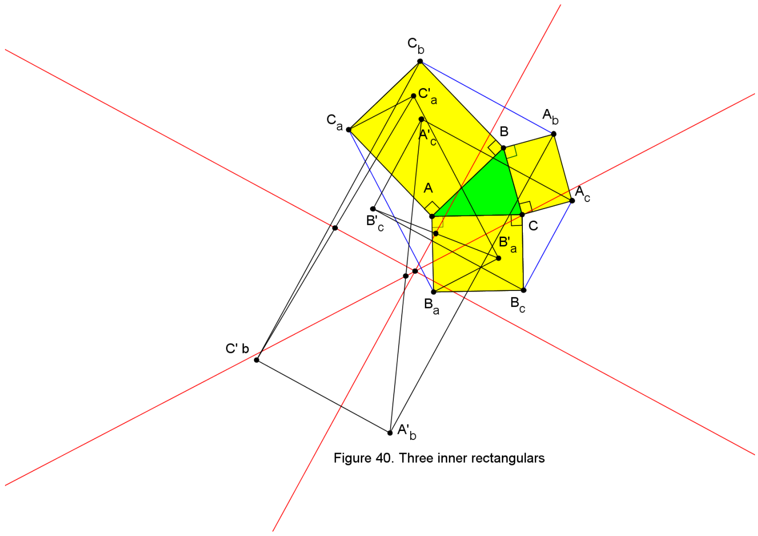

Figure 40. Three inner rectangulars

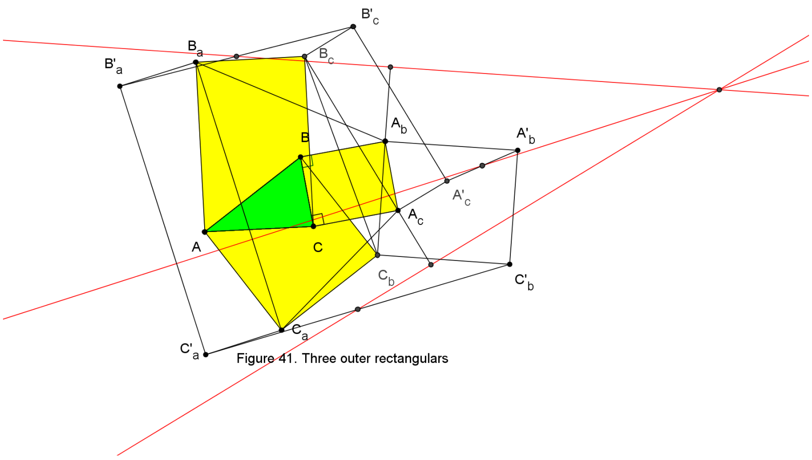

Figure 41. Three outer rectangulars

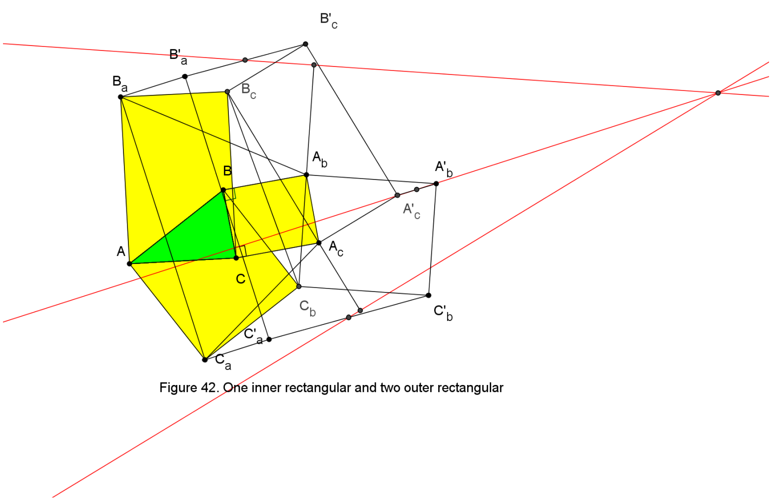

Figure 42. One inner rectangular and two outer rectangular

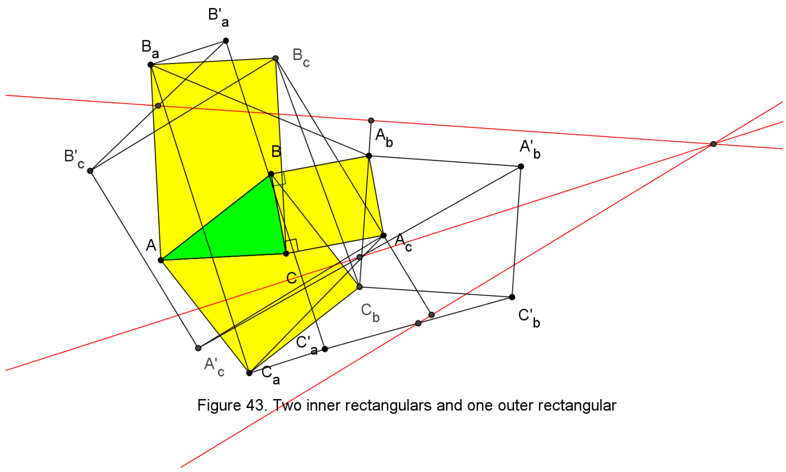

Figure 43. Two inner rectangulars and one outer rectangular

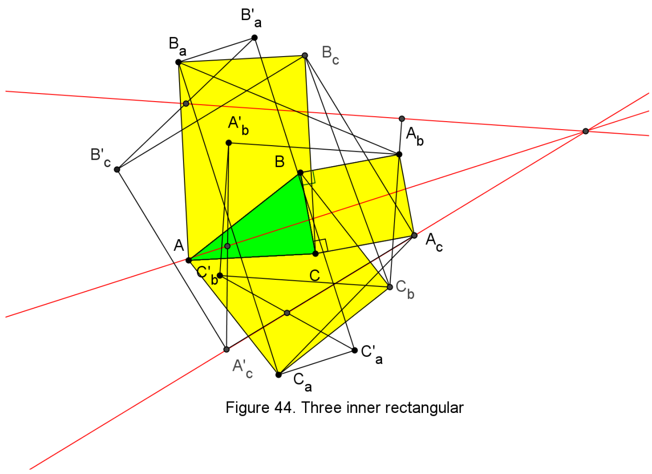

Figure 44. Three inner rectangular

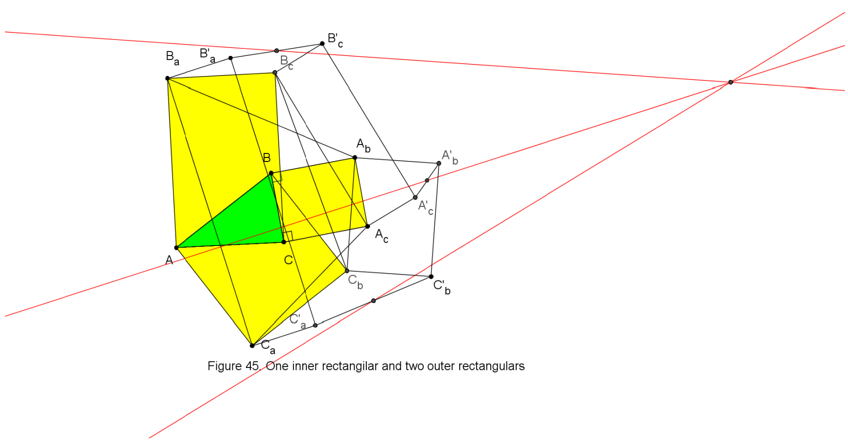

Figure 45. One inner rectangilar and two outer rectangulars

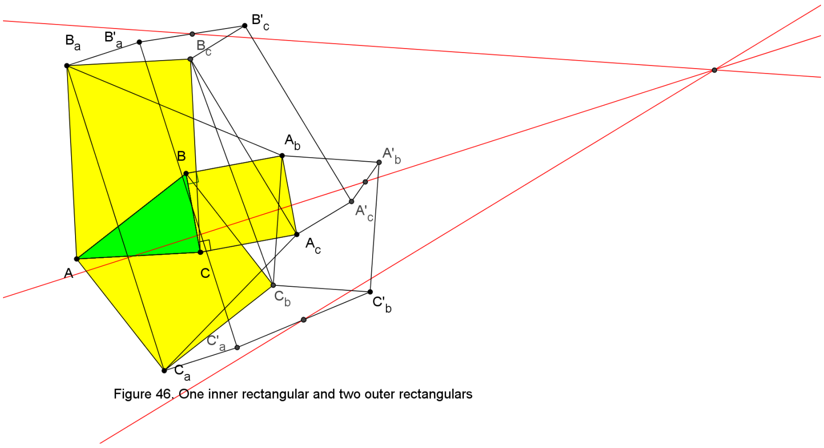

Figure 46. One inner rectangular and two outer rectangulars

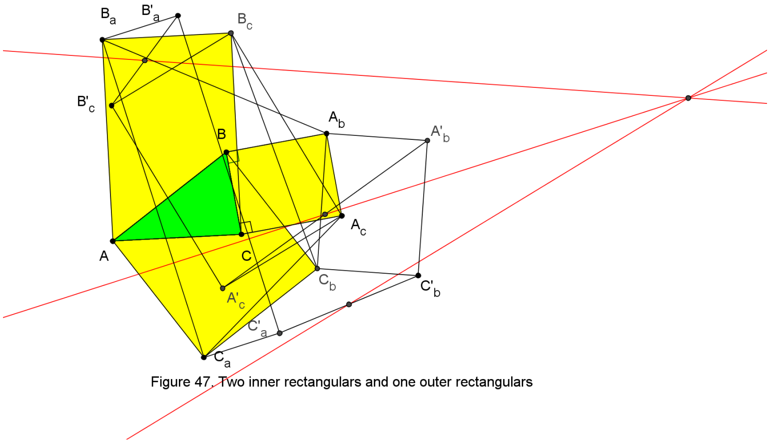

Figure 47. Two inner rectangulars and one outer rectangulars

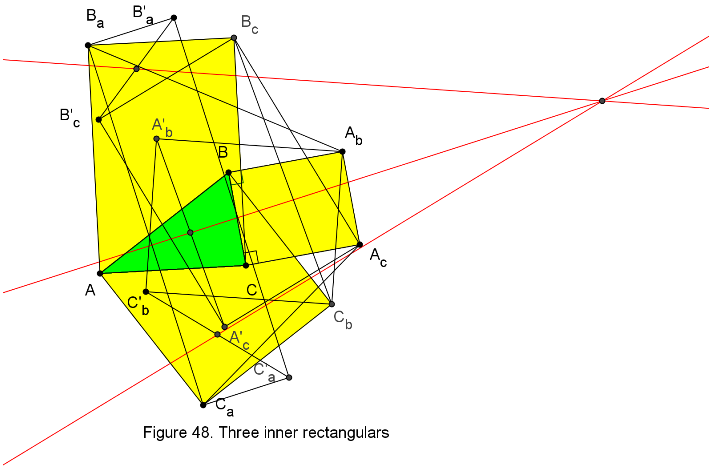

Figure 48. Three inner rectangulars

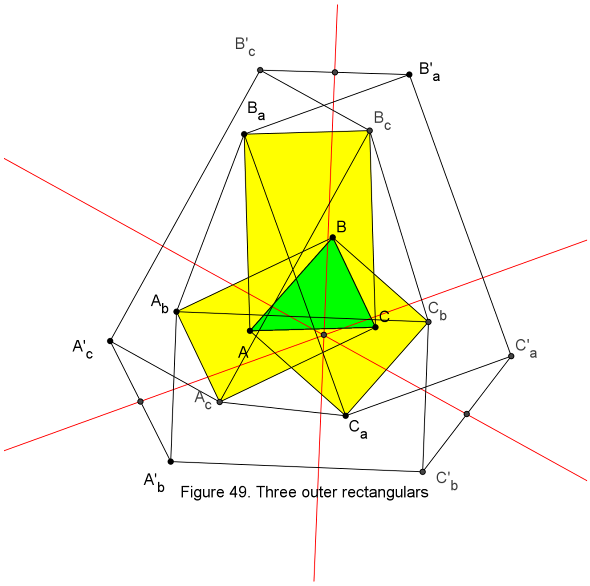

Figure 49. Three outer rectangulars

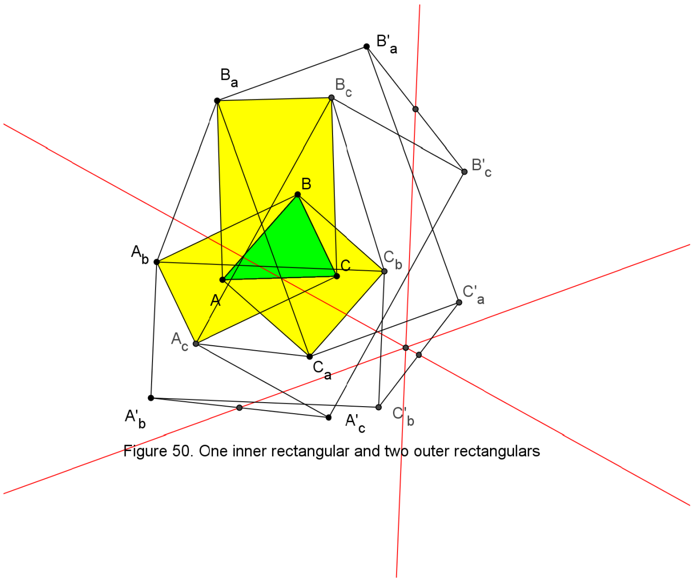

Figure 50. One inner rectangular and two outer rectangulars

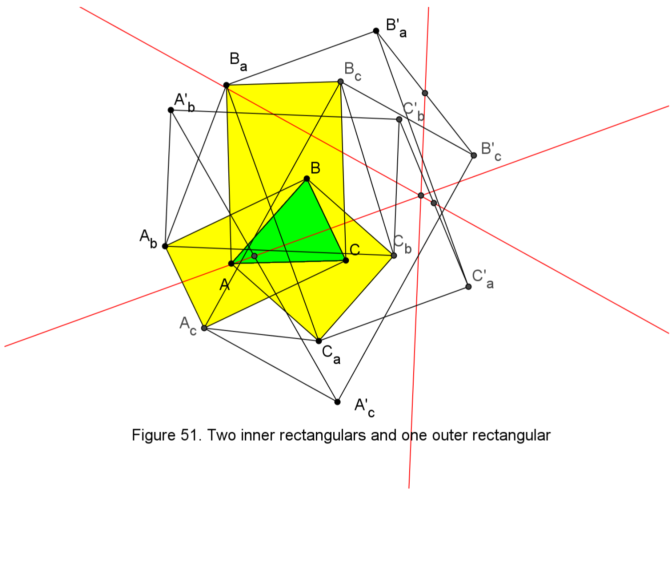

Figure 51. Two inner rectangulars and one outer rectangular

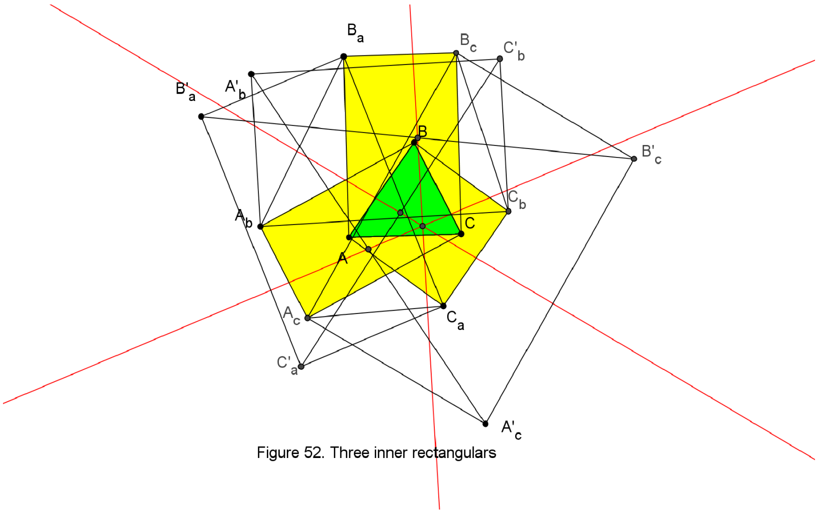

Figure 52. Three inner rectangulars

Theorem 2.11. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of sides CA, AB, BC perpendicularly to C_bA_b, A_cB_c, B_aC_a respectively are concurrent.*

See figures 53, 54, 55, 56.

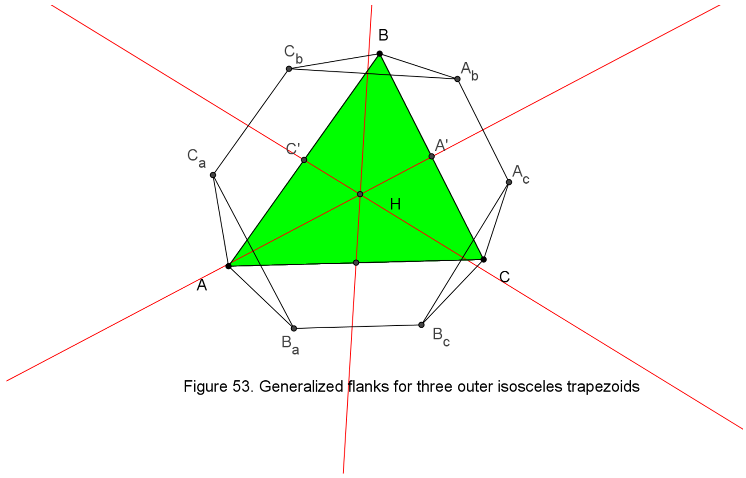

Figure 53. Generalized flanks for three outer isosceles trapezoids

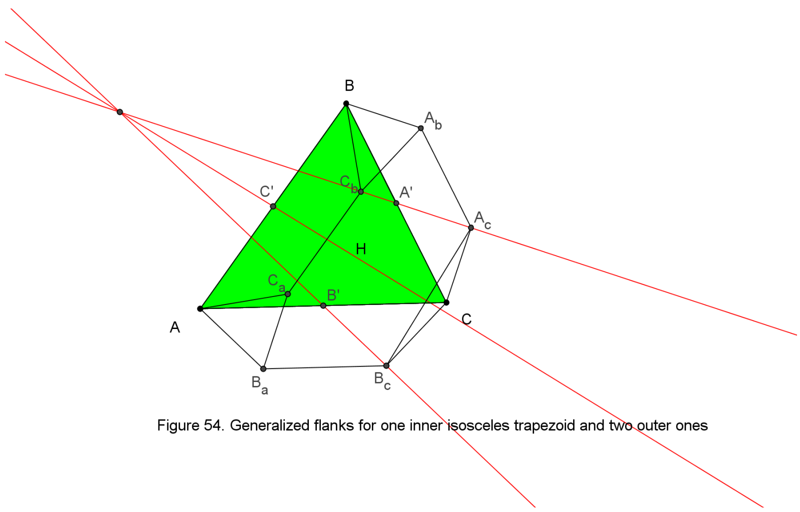

Figure 54. Generalized flanks for one inner isosceles trapezoid and two outer ones

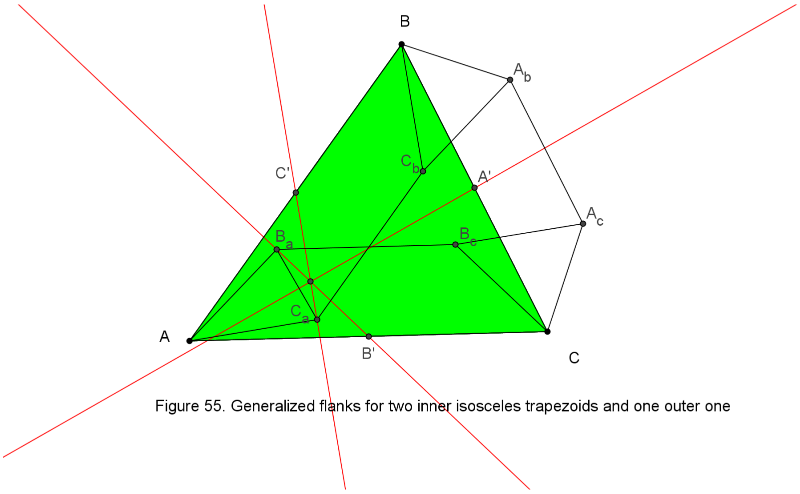

Figure 55. Generalized flanks for two inner isosceles trapezoids and one outer one

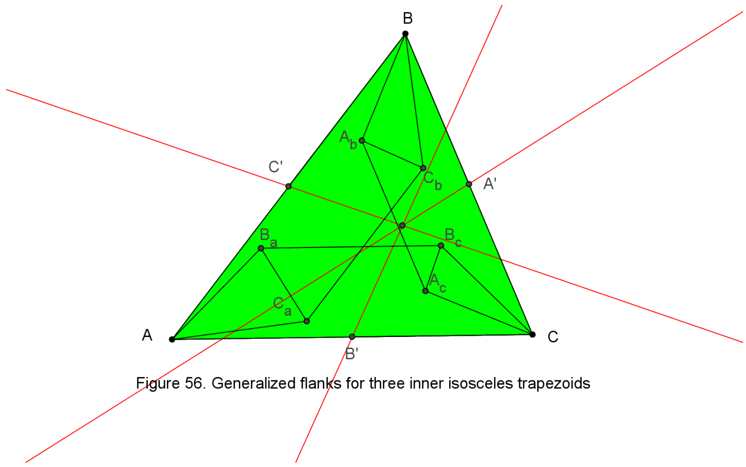

Figure 56. Generalized flanks for three inner isosceles trapezoids

Theorem 2.12. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the midpoints of CA, AB, BC perpendicularly to A_cC_a, A_bB_a, B_cC_b respectively are concurrent.*

See figures 57, 58, 59, 60.

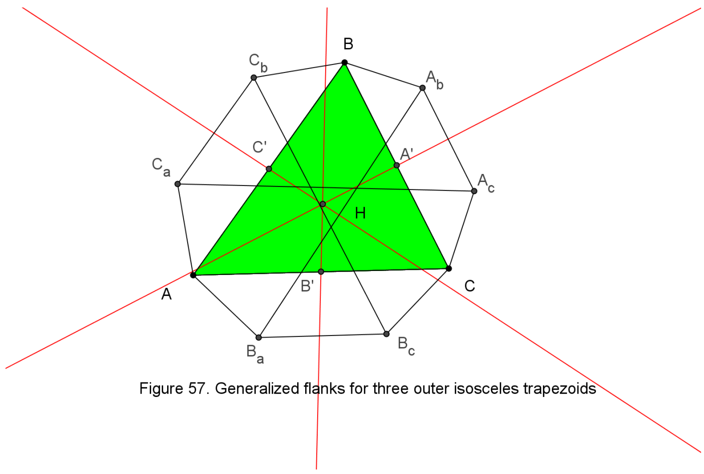

Figure 57. Generalized flanks for three outer isosceles trapezoids

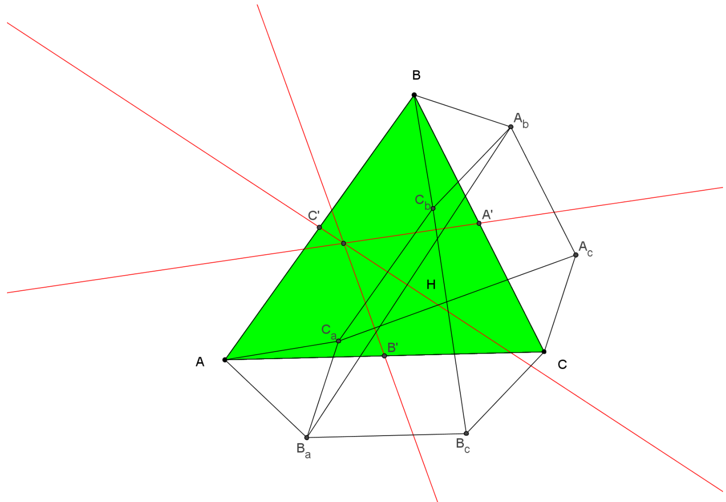

Figure 58. Generalized flanks for one inner isosceles trapezoid and two outer ones

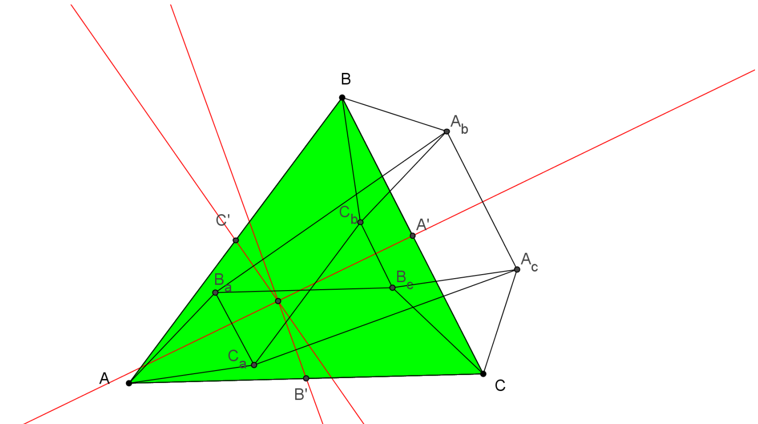

Figure 59. Generalized flanks for two inner isosceles trapezoids and one outer one

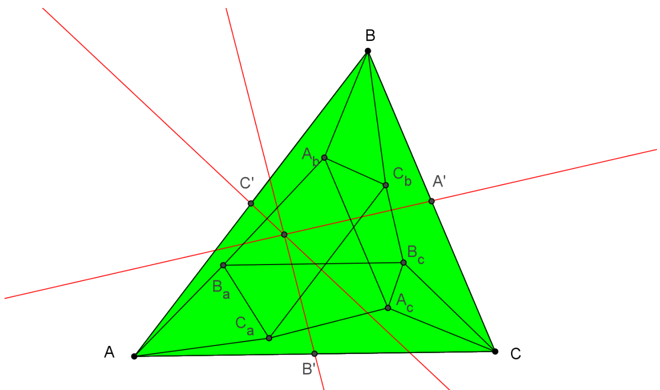

Figure 60. Generalized flanks for three inner isosceles trapezoids

Theorem 2.13. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three lines passing through the mid-points of A_cC_a, A_bB_a, B_cC_b perpendicularly to CA, AB, BC respectively are concurrent.*

See figures 61, 62, 63, 64.

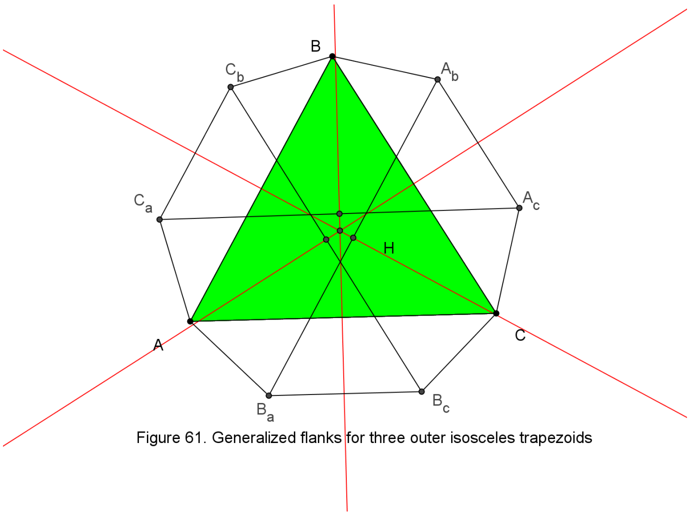

Figure 61. Generalized flanks for three outer isosceles trapezoids

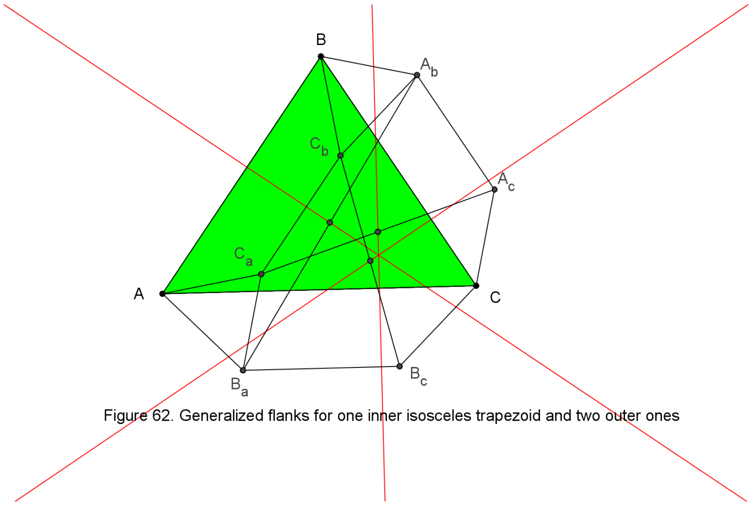

Figure 62. Generalized flanks for one inner isosceles trapezoid and two outer ones

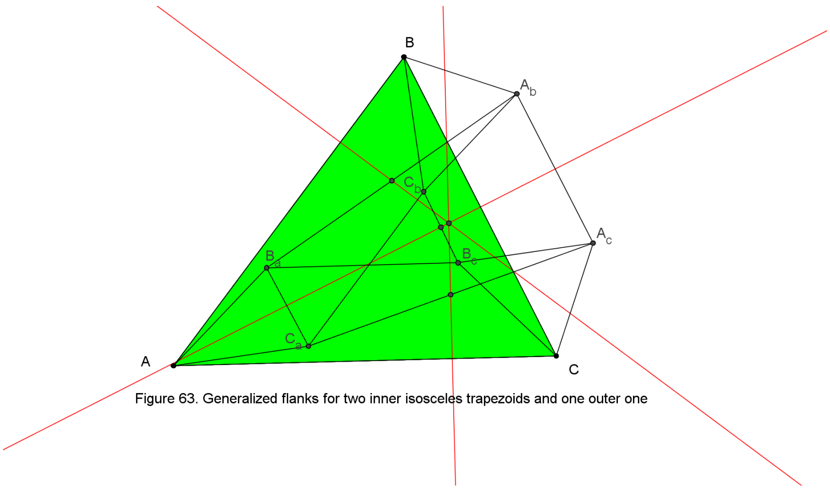

Figure 63. Generalized flanks for two inner isosceles trapezoids and one outer one

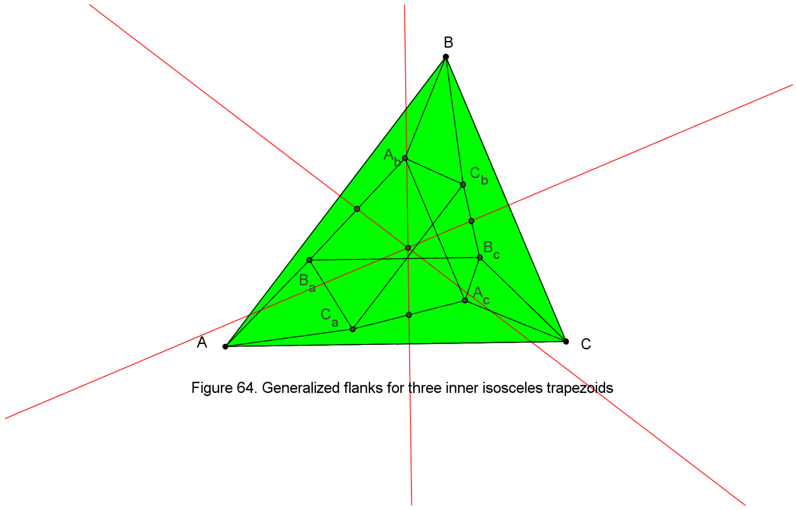

Figure 64. Generalized flanks for three inner isosceles trapezoids

Theorem 2.14. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Then three mid-perpendiculars of segments A_cC_a, A_bB_a, B_cC_b are concurrent.*

See figures 65, 66, 67, 68.

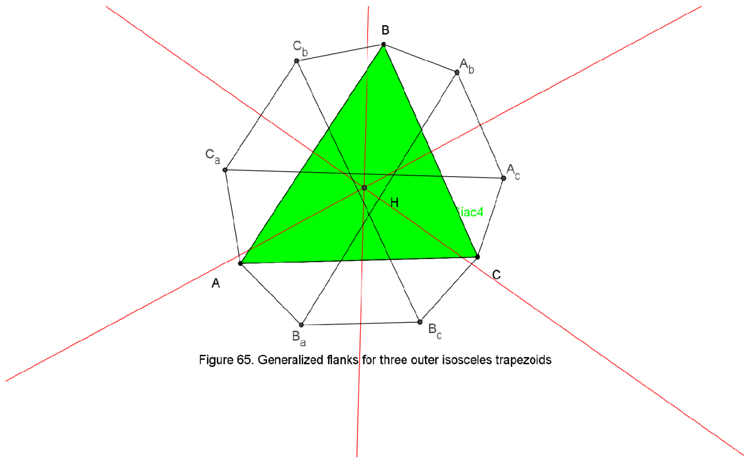

Figure 65. Generalized flanks for three outer isosceles trapezoids

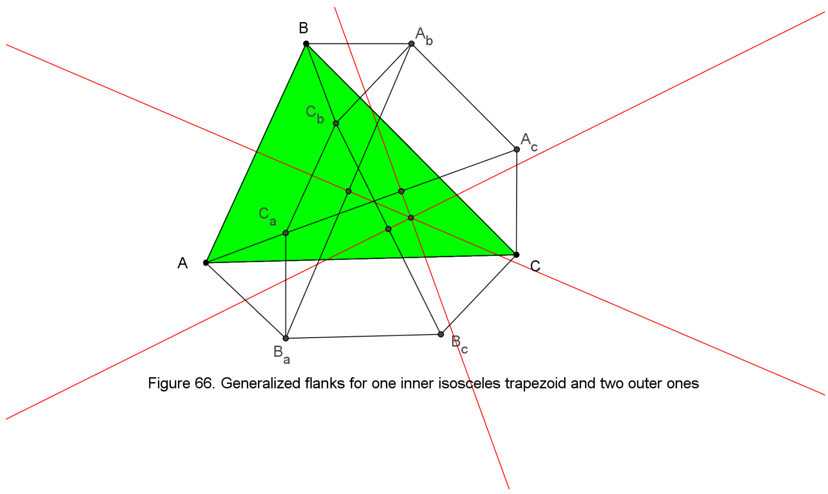

Figure 66. Generalized flanks for one inner isosceles trapezoid and two outer ones

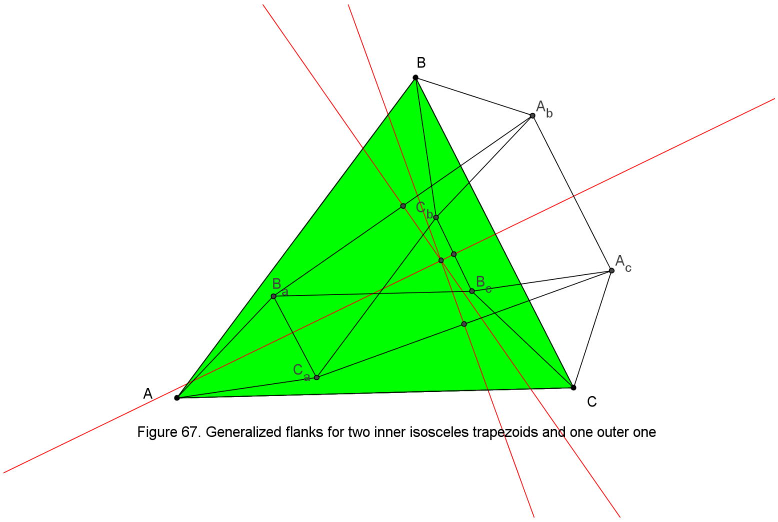

Figure 67. Generalized flanks for two inner isosceles trapezoids and one outer one

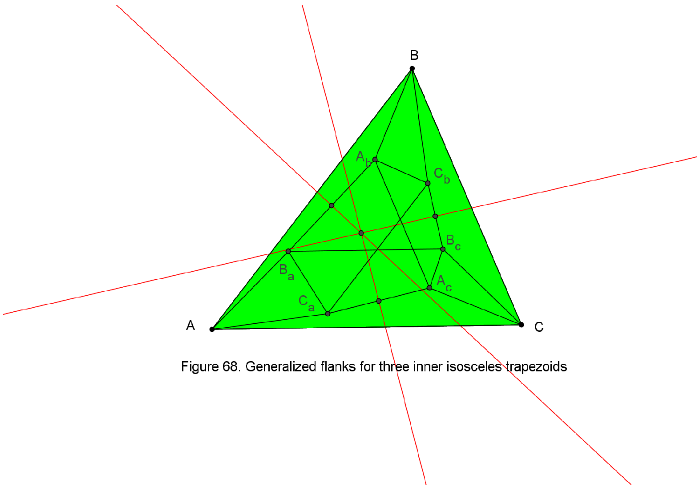

Figure 68. Generalized flanks for three inner isosceles trapezoids

Theorem 2.15. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines AA', BB', CC' are concurrent, respectively.*

See figures 69, 70, 71, 72.

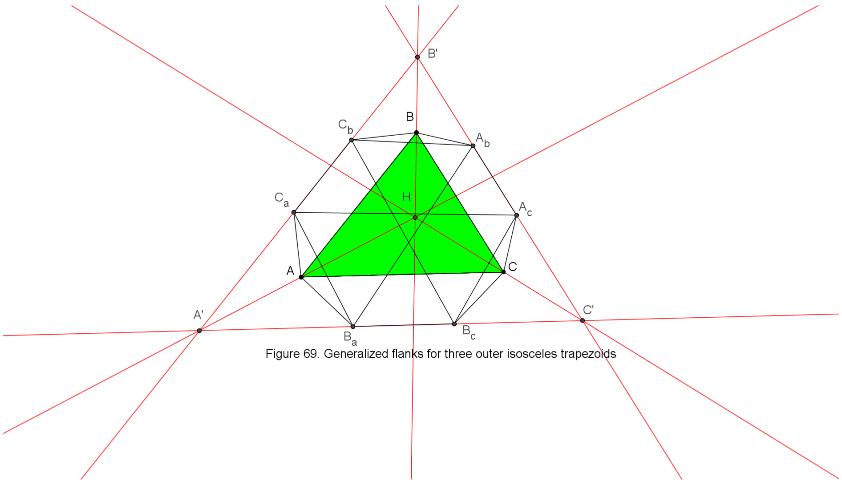

Figure 69. Generalized flanks for three outer isosceles trapezoids

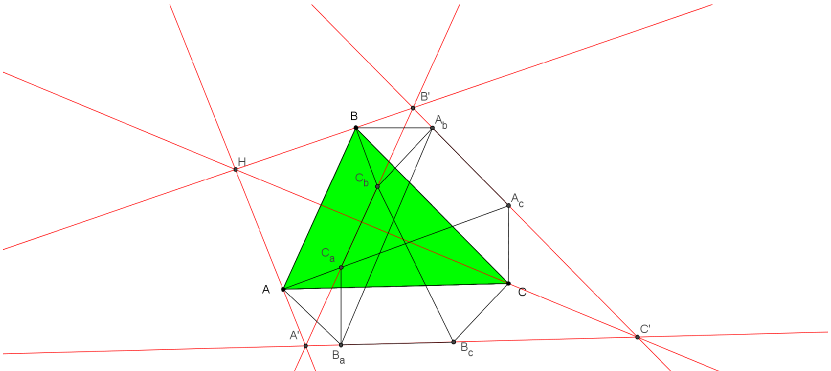

Figure 70. Generalized flanks for one inner isosceles trapezoid and two outer ones

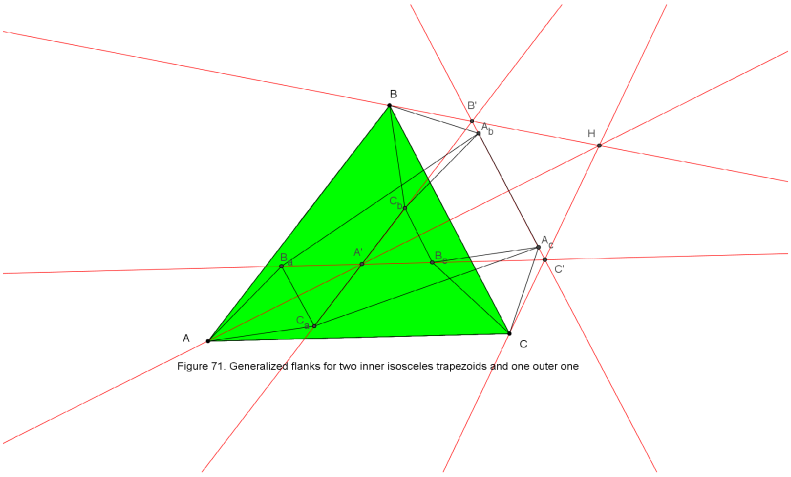

Figure 71. Generalized flanks for two inner isosceles trapezoids and one outer one

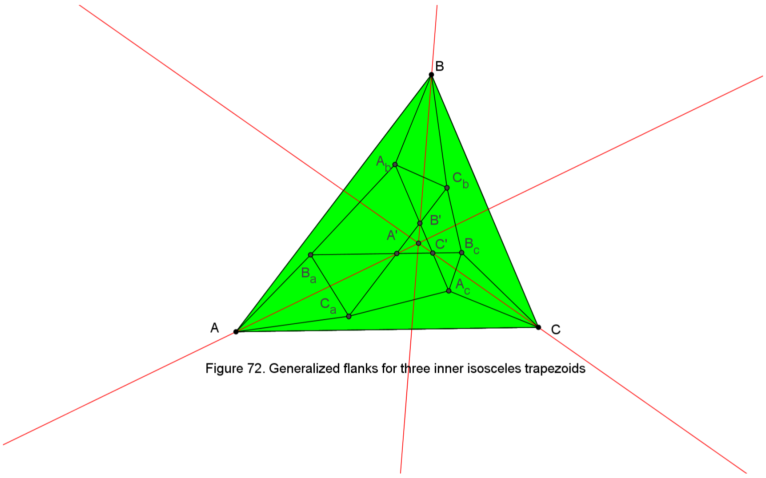

Figure 72. Generalized flanks for three inner isosceles trapezoids

Theorem 2.16. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines passing through B', C', A' perpendicularly to A_bC_b, B_cA_c, C_aB_a respectively are concurrent.*

See figures 73, 74, 75, 76.

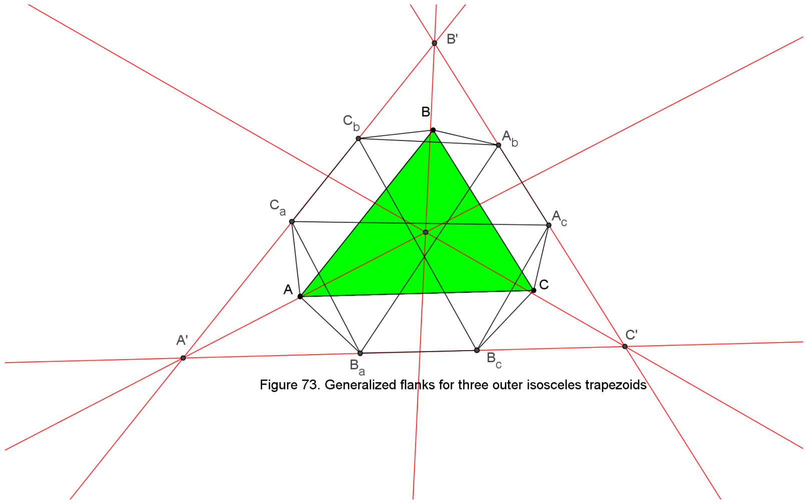

Figure 73. Generalized flanks for three outer isosceles trapezoids

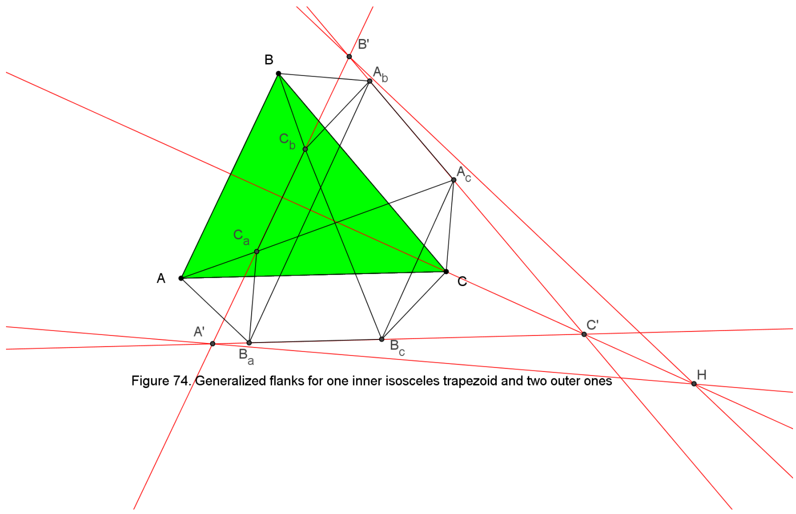

Figure 74. Generalized flanks for one inner isosceles trapezoid and two outer ones

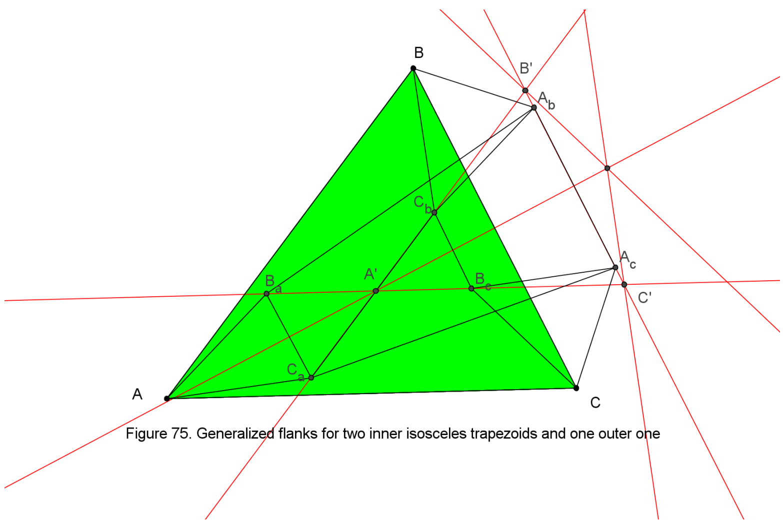

Figure 75. Generalized flanks for two inner isosceles trapezoids and one outer one

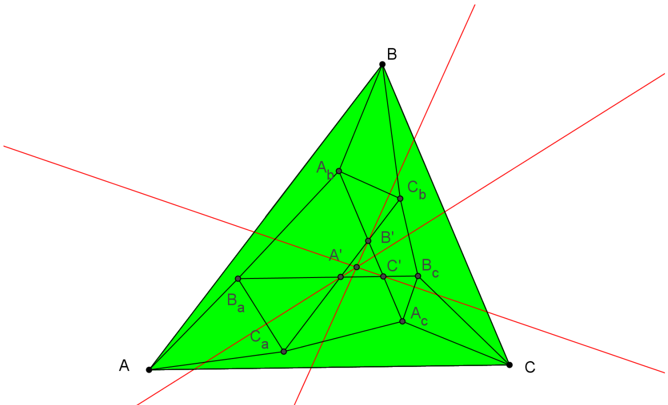

Figure 76. Generalized flanks for three inner isosceles trapezoids

Theorem 2.17. *Given a triangle ABC. Three similar isosceles trapezoids ABC_bC_a, BCA_cA_b, CAB_aB_c are constructed on three sides having the arbitrary orientation (inner or outer orientation). Let $B' = A_bA_c \cap C_bC_a$; $C' = A_bA_c \cap B_aB_c$; $A' = B_aB_c \cap C_aC_b$. Then three lines passing through B', C', A' perpendicularly to A_cC_a, B_aA_b, C_bB_c respectively are concurrent.*

See figures 77, 78, 79, 80.

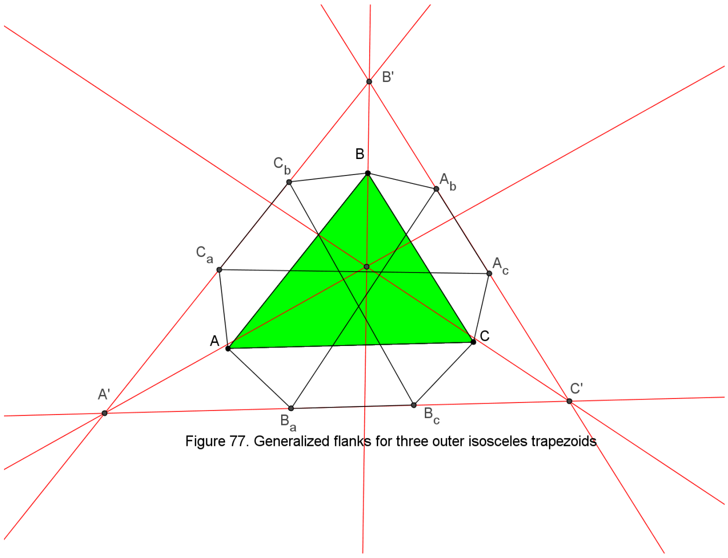

Figure 77. Generalized flanks for three outer isosceles trapezoids

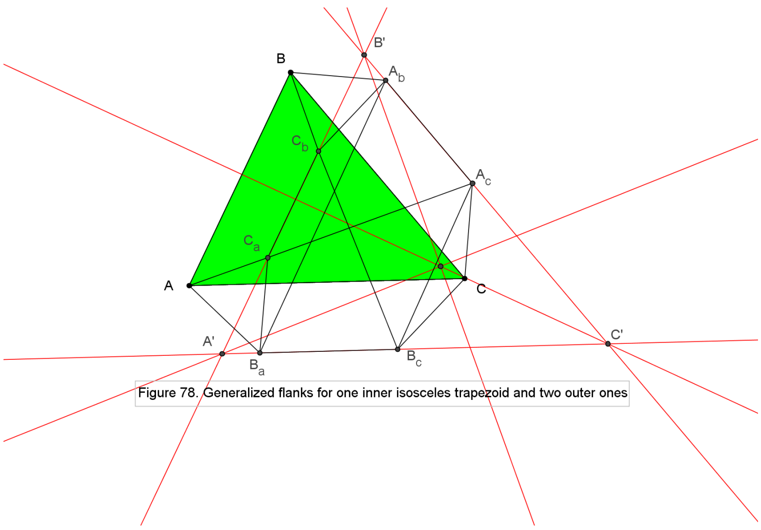

Figure 78. Generalized flanks for one inner isosceles trapezoid and two outer ones

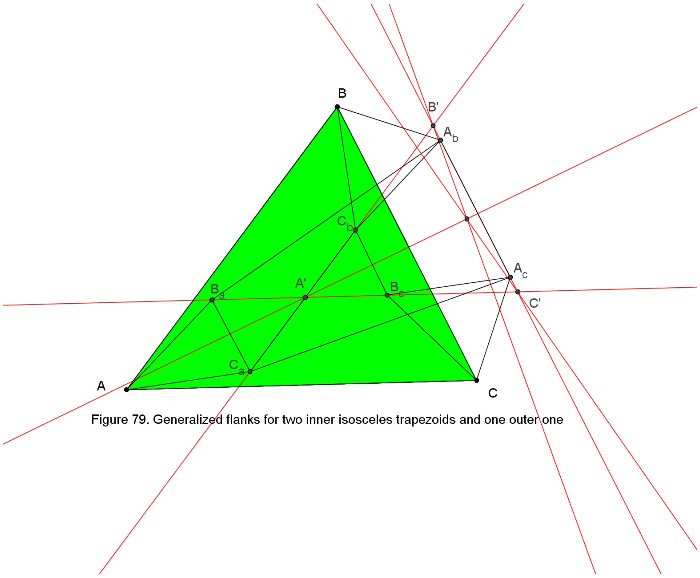

Figure 79. Generalized flanks for two inner isosceles trapezoids and one outer one

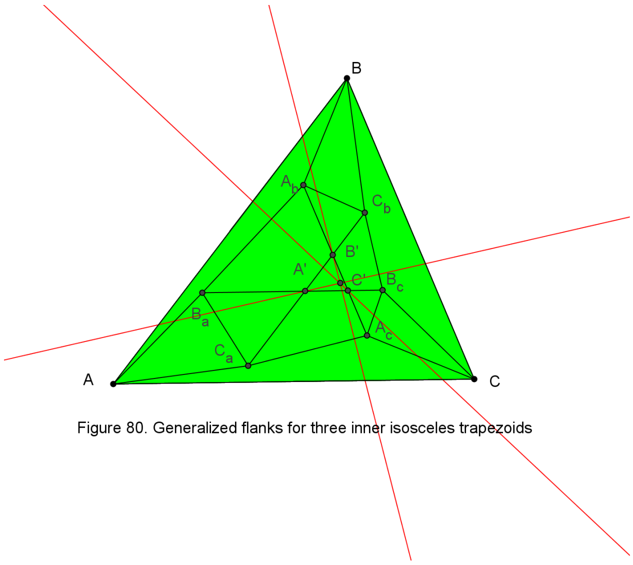

Figure 80. Generalized flanks for three inner isosceles trapezoids

Theorem 2.18. *Given a triangle ABC and P is a point not lying on lines BC, CA or AB. Draw lines passing through P parallel to BC, CA, AB meet these sides respectively, we obtain parallel segments C_2B_2, A_2C_1, B_1A_1. Errect similar rectangulars externally on the parallel segments C_2B_2, A_2C_1, B_1A_1 with their centers are O_a, O_b, O_c, respectively. Prove that AO_a, BO_b, CO_c are concurrent at a point.*

See figure 81.

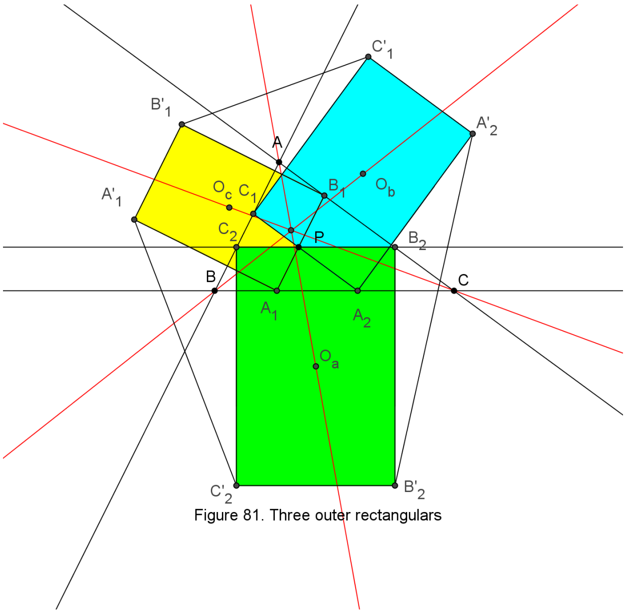

Figure 81. Three outer rectangulars

Theorem 2.19. *Given a triangle ABC and P is a point not lying on lines BC, CA or AB. Draw lines passing through P parallel to BC, CA, AB meet these sides respectively, we obtain parallel segments C_2B_2, A_2C_1, B_1A_1. Errect similar rectangulars $C_2B_2B_2'C_2'$, $A_2C_1C_1'A_2'$, $B_1A_1A_1'B_1'$ externally on the parallel segments C_2B_2, A_2C_1, B_1A_1. Prove that lines passing through the midpoints of $B_1'C_1'$, $A_1'C_2'$, $B_2'A_2'$ perpendicularly to BC, CA, AB respectivley are concurrent.*

See figures 82, 83, 84, 85.

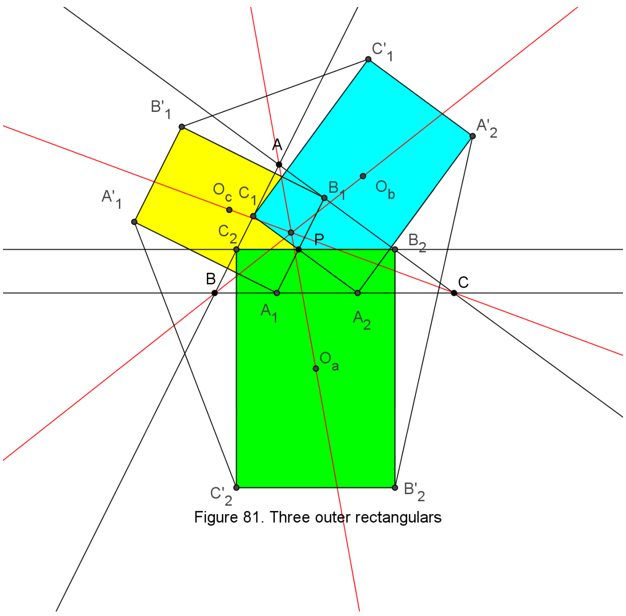

Figure 81. Three outer rectangulars

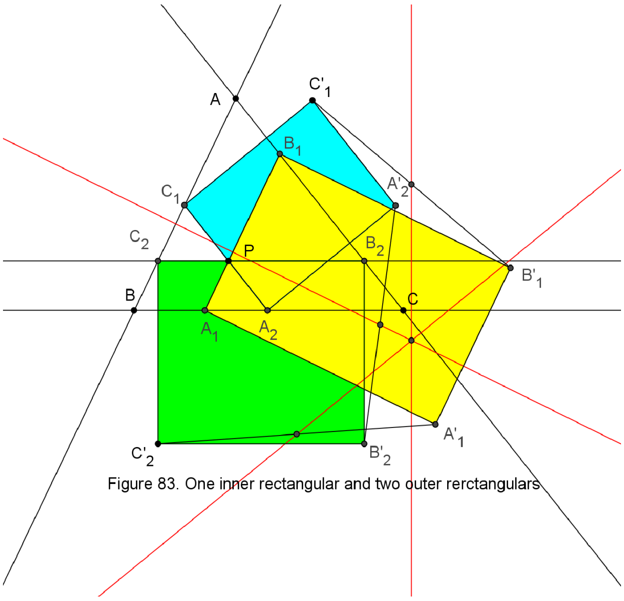

Figure 83. One inner rectangular and two outer rerctangulars

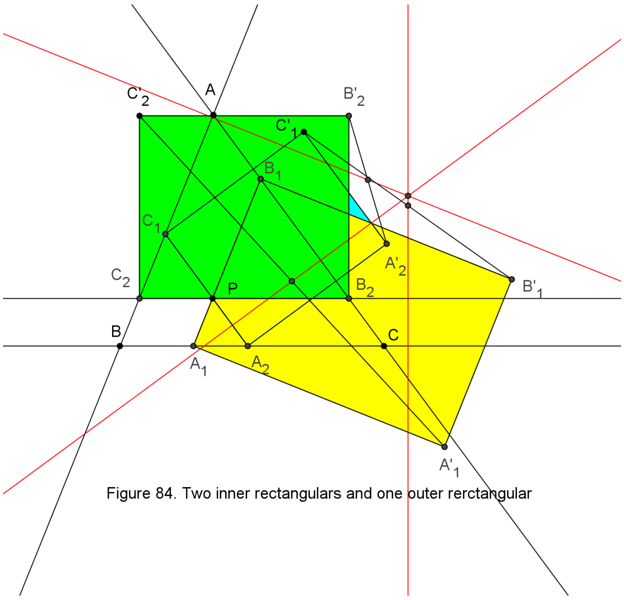

Figure 84. Two inner rectangulars and one outer rerctangular

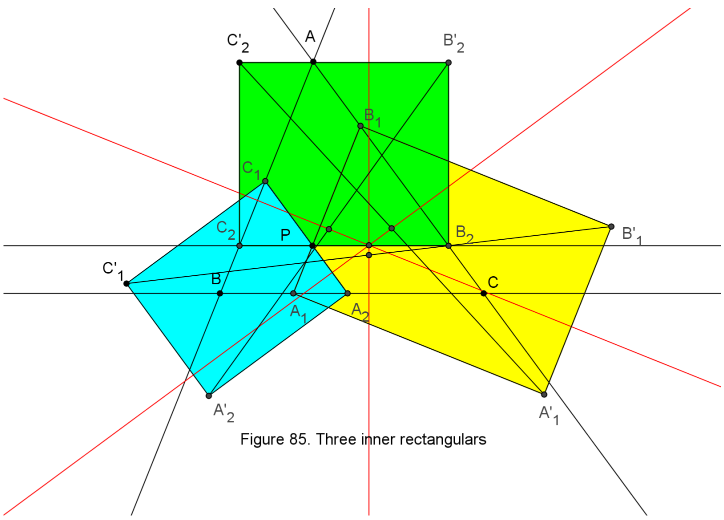

Figure 85. Three inner rectangulars

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[1] Nikolaos Dergiades and Floor van Lamoen, Rectangles Attached to Sides of a Triangle, *Forum Geometricorum*, 3 (2003) 145 – 159.

[2] Nguyen Ngoc Giang, Flanks, new flanks, generalized flanks and their properties, *International Journal of Computer Discovered Mathematics*, 2 (2017), 146 – 184.

[3] Romanian Forum: *Romanian Mathematical Magazine RMM*